A Macroeconomic Model of the Cross-Section of Currencies*

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Abstract

We study the cross-section of currencies using a quantitative macroeconomic model with heterogeneous countries, segmented asset markets, and risk-averse financial intermediaries. The model admits a two-factor structure of bilateral exchange rates. The factors combine global business cycle shocks and financial shocks that reprice the business cycle risk. Countries' exposures to these factors are heterogeneous and depend on their reliance on commodity exports and dollar asset holdings. Increases in risk aversion lead to capital flows that induce a depreciation of high-commodity currencies against low-commodity ones and an appreciation of the dollar against the rest of the world. We estimate the resulting factor structure empirically and use it to discipline the model. Our results indicate substantial heterogeneity in exchange rate drivers across countries, primarily determined by risk exposure. Riskier currencies are those of high-commodity countries that are most affected by the global business cycle and low-dollar countries that are most affected by shocks to the dollar value originating in the US. Currencies of countries combining both risk exposures are mostly driven by financial shocks. Currencies of countries with low risk exposures move less relative to the dollar, and their fluctuations are mostly determined by idiosyncratic shocks.

Key Words: currencies, exchange rates, risk, macroeconomy

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1 Introduction

Bilateral exchange rates exhibit significant fluctuations that follow systemic patterns, characterized by differential sensitivities across countries. A notable example is the global financial crisis of 2008, when most currencies depreciated against the US dollar, but with large variation in magnitudes. For instance, the Australian dollar depreciated by 39%, the Hungarian forint by 25%, and the Thai baht by 15%. A core question in international macroeconomics and finance is what drives these currency movements in the cross-section. This topic has primarily been analyzed in the international finance literature, which documents that most of the variation in bilateral exchange rates can be accounted for by two factors to which countries have heterogeneous exposures (Verdelhan, 2018; Lustig and Richmond, 2020).

In this paper, we develop a macroeconomic model of the cross-section of currencies that provides a structural interpretation of the factor structure of exchange rates. The framework features heterogeneous countries, segmented asset markets, and global financial intermediaries. In the model, exchange rates are a function of a "dollar" and a "commodity" factor, with country-specific loadings. The dollar factor reflects a combination of shocks to US non-traded output and to the risk-bearing capacity of intermediaries, and the commodity factor reflects a combination of the risk-taking shocks and shocks to global traded output. Countries' loadings on the factors depend on their dollar asset holdings and on the incidence of commodities in their production.

We use empirical estimates of this factor structure to discipline and validate the model. Our quantitative analysis indicates substantial heterogeneity in exchange rate drivers across countries. The relative importance of global macroeconomic and financial shocks and idiosyncratic shocks depends on countries' dollar assets and commodity exposure. In the case of exchange rates relative to the US dollar, the relevance of risk shocks stands out because they move relative prices in the US and in the other countries in different directions.

We start by developing a macroeconomic model of the cross-section of exchange rates. Our model introduces heterogeneous countries into a world economy with segmented asset markets and financial intermediaries with limited risk-bearing capacity (as in, e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021), embedded in a canonical structure with tradable and non-tradable goods (as in, e.g., Schmitt-Grohé and Uribe, 2017). Aggregate shocks affect global output and the risk-taking capacity of intermediaries, broadly capturing the global business and financial cycle.

Countries feature two dimensions of ex-ante heterogeneity: they differ in their endowments of commodities and in their holdings of dollar assets.

Commodity exporters are more exposed to commodity-biased fluctuations in global tradable output. An expansion in global tradable output increases tradable consumption and the relative price of non-tradable goods in all countries, with larger increases occurring in commodity-intensive countries. Countries are also affected by US-specific fluctuations, with exposures determined by their dollar asset holdings. Contractions in US non-tradable output increase prices of non-tradables in the US because they reduce the supply of these goods. These shocks affect other countries' non-tradable prices through a realignment of capital flows. Countries with large dollar assets increase their consumption of tradable goods because they benefit from capital gains on their dollar holdings. This increase in consumption is financed by capital inflows from countries with low dollar assets, whose tradable consumption falls to clear the goods market.

Global financial intermediaries facilitate capital flows, and the global financial cycle in our model is generated by shocks to their risk-bearing capacity, which leads to repricing of risk generated by output shocks. In equilibrium, intermediaries absorb the global demand for dollar reserves by issuing dollar bonds and invest the proceeds in the bonds of all other currencies. Currencies endogenously have different risk profiles, which determine the intermediaries' portfolio and how capital flows respond to risk shocks. Intermediaries operate with a stochastic discount factor that combines marginal utility of tradable consumption of all countries. Commodity exporters are a poor hedge because their currencies depreciate during periods of low global output, which is when marginal utility of consumption is high. Countries with low dollar assets are a poor hedge because their currencies depreciate more following contractions in US non-tradable output, which is also when global marginal utility is high. On the other hand, the US, other dollarized countries, and non-commodity producers are a good hedge because their currencies appreciate during these episodes. Following a shock that increases risk aversion, intermediaries rebalance away from risky to safe countries countries, causing the currencies of risky countries to depreciate through capital flight, and the dollar and other safe currencies to appreciate due to a flight to safety.

We analytically characterize equilibrium exchange rates and allocations under a particularly tractable parameterization. The exchange rate between the currencies of countries i and j is given by the ratio of their non-tradable goods prices. It depends on the three aggregate shocks and the idiosyncratic shocks of both countries. An expansion in global tradable output appreciates currency i relative to j if i has more commodities than j. Similarly, an expansion of US non-tradable output appreciates the currency i relative to j if i has more dollar assets than j. Finally, the exposure of i and j's exchange rate to the global risk shock is a weighted average of their relative commodity exposure and their dollar assets.

In the case of exchange rates relative to the US dollar, we show that they admit a two-factor structure, with a dollar and a commodity factor. The dollar factor is the average depreciation of all currencies relative to the dollar, and it structurally reflects a combination of the shock to US non-tradable output and part of the risk shock that interacts with dollar assets. The commodity factor is the average depreciation of currencies with above-median commodity incidence to those below the median. It structurally reflects a combination of the shock to global tradable output and the other part of the risk shock that loads on commodity incidence. The countries' loadings on the dollar and commodity factors depend on their dollar assets and commodity incidence, respectively. Finally, the unexplained innovations to exchange rates in the factor structure structurally map to the countries' idiosyncratic shocks to non-traded output.

In the second part of the paper, we estimate the factor structure implied by the model using data on bilateral exchange rates for 32 countries. The two-factor structure empirically explains approximately half of the variation of monthly changes in bilateral exchange rates relative to the US dollar, and its explanatory power is almost as good as that of the first two principal components — an upper bound on how much any two global factors can explain. We also show that, consistent with the model's predictions, countries' loadings on the dollar factor are positively related to their share of dollar assets in the data, and their loadings on the commodity factor are positively related to the share of commodities in their total exports. These results are largely in line with the findings of the literature (e.g., Ready et al., 2017; Verdelhan, 2018; Richmond, 2019). In particular, the commodity factor captures similar cross-sectional variation as alternative factors that exploit cross-country heterogeneity in interest rate differentials or trade network centrality.

The empirical counterparts of the factors contain useful information that we use to discipline our model quantitatively. Because the dollar factor reflects US non-traded output and global risk premium shocks, and the commodity factor follows global tradable output and global risk premia, the variances of the three aggregate shocks can be read off the factors' covariance matrix. In particular, in the data the dollar factor is more volatile than the commodity factor, which points to a higher relevance of US non-traded output shocks relative to global tradable output. The factors are also positively correlated in the data, which points to the importance of risk premium shocks. We calibrate our model by targeting these moments along with other macro moments commonly used in the literature, such as the Backus-Smith correlation and the volatility of exchange rates.

We then use the model to study the drivers of currencies in the cross-section. Following prior literature, we focus on bilateral exchange rates relative to the US dollar. We highlight two key takeaways from our quantitative analysis. First, we estimate substantial heterogeneity in exchange rate drivers across countries. On the one hand, countries with either low commodity incidence or high dollar assets have a similar exposure to global shocks as the US and, hence, their bilateral exchange rates are mostly driven by idiosyncratic shocks and less so by global shocks. On the other hand, the exchange rates of commodity-intensive countries are strongly influenced by global tradable output and risk shocks, and the exchange rates of countries with low dollar assets are mostly influenced by US non-traded output and risk shocks. Second, risk shocks stand out as a relevant driver of exchange rates relative to the US dollar because these realign capital flows between the US and the majority of the other countries, moving non-traded prices in opposite directions and significantly affecting bilateral exchange rates.

We use our model to analyze in retrospect the opening examples. The relatively mild depreciation of the Thai baht during the global financial crisis is congruent with the high asset dollarization and low commodity exposure of Thailand, which implies that its dollar exchange rate should exhibit moderate responses to global shocks. On the other hand, the forint's large depreciation is consistent with low asset dollarization in Hungary, which exposes its currency to US fluctuations and risk shocks. Finally, the sharp depreciation of the Australian dollar is reconciled with the large incidence of commodities and relatively low dollar assets in Australia. This exposes the Australian currency to both main of global risk and leads financial shocks to dominate its exchange rate against the dollar.

Related Literature Our paper lies at the intersection of the international finance literature that studies the cross section of currencies and the international macroeconomics literature that develops quantitative macroeconomic models of the exchange rate.

An extensive literature in international finance studies the cross-section of exchange rates. A strand of this literature identifies factors that explain the cross-section of excess returns in currency markets and studies the pricing of currency risk (see e.g., Lustig and Verdelhan, 2007; Lustig et al., 2011; Menkhoff et al., 2012; Lustig et al., 2014; Chernov et al., 2023, 2025). Other papers have documented that the variation in bilateral exchange rates exhibits strong common components with heterogeneous exposures across countries (Verdelhan, 2018; Lustig and Richmond, 2020). Motivated by this empirical literature, a body of work develops theories that explain the origins of currency risk and how countries differ in their exposure to risk (Hassan, 2013; Maggiori, 2017; Colacito et al., 2018; Richmond, 2019). Hassan and Zhang (2021) survey recent advances in this literature. Our paper relates to Ready et al. (2017), who develop a model of international trade and currency pricing in which currencies of commodity producers are more exposed to global risk. Our model builds on this idea by introducing country heterogeneity in the incidence of commodity production, which gives rise to a commodity factor, and expands on it by modeling a dollar factor that emerges because of country heterogeneity in dollar asset holdings. Closest to our work, Verdelhan (2018) documents that most of the observed monthly variation in bilateral exchange rates can be explained with a two-factor empirical model and proposes a model of the stochastic discount factors that can replicate this structure. We contribute to this literature by developing a quantitative macroeconomic model that admits a two-factor structure tightly linked to that in Verdelhan (2018), and proposes a structural interpretation of the drivers of factors and exchange rates in the cross-section.

Second, our paper builds on the growing literature that develops quantitative macroeconomic models of the exchange rate (see the recent survey by Itskhoki and Mukhin, 2025a, and references therein). A strand of this literature has studied the quantitative drivers of exchange rates (see, e.g., Stockman and Tesar, 1995; Engel and West, 2005; Itskhoki and Mukhin, 2021, 2025b; Chahrour et al., 2024; Engel and Wu, 2024; Kekre and Lenel, 2024a; Bodenstein et al., 2024). Another body of work studies the flight to safety properties of the US dollar (see, e.g., Jiang et al., 2024; Kekre and Lenel, 2024b; Bodenstein et al., 2023). We contribute to this literature by introducing country heterogeneity aimed at providing a structural explanation of the dynamics of the cross-section of exchange rates. In this sense, our paper also relates to the recent work by Kekre and Lenel (2025), who study the heterogeneous spillovers of US monetary policy across countries, and the literature that studies the heterogeneous effects of the global financial cycle, such as Morelli, Ottonello, and Perez (2022) and Oskolkov (2025).

Finally, our paper is related to the literature that studies the relationship between exchange rates and international asset markets imperfections (see Maggiori, 2022, for a recent review).

Following the original work of Backus and Smith (1993), a set of papers analyzes how segmentation in international asset markets shapes the equilibrium properties of exchange rates (see, e.g., Lustig and Verdelhan, 2019; Chernov et al., 2024; Jiang et al., 2023). A related body of work studies the role of imperfect financial intermediation. Gabaix and Maggiori (2015) develop a theory of exchange rate dynamics and frictional financial intermediaries that can account for various properties of exchange rates. Lettau et al. (2014) study the role of financial intermediaries in the pricing of cross-sectional currency risk. Our paper builds on the insights from this literature by incorporating global financial intermediaries with fluctuating risk-bearing capacity that are central to explaining exchange rate movements in the cross section.

Outline. Section 2 describes the model and characterizes the equilibrium. Section 3 presents our empirical analysis. Section 4 conducts the quantitative analysis and decomposes exchange rates into fundamental drivers in cross-section. We conclude in Section 5.

2 A Model of the Cross-Section of Exchange Rates

In this section, we develop a macroeconomic model of the cross-section of currencies. We model the global economy composed of heterogeneous countries and a representative global financial intermediary. The model features a canonical structure with tradable and non-tradable goods, as in Backus and Smith (1993) and Schmitt-Grohé and Uribe (2017). Asset markets are segmented, following modern macroeconomic models of the exchange rate such as Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021). We consider fluctuations in countries' economic activity, broadly capturing the global business cycle, and fluctuations in the risk-bearing capacity of intermediaries, capturing the global financial cycle.

2.1 Environment

Heterogeneous economies. There is a continuum of small open economies indexed by $i \in [0,1]$, and the US, indexed by u, which is a mass point. Each economy is a representative household and a central bank. Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{t=\infty} \beta^t \frac{C_{it}^{1-\rho}}{1-\rho}, \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, ρ is the inverse elasticity of intertemporal substitution, and C_{it} is an aggregator that uses C_{it}^T units of tradable and C_{it}^N units of non-tradable goods:

$$C_{it} = \left(\frac{C_{it}^N}{\alpha}\right)^{\alpha} \left(\frac{C_{it}^T}{1-\alpha}\right)^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the weight on local non-tradables. The price of tradables is normalized to one, the price of non-tradable is P_{it}^N, and $Q_{it} = (P_{it}^N)^{\alpha}$ is the ideal price index of the consumption aggregator, in terms of tradable goods. For brevity, we will refer to countries' consumption baskets as "local currency". The relative price of consumption baskets between any two countries is their bilateral real exchange rate. For example, $S_{iut} = Q_{ut}/Q_{it}$ is the bilateral real exchange rate of country i against the dollar (i.e., the price of the US consumption basket in terms of the consumption basket in country i). Whenever S_{iut} increases (decreases) country's i currency depreciates (appreciates) against the dollar.

Households are endowed with both goods. The non-traded endowment is $Y_{it}^N = N(1 + x_{it})$, where x_{it} is a mean-zero idiosyncratic shock process. The endowment of the undifferentiated tradable goods is $Y_{it}^T = 1 + e_i z_t$, where z_t is an aggregate disturbance in tradable endowments and e_i is country i's exposure to this shock, which is a source of permanent heterogeneity across countries. Appendix D provides a micro-foundation for this formulation of the traded endowment. It shows that this functional form of Y_{it}^T arises in a setup with two types of tradables: final goods and primary inputs (commodities). Final tradable goods are produced using labor and commodities, and z_t is a commodity-biased productivity shock. Exposures e_i map one-to-one to countries' commodity endowments: those with large e_i receive more tradable income following an increase in z_t. The micro-foundation also maps e_i into the share of commodities in country i's exports, a moment we use in the next sections for the empirical and quantitative analysis.

Households can save and borrow in one-period bonds denominated in local currency. We denote their bond purchases at time t by $B_{i,t+1}$. The bonds pay an interest rate $R_{i,t+1}$, known at time t, at maturity. The household's budget constraint, expressed in tradable goods, is

$$C_{it}^T + P_{it}^N C_{it}^N = Y_{it}^T + P_{it}^N Y_{it}^N + R_{it} Q_{it} B_{it} - Q_{it} B_{i,t+1} + T_{it} + \Pi_{it}, \tag{2}$$

where T_{it} are transfers from the central bank and Π_{it} profit rebates from financial intermediaries.

Households choose sequences $\left\{C_{it}^T, C_{it}^N, B_{i,t+1}\right\}_{t \geq 0}$ to maximize equation (1) subject to equation (2). The optimality conditions are $\alpha C_{it}^T = (1 - \alpha) P_{it}^N C_{it}^N$ and $C_t^{-\rho} = \beta R_{it} \mathbb{E}_t C_{t+1}^{-\rho}$.

The central bank in each small open economy accumulates foreign reserves, denominated in dollars, and transfers returns to the households. At time t, the central bank of country i buys $M_{i,t+1}$ dollar denominated bonds. Its rebate to households is

$$T_{it} = R_{ut} Q_{ut} M_{it} - Q_{ut} M_{i,t+1}.$$

We assume the following policy for reserve accumulation:

$$Q_{ut} M_{i,t+1} = \tau \cdot Q_t M_i + (1-\tau) \cdot Q_{ut} M_i,$$

where $Q_t \equiv \int Q_{it} \mathrm{d}i$ is the global average of consumption basket prices. The policy is a linear combination between targeting a constant value of reserves in the global basket of currencies and in dollars, with a weight $\tau \in (0,1)$ on the former. This weight governs the degree to which the central banks engages in exchange rate stabilization: when the dollar is strong against the global basket of currencies, a high level of τ prescribes selling more dollars to realize capital gains and strengthen domestic currency. The target level of dollar reserves is M_i is exogenous and is the second and last source of permanent heterogeneity across countries. A related literature studies the determinants of asset dollarization across countries, which include varying levels of domestic policy risk and exposure to roll-over risk (see, e.g., Bianchi et al., 2018; Drenik et al., 2022).

Finally, the problem of US households is symmetric to that of small open economies. In the US, the central bank's policy of dollar asset accumulation is irrelevant as households choose their bond holdings to be on the Euler equation for dollar bonds at all times. Based on this, we henceforth assume that the US does not maintain dollar reserves.

Global financial intermediaries. There is a representative global intermediary that trades bonds in all currencies. Let $A_{i,t+1}$ and $A_{u,t+1}$ be its purchases of currency-i and dollar bonds at t. The intermediary starts with no initial capital, so its balance sheet is given by

$$\int Q_{i,t} A_{i,t+1} \mathrm{d}i + Q_{u,t} A_{u,t+1} = 0, \tag{3}$$

The intermediary's preferences are defined over $t+1$ risk-adjusted returns from investing in t. Let $X_{i,t+1}$ be the realized excess returns of investing in i's bonds relative to dollar bonds:

$$X_{i,t+1} \equiv \frac{R_{i,t+1} Q_{i,t+1}}{Q_{it}} - \frac{R_{u,t+1} Q_{u,t+1}}{Q_{ut}}.$$

We can express total realized profits, expressed in tradable goods, as $\int Q_{i,t} A_{i,t+1} X_{i,t+1} \mathrm{d}i$, where we have substituted out the dollar position using the balance-sheet equation (3). The intermediary chooses $\{A_{i,t+1}\}$ for all i and t to maximize

$$\mathbb{E}_t \left[\Lambda_{t+1} \int Q_{it} A_{i,t+1} X_{i,t+1} \mathrm{d}i \right] - \frac{\beta}{2\chi} \int \left(Q_{it} A_{i,t+1} - L_{t+1} \right)^2 \mathrm{d}i. \tag{4}$$

We assume that the intermediary uses the average stochastic discount factor of all countries. Define $\Lambda_{t+1}^{\text{ave}} = \int \Lambda_{i,t+1} \mathrm{d}i + \Lambda_{u,t+1}$, where $\Lambda_{i,t+1}$ is country's i stochastic discount factor tradable goods:

$$\Lambda_{i,t+1} = \beta \frac{\partial u(C_{i,t+1})}{\partial C_{i,t+1}^T} \left(\frac{\partial u(C_{it})}{\partial C_{it}^T} \right)^{-1},$$

and $\Lambda_{u,t+1}$ is similarly defined for the US. The intermediary's stochastic discount factor is

$$\Lambda_{t+1} = \Gamma_t \cdot \left[\Lambda_{t+1}^{\text{ave}} \right] + (1 - \Gamma_t) \cdot \mathbb{E}_t \left[\Lambda_{t+1}^{\text{ave}} \right],$$

This stochastic discount factor is a weighted average of a random and a deterministic component. The first term is state-dependent, aggregating the stochastic discount factors of all households, including the US. This implies that the intermediary inherits the households' risk aversion. The second term is known at time t and equals the average of $\mathbb{E}_t[\Lambda_{i,t+1}]$ over all countries, including the US. The weight Γ_t governs how much of household's risk aversion is passed to the intermediary. We assume $\Gamma_t = \Gamma(1 + \gamma_t)$, where γ_t is an aggregate shock to the risk-bearing capacity.

The intermediary faces portfolio management costs, the second term in equation (4). These costs capture frictions that make capital flows slow-moving and less elastic to risk-adjusted returns than predicted by frictionless models (as estimated in, e.g., Koijen and Yogo, 2020). The cost decreases in χ and penalizes deviations from a portfolio that issues dollar bonds to satisfy the demand for reserves by all countries and invests the proceedings uniformly in all other countries. That is, the reference level of liabilities is $L_{t+1} = \int Q_{ut} M_{i,t+1} \mathrm{d}i$. The optimal portfolio is

$$Q_{it} A_{i,t+1} = L_{t+1} + \beta^{-1} \chi \left(\mathbb{E}_t \left[\Lambda_{t+1}^{\text{ave}} \right] \mathbb{E}_t [X_{i,t+1}] + \Gamma_t \mathbb{C}_t \left[\Lambda_{t+1}^{\text{ave}}, X_{i,t+1} \right] \right).$$

The intermediary will allocate more funds to currencies that promise higher expected excess returns and have higher covariance with the global average marginal utility of tradable consumption. The intermediary's dollar position is given by

$$Q_{u,t} A_{u,t+1} = -L_{t+1} + \beta^{-1} \chi \left(\Gamma_t \mathbb{C}_t \left[X_{t+1}, \Lambda_{t+1}^{\text{ave}} \right] - \mathbb{E}_t \left[\Lambda_{t+1}^{\text{ave}} \right] \mathbb{E}_t \left[X_{t+1} \right] \right),$$

where $X_{t+1} = \int X_{i,t+1}\mathrm{d}i$. The optimal intermediary's demand for dollar bonds in excess of total reserve demand is proportional the average excess returns in foreign currencies and to the covariance between the average foreign currency return and the global marginal utility of tradables.

Finally, we assume that profits are rebated back to each country according to

$$\Pi_{it} = Q_{i,t-1}A_{it}\left(R_{it}\frac{Q_{it}}{Q_{i,t-1}} - \frac{Q_{ut}}{Q_{u,t-1}}\right) - Q_{ut}M_{it}(R_{ut}-1).$$

Each country receives returns that the intermediary realizes on its bond, including interest income and exchange rate appreciation, net of interest payments on reserves. The US only receives the first part, since it does not maintain reserves.

Aggregate shocks. The model features three aggregate shocks: the global tradable endowment shock z_t, the US non-tradable endowment shock x_{ut}, and the risk aversion shock γ_t. The US non-tradable endowment is an aggregate shock because US is economically large and all countries accumulate reserves denominated in dollars. We assume that all shocks are stationary and follow the following processes with independent increments:

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon_{z,t+1},$$

$$\gamma_{t+1} = \rho_\gamma \gamma_t + \sigma_\gamma \epsilon_{\gamma,t+1},$$

$$x_{u,t+1} = \rho_x x_{ut} + \sigma_x \epsilon_{u,t+1}.$$

Finally, we assume that the idiosyncratic non-traded endowments of individual countries, $x_{i,t+1}$, follow the same process as the US non-traded endowment.

Equilibrium. An equilibrium is a set of processes for prices $\{P^N_{it}, R_{it}, Q_{it}\}_{t\geq 0}$ and quantities $\{C^N_{it}, C^T_{it}, B_{i,t+1}, A_{i,t+1}\}_{t\geq 0}$ such that, given reserve policy $\{M_{i,t+1}\}_{t\geq 0}$, profits, and transfers, all quantities are chosen optimally given prices, and the following market clearing conditions hold:

non-tradable good for all i: $$C^N_{it} = Y^N_{it},$$

tradable good: $$\int C^T_{it}\mathrm{d}i + C^T_{ut} = \int Y^T_{it}\mathrm{d}i + Y^T_{ut},$$

local currency bonds for all i: $$A_{i,t+1} + B_{i,t+1} = 0,$$

dollar bonds: $$\int M_{i,t+1}\mathrm{d}i + A_{u,t+1} + B_{u,t+1} = 0.$$

2.2 Equilibrium Characterization

We take the linear approximation of the model around a deterministic steady state. Formally, we look for equilibrium exchange rates in the following form:

$$q_{it} = \eta_i x_{it} + \underbrace{\theta_i z_t + \xi_i x_{ut} + \mu_i \gamma_t}_{\text{exogenous global states}} + \underbrace{\omega_i l_{it} + \delta_i l_{ut} + \zeta_i m_t}_{\text{endogenous states}} \tag{5}$$

$$q_{ut} = \theta_u z_t + \xi_u x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} + \zeta_i m_t \tag{6}$$

Small letters denote first-order log deviations, unless otherwise noted. Individual countries' currencies are exposed to local non-traded shocks x_{it}, aggregate exogenous shocks (z_t, γ_t, x_{ut}), and endogenous states (l_{it}, l_{ut}, m_t). Country i's capital inflows from bonds in their currency, denoted by l_{it}, and its reserve holdings, denoted by m_t, are idiosyncratic states that affect the country's exchange rate through its balance of payments. Reserve holdings do not have a subscript i because the policy we specify above leads to the same reaction of reserves in all countries

$$m_{t+1} = \tau(q_t - q_{ut}). \tag{7}$$

Central banks realize the same share of their capital gains on the dollar, as measured against the global aggregate of currencies. Finally, the dollar inflows to the US, denoted by l_{ut}, is also an aggregate state because it affects the US balance of payments.[1]

The portfolio of intermediaries is not well-defined in the steady state. We follow prior literature and analyze the portfolio that arises when we set the variance of shocks to zero and scale up intermediaries' risk aversion so that non-zero risk premia persist even in the linear approximation. Specifically, we consider the following double limit $(\sigma_z, \sigma_\gamma, \sigma_x) \longrightarrow 0$ and $\Gamma \longrightarrow \infty$, where $(\Gamma\sigma_z^2, \Gamma\sigma_x^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_x, \Gamma_\gamma)$, which is akin to one used by Borovicka and Hansen (2013) and commonly used in the international macroeconomics literature. It lets us keep non-zero risk premia even in the linear approximation. The intermediary prices risk associated with shocks to the global tradable and the US non-tradable endowments, z_t and x_{ut}, and potentially shocks to its own risk-bearing capacity γ_t. The risk premia are functions of $(\Gamma_z, \Gamma_x, \Gamma_\gamma)$, as well as of the exposures of exchange rates to shocks, which determines the limiting conditional covariance of returns with

[1] Formally, l_{it} is the linear deviation of $L_{i,t+1} \equiv Q_{i,t} A_{i,t+1}$ from the steady state inflows to country i; l_{ut} is similarly defined for the US, where inflows are given by $L_{u,t+1} \equiv Q_{i,t} A_{i,t+1} - \int Q_{u,t} M_{i,t+1} \mathrm{d}i$; and m_t is the log-linear deviation from the steady state of the capital gains on reserves, $\tau Q_t + (1-\tau) Q_{ut}$.

the intermediary's stochastic discount factor. An important object that determines this covariance is $\lambda^{\text{ave}}_{t+1}$, the log deviation of the average stochastic discount factor $\Lambda^{\text{ave}}_{t+1} = \int \Lambda_{i,t+1} \mathrm{d}i + \Lambda_{u,t+1}$.

We start the characterization with a lemma that describes exchange rate determination.

Lemma 1. *The price of i's consumption basket in equilibrium is*

$$q_{it} = \alpha e_i z_t - \alpha x_{it} \underbrace{+ \alpha \Delta l_{i,t+1}}_{\textit{inflows}} \underbrace{- \alpha M_i \Delta m_{t+1}}_{\textit{reserve accumulation}} .$$

The intermediary's investment in i is determined by

$$l_{i,t+1} = L m_{t+1} + \chi\varphi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \chi \mathcal{C}_i \gamma_t,$$

where $\mathcal{C}_i = \lim_{(\sigma_z,\sigma_\gamma,\sigma_x)\to 0,\ \Gamma\to\infty} \Gamma \cdot \mathbb{C}_t[\lambda^{ave}_{t+1}, \Delta q_{i,t+1} - \Delta q_{u,t+1}]$ *is country i's limiting risk premium.*

The price of i's consumption basket rises when the tradable endowment increases, and a higher exposure e_i makes it appreciate more in times of high z_t. The non-tradable shock x_{it} makes local goods more abundant and leads to a depreciation. Inflows $\Delta l_{i,t+1}$ lead to appreciation because they increase disposable income, which increases the demand for non-tradables and its equilibrium price. This is the channel through which financial shocks affect exchange rates: inflows are caused by shocks to the intermediary's risk aversion, and their sign depends on the country's riskiness, as measured by $\mathcal{C}_i$. When intermediaries' risk aversion increases, they rebalance portfolios away from countries with a negative covariance $\mathcal{C}_i$ between their exchange rate and the average stochastic discount factor. These countries are risky, and their currencies depreciate in times of high risk aversion through capital outflows. Reserve accumulation Δm_{t+1}, scaled by the steady state position M_i, leads to a depreciation by decreasing disposable income as well. This is the channel through which reserve policy stabilizes currencies: the government generates outward flows when the dollar depreciates and inward flows when the dollar is expensive, counteracting its movement.

We next characterize the intermediary's stochastic discount factor. Specifically, we derive its linearized version $\lambda^{\text{ave}}_{t+1}$ of the average stochastic discount factor of households. The exposure of $\lambda^{\text{ave}}_{t+1}$ to shocks determines the riskiness of individual currencies through the covariance in Lemma 1.

Proposition 1. *In the first order, the average stochastic discount factor is*

$$\lambda^{ave}_{t+1} = -\alpha\varphi(e + e_u)\Delta z_{t+1} - (\rho - 1)\alpha \Delta x_{u,t+1}.$$

The average stochastic discount factor captures the growth of marginal utility of tradable consumption. The first term in Proposition 1 shows that, unsurprisingly, marginal utility of tradable consumption decreases in tradable output itself. The implication is that a currency is a bad hedge if it appreciates against the dollar when global tradable output is high. This makes countries with high exposure to tradable output shocks e_i risky.

The second term shows how marginal utility of tradable consumption changes with non-tradable output. This term has two important properties. First, non-tradable output of regular countries integrates out on average, and only US non-tradable output remains due to a positive weight that $\Lambda_{t+1}^{\text{ave}}$ puts on the US. Second, marginal utility of tradable consumption can either increase or decrease in non-tradable output depending on the inverse elasticity of intertemporal substitution ρ. More precisely, what matters is how the elasticity of intertemporal substitution compares to the substitution elasticity between the two goods within periods. More non-tradable consumption can in principle increase marginal utility of tradables through within-period complementarities. However, if the intertemporal elasticity is sufficiently low (ρ is sufficiently high), more non-tradable consumption decreases marginal utility of tradables through higher overall consumption. If $\rho > 1$, which is standard in the literature, this force dominates, and low non-tradable output in the US means high marginal utility of tradables. Since the dollar appreciates when the US non-tradable output is low, strong dollar coincides with high values of the stochastic discount factor. Currencies that depreciate when the dollar is strong are hence risky.

Exchange rates. We next turn to bilateral exchange rates and their factor structure. To derive analytical results, we make simplifying parametric assumptions. We take persistence of output shocks to one: $\rho_z = \rho_x = 1$, and make risk aversion shocks transitory: $\rho = 0$. We also take the intermediaries' portfolio management costs to zero: $\chi \longrightarrow \infty$. This makes the expressions for exchange rates tractable by eliminating endogenous states. At the same time, this limit eliminates exchange rate movements resulting from capital gains on reserves. To keep the reserve management channel operational, we increase the stock of reserves at an appropriate rate: $M_i = \sqrt{\chi} m_i$. With these simplifying assumptions, we can now characterize bilateral exchange rate depreciation $\Delta s_{i,t+1} = \Delta q_{u,t+1} - \Delta q_{i,t+1}$ of country i against the dollar in closed form.

Proposition 2. *The bilateral exchange rate depreciation of country i against the dollar is*

$$\Delta s_{i,t+1} = (e_u - e_i) \cdot \alpha \Delta z_{t+1} + (\tau \phi (m_i - m) - 1) \cdot \alpha \Delta x_{u,t+1} + \kappa_i \cdot \Delta \gamma_{t+1} + \alpha \Delta x_{i,t+1}.$$

The loading κ_i *of the risk aversion shock is given by*

$$\kappa_i = (e_i - e_u) \cdot (e + e_u)\alpha^2\Gamma_z + (1 - \tau\phi(m_i - m)) \cdot \alpha\phi^2\Gamma_x(\rho - 1),$$

where $\Gamma_z = \lim_{\sigma_z \to 0,\ \Gamma \to \infty} \Gamma\sigma_z^2$*,* $\Gamma_x = \lim_{\sigma_x \to 0,\ \Gamma \to \infty} \Gamma\sigma_x^2$*, and* $\phi = \sqrt{\alpha/\varphi}$.

The coefficient of $\Delta s_{i,t+1}$ on Δz_{t+1} reflects the relative exposure of country i and the US to the tradable output shocks. The coefficient on $\Delta x_{u,t+1}$ has two parts. First, the -1 reflects the direct effect of an increase in US non-tradable output: it becomes more abundant, so the relative price of the US consumption basket falls. The part $\tau\phi(m_i - m)$ reflects reserve management. When an increase in x_{ut} makes the dollar depreciate, countries increase their reserve holdings, and their currencies depreciate in proportion to m_i. At the same time, the rise in demand for dollar bonds leads to inflows into all countries when the intermediary recycles this demand into local currency bonds. These inflows offset reserve purchases on average, making the net foreign assets change in proportion to $m_i - m$.

The coefficient κ_i on the risk aversion shock $\Delta\gamma_{t+}$ comes from the limiting covariance of $\Delta s_{i,t+1}$ and the average stochastic discount factor $\lambda_{t+1}^{\text{ave}}$ that we characterize in Proposition 1. Two output shocks, Δz_{t+1} and $\Delta x_{u,t+1}$, load on the average stochastic discount factor, so there are two sources of priced risk: global tradable output and US non-tradable output. Countries are risky with respect to the tradable output shock if $e_i - e_u > 0$, meaning their tradable output is more procyclical than that in the US, and their bilateral exchange rate against the dollar is more procyclical than average. The US-specific shock generates risk for two reasons. First, all countries are risky because their currencies depreciate against the dollar in times of low US non-traded output. This is due to the direct effect of $\Delta x_{u,t+1}$ on $\Delta s_{i,t+1}$. Second, countries can be more or less risky depending on their dollar assets. Currencies with high m_i depreciate less against the dollar when it is broadly strong, which makes them less risky.

The fact that the coefficient κ_i inherits the shape of the loadings of Δz_{t+1} and $\Delta x_{u,t+1}$ on $\Delta s_{i,t+1}$ makes it easy to see the how common components in exchange rates align with the two sources of heterogeneity in cross-section. Grouping the coefficients in Proposition 2, we obtain:

$$\begin{aligned}\Delta s_{i,t+1} = &(\alpha\Delta z_{t+1} - \alpha^2(e + e_u)\Gamma_z\Delta\gamma_{t+1}) \cdot (e_u - e_i) \\ &+ (\alpha\Delta x_{u,t+1} - \alpha\phi^2(\rho - 1)\Gamma_x\Delta\gamma_{t+1}) \cdot (\tau\phi(m_i - m) - 1) \\ &+ \alpha\Delta x_{i,t+1}.\end{aligned}$$

The fist term in the equation above shows the heterogeneity along the commodity exposure dimension. Currencies with different e_i react heterogeneously to the tradable output shocks Δz_{t+1}, and they react in the same way to the risk aversion shocks $\Delta\gamma_{t+1}$ because these lead to repricing of the commodity risk emanating from Δz_{t+1}. The second term shows the heterogeneity along the dollar assets dimension. Currencies with different m_i react heterogeneously to US non-tradable output shocks $\Delta x_{u,t+1}$, and they react in the same way to the risk aversion shocks $\Delta\gamma_{t+1}$ because these lead to repricing of the dollar risk emanating from $\Delta x_{u,t+1}$. The last term is an idiosyncratic shock that increases non-tradable endowment in country i and decreases its relative price.

The two common components have natural interpretations as a commodity and a dollar factor. They can easily be isolated by integration, assuming that (e_i, m_i) are uncorrelated in the cross-section and that the exposure of the US to the global tradable shock is equal to the average exposure of the small open economies, $e_u = e$, where $e = \int e_i \mathrm{d}i$.

Definition 1. *The commodity and dollar factors are*

$$\begin{aligned} comm_{t+1} &= 2\int_{i:\ e_i > med\{e\}} \Delta s_{i,t+1}\, di - 2\int_{i:\ e_i < med\{e\}} \Delta s_{i,t+1}\, di \\ doll_{t+1} &= \int \Delta s_{i,t+1}\, di \end{aligned}$$

With this definition, we can formulate our main result. Denote by e_H and e_L the average values of e_i above and below the median, respectively.

Proposition 3. *Bilateral exchange rates have the following factor structure:*

$$\Delta s_{i,t+1} = \frac{e_i - e}{e_H - e_L} \cdot comm_{t+1} + (1 - \tau\phi(m_i - m)) \cdot doll_{t+1} + \alpha\sigma_x \epsilon_{i,t+1}. \tag{8}$$

The factors are

$$\begin{aligned} comm_{t+1} &= 2\alpha^2 e(e_H - e_L)\Gamma_z \Delta\gamma_{t+1} - (e_H - e_L)\alpha\Delta z_{t+1}, \\ doll_{t+1} &= \alpha\phi^2(\rho - 1)\Gamma_x \Delta\gamma_{t+1} - \alpha\Delta x_{u,t+1}. \end{aligned}$$

This proposition highlights that the dollar and the commodity factors are two aggregations that effectively integrate out one of the three aggregate shocks. By computing the average change in all the exchange rates against the dollar, the dollar factor is effectively abstracting from the shock to global tradable output, since the commodity exposure of the US is the world's average exposure. By computing the average change in the exchange rate of high-minus-low commodity producers, the commodity factor is effectively abstracting from the shock to the US non-tradable output. The risk aversion shock γ_t is reflected in both factors to the extent that both z_t and x_{ut} are priced risks, and movements in γ_t lead to risk repricing. The factor loadings hence reflect the exposure of each currency to the underlying real shocks.

2.3 Identification from the Cross-section of Currencies

To understand what drives the cross-section of currencies in the data, we need to know the strength of the real and financial shocks. Our factor perspective gives us access to new parameters that are informative about the size of these shocks, that is, the parameters $(\sigma_z, \sigma_x, \sigma_\gamma)$. To identify them, we make explicit the fact that $\Gamma_z = \Gamma\sigma_z^2$ and $\Gamma_x = \Gamma\sigma_x^2$ in the limit.

PROPOSITION 4. *The covariance matrix of the factors is given by*

$$\begin{aligned}
\mathbb{V}[comm_t] &= \alpha^2(e_H - e_L)^2\sigma_z^2 + 8\alpha^2 e^2(e_H - e_L)^2\sigma_z^4 \cdot (\Gamma\sigma_\gamma)^2, \\
\mathbb{V}[doll_t] &= \alpha^2\sigma_x^2 + 2\alpha^2\phi^4(\rho - 1)^2\sigma_x^4 \cdot (\Gamma\sigma_\gamma)^2, \\
\mathbb{C}[comm_t, doll_t] &= 4\alpha^3\phi^2 e(e_H - e_L)(\rho - 1) \cdot (\Gamma\sigma_\gamma)^2.
\end{aligned}$$

The mapping $(\sigma_z^2, \sigma_x^2, (\Gamma\sigma_\gamma)^2) \mapsto (\mathbb{V}[comm_t], \mathbb{V}[doll_t], \mathbb{C}[comm_t, doll_t])$ *is invertible.*

Using the three second moments of the factors, we can recover $(\sigma_x, \sigma_z, \Gamma\sigma_\gamma)$ in closed form. The expression is quite large, and we provide it in the proof. However, two things are clear from the statement of Proposition 4. First, the covariance between the two factors is only due to financial shocks. The size of the correlation is hence informative about their importance. A positive correlation would also be indicative of $\rho > 1$, validating the correspondence between strong dollar and high global marginal utility, which is necessary to get the sign of the dollar risk correctly. Second, the variance of the factors provides a comparison between the two real shocks. A relatively large variance of the dollar factor compared to the commodity factor would point to a higher importance of the US non-tradable shock.

3 Empirical Analysis

In the first part of this section, we estimate the factor structure implied by the model and document the relationship between the factors and empirical proxies of the model's structural shocks. We then discuss how the factors are related to other factors estimated in the literature. In the second part, we relate the estimated loadings on the two factors to country characteristics in the cross-section, as suggested by the model.

3.1 Factor Structure of Exchange Rates

We use monthly data on bilateral exchange rates vis-a-vis the US dollar for a sample of $N = 32$ countries from 2000 to 2019.[2] Exchange rates are expressed in units of local currency per US dollar, unless noted otherwise. A rising exchange rate means dollar appreciation. For each currency i, we estimate the empirical model analog of equation (8), given by

$$\Delta s_{i,t+1} = \alpha_i + \beta_{i,\text{comm}} \text{comm}_{i,t+1} + \beta_{i,\text{doll}} \text{doll}_{i,t+1} + \epsilon_{i,t+1}, \tag{9}$$

where $\Delta s_{i,t+1}$ is the monthly change in the log exchange rate of currency i per dollar in period $t+1$, $\text{doll}_{i,t+1}$ is the dollar factor, and $\text{comm}_{i,t+1}$ is the commodity factor. The factors are indexed by currency i because we exclude currency i when constructing them to avoid a mechanical relationship between the left-hand side and the right-hand side of equation (9). Paralleling its model counterpart, we construct the commodity factor as the average change in the log exchange rate of high-versus-low commodity producers

$$\text{comm}_{i,t+1} \equiv \frac{2}{N-1} \sum_{j \in \mathcal{I}_{h,i}} \Delta s_{j,t+1} - \frac{2}{N-1} \sum_{j \in \mathcal{I}_{l,i}} \Delta s_{j,t+1}, \tag{10}$$

where $\mathcal{I}_{h,i}$ and $\mathcal{I}_{l,i}$ are the sets of countries in which the time-average share of commodity exports in total exports is above and below the median across countries excluding country i, respectively. We construct the dollar factor as the average change in the log exchange rates of all currencies against the dollar, excluding currency i:

$$\text{doll}_{i,t+1} \equiv \frac{1}{N-1} \sum_{j \neq i} \Delta s_{j,t+1}. \tag{11}$$

We provide further details on the construction of these variables in Appendix A.

Panel (a) of Figure 1 reports the R^2 from the estimation of equation (9) for all currencies. The factor regressions have high explanatory power, with an average R^2 of 43% and as high as 74%

[2] The sample of countries includes Albania, Australia, Brazil, Canada, Switzerland, Chile, China, Colombia, Costa Rica, Czech Republic, Euro Area, United Kingdom, Hungary, Indonesia, Israel, India, Iceland, Japan, South Korea, Mexico, Malaysia, Norway, Peru, Philippines, Poland, Russia, Sweden, Singapore, Thailand, Turkey, Uruguay, and South Africa.

for some countries. As a benchmark for comparison, panel (a) of Appendix Figure A.1 shows that the R^2 are close to those obtained from regressing changes in log exchange rates on their first two principal components, which is an upper bound on how much variation can be explained by any two variables. The average R^2 from the two-principal-components regressions is 51%.

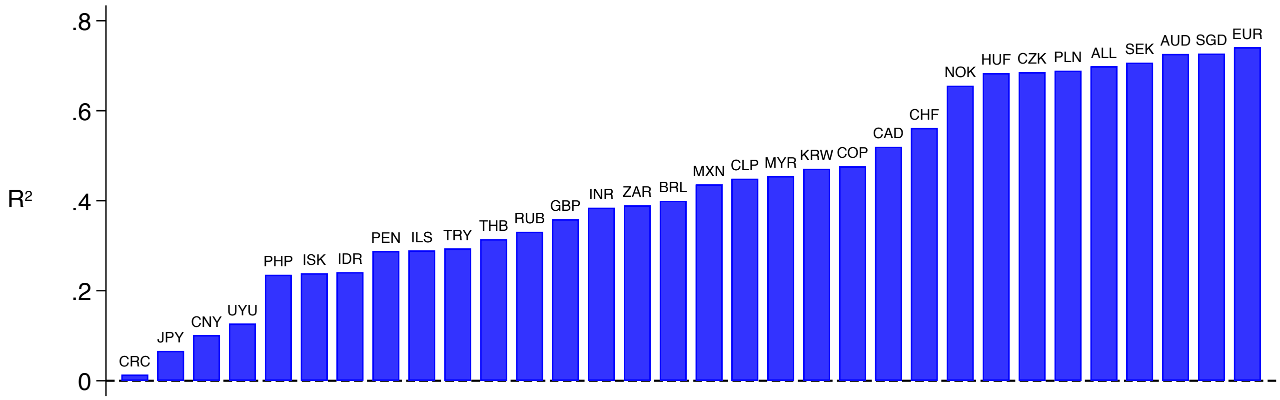
Panel (b) of Figure 1 reports the loadings on the commodity factor by currency, which exhibit significant cross-country heterogeneity, with roughly half of the countries loading positively and the other half negatively. Panel (c) reports the loadings on the dollar factor, which are all positive and also exhibit significant heterogeneity across countries. Tables A.1 and A.2 report these regression estimates along with their Newly-West adjusted standard errors.

Macroeconomic determinants of factors. Motivated by the model's structural interpretation of the factors summarized in Proposition 3, we estimate the relationship between the factors and empirical proxies of the structural shocks that determine them. We first regress the commodity factor on the changes of a global commodity price index (as a proxy for the global tradable output shocks) and empirical measures of global risk, namely, the VIX, the excess bond premium from Gilchrist and Zakrajšek (2012), and (the negative) of global intermediaries' equity capital ratio from He et al. (2017). Panel (a) of Appendix Table A.3 shows that, consistent with the model's prediction, the commodity factor is negatively related to the global commodity price index and positively related to all the empirical measures of global risk. We also regress the dollar factor on the changes in US employment in the service sector (as a proxy for US non-tradable output shocks) and the same measures of global risk. Panel (b) of Appendix Table A.3 reports that the dollar factor is positively related to changes in the employment rate in the service sector and changes in the measures of global risk. We provide further details on the description of variables used in the regression analysis in Appendix A.

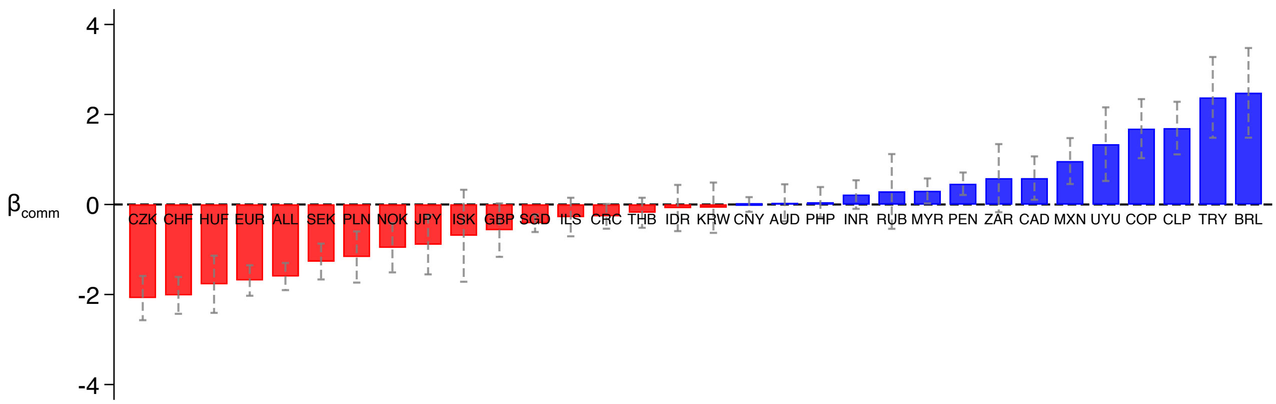
Commodity factor and other factors. In this subsection, we study how the commodity factor relates to alternative factors. We first compare it with two factors previously studied in the literature. One is the carry factor proposed by Verdelhan (2018), computed in a similar way to the commodity factor but sorting countries by their local-currency interest rates; the other is the trade-centrality factor proposed by Richmond (2019), also computed similarly to the commodity factor but sorting countries according to their trade centrality (see Appendix A for further details

Figure 1: Factor Structure of Exchange Rates: Country Regressions

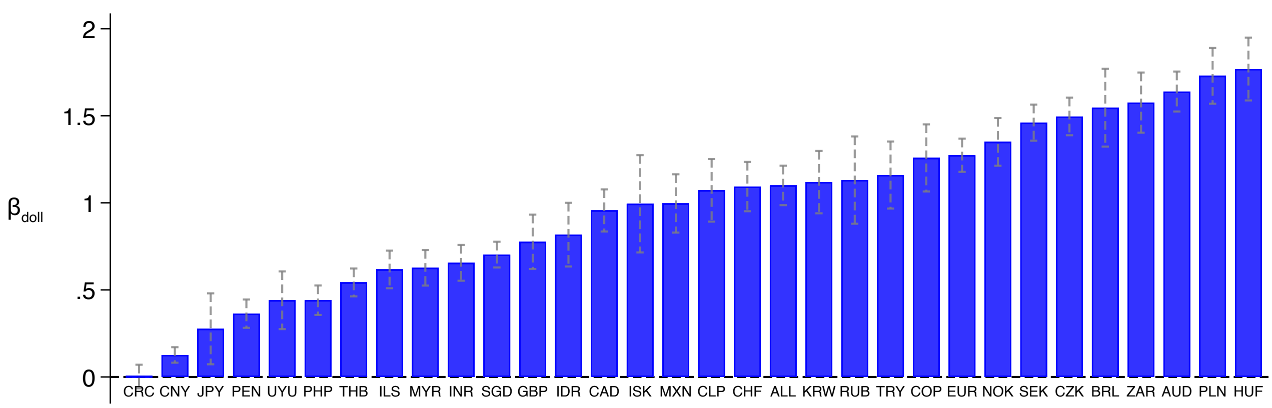
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



Notes: Panel (a) shows the R^2, representing the fraction of variation in $\Delta s_{i,t+1}$ explained by our empirical model in equation (9) for each currency i. Panel (b) shows the estimated loading of the commodity factor $\text{comm}_{i,t+1}$ for each currency i from our empirical model in equation (9), with 90% confidence interval in gray, calculated using Newey-West standard errors. Panel (c) shows estimated loading of the dollar factor $\text{doll}_{i,t+1}$ for each currency i from our empirical model in equation (9), with 90% confidence interval in gray, calculated using Newey-West standard errors.

on the construction of these factors). Panel (a) of Appendix Figures A.5 and A.6 show that the explanatory power of our baseline two-factor specification is very similar to alternative two-factor specifications in which we replace the commodity factor with the carry factor or the trade centrality factor. Additionally, Panel (b) of Appendix Figures A.5 and A.6 shows that the country-loadings on the commodity factor are tightly correlated with the loadings on the carry and the trade-centrality factors. This analysis suggests that the commodity factor captures similar variation as the carry and trade-centrality factors, consistent with the findings of Ready et al. (2017).

Second, we document that the commodity factor is tightly related to a factor constructed based on cross-country variation in the sensitivity of tradable output to a global commodity price index. As suggested by the model, this "tradable-output-sensitivity factor" captures the mechanism through which heterogeneity in commodity incidence affects risk exposure of countries. To construct this factor, we first obtain country-specific sensitivities to a global commodity price index, $\beta_{i,\text{tos}}$ by estimating the following regression:

$$\Delta y^T_{i,t+1} = \alpha_i + \beta_{i,\text{tos}} \Delta p_{\text{comm},t+1} + \upsilon_{i,t}, \tag{12}$$

where $\Delta y^T_{i,t+1}$ is the annual log change in tradable output from country i in period $t+1$, and $\Delta p_{\text{comm},t+1}$ is the annual log change in the global commodity price index. We then construct the tradable-output-sensitivity factor as the differential average change in the log exchange rate of above-median relative to below-median tradable-output-sensitivity-countries (see Appendix A for further details).

Appendix Figure A.7 shows that the regression in which we substitute the commodity factor for the tradable-output-sensitivity factor has a similar explanatory power to the baseline regression. It also shows that the tradable-output-sensitivity factor captures similar country variation as the baseline commodity factor.

The role of dollar basis. We also argue that the dollar factor captures a global factor and not a local US factor that arises simply because we express currencies relative to the US dollar. We perform two related exercises to support this conclusion. First, consider expressing exchange rates relative to a base currency j instead of the dollar. In the model, these exchange rates admit a three-factor structure in which, in addition to the dollar and commodity factor, there is a third "$j-$base" factor, defined in the same way as the dollar factor but expressing all exchange rates relative base currency j. This new factor captures the idiosyncratic shocks to non-traded output of country j, which affect non-traded prices in that country and all bilateral exchange rates. Proposition 6 in Appendix B specifies the exact factor structure and the structural definition of the factors. According to this result, switching to base currency j should shift the dollar factor loadings by 1 for all currencies, the commodity factor loadings should be unchanged, and the base currency factor should have a loading of 1 on all currencies.

We first focus on the British pound as an example of another base. Appendix Figure A.2 shows the three factor loadings of regressing the changes in bilateral exchange rates vis-a-vis the pound on the commodity factor, the dollar factor, and a British pound factor as independent variables, and compares them to the same regression in which we express the exchange rates vis-a-vis the dollar. Consistent with the model, the dollar factor loadings display similar heterogeneity across countries when estimated with exchange rates vis-a-vis the British pound and vis-a-vis the dollar. The dollar loadings remain statistically significant for most countries when estimated on exchange rates vis-a-vis the pound, and are shofted by approximately 1 relative to the specification with USD as the base currency. In contrast, the loadings on the pound factor are positive and fluctuate around 1 in all countries when estimated on exchange rates vis-a-vis the pound, but are close to zero when estimated on exchange rates vis-a-vis the dollar. The commodity loadings are similar when estimated using both bases.

We nxt scale the analysis and estimate the same regressions for all possible currency bases. Appendix Table A.4 reports in how many regressions in a given currency base the dollar factor is statistically significant. When we pool all the currency bases, the dollar factor remains a statistically significant factor in 682 out of 992 country-currency-base regressions, suggesting that the dollar factor plays a global role over and above the mechanical effect of using it as a currency base.

Second, we show that the reverse is not true. Other currency bases do not have as strong explanatory power as the dollar factor. We illustrate this by estimating a similar specification to the baseline with exchange rates relative to the dollar as dependent variables, in which we add an additional base-currency factor, for all possible currency bases. As reported in the last column of Appendix Table A.4, all other currency bases are statistically significant in only 268 out of the 992 regressions.

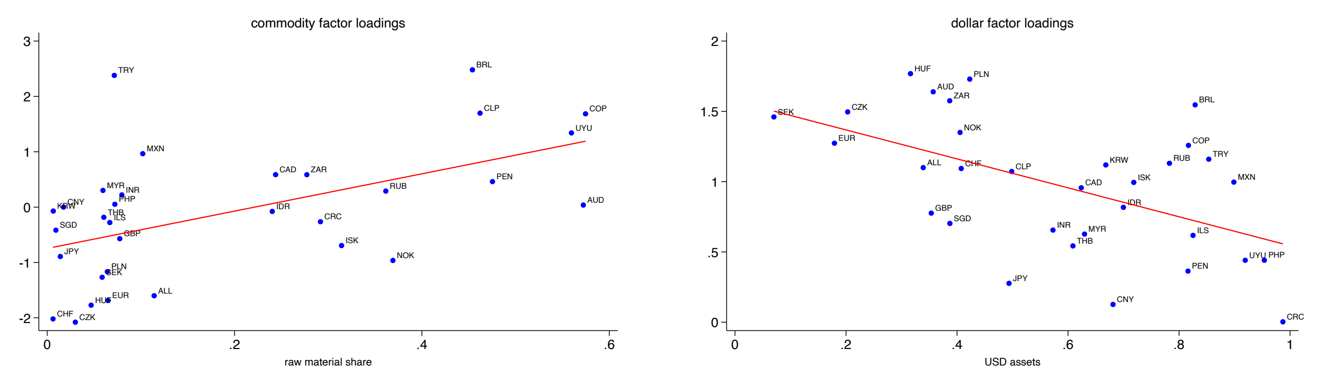
3.2 Determinants of Country-Factor Loadings

We now analyze how the estimated country-loadings from equation (9) are related to certain observable country-characteristics, as suggested by the model. According to Proposition Proposition 3, the loadings on the commodity factor should be positively related to the commodity incidence of a country, and the loadings on the dollar factor should be negatively related to the incidence of dollar assets of a country. Panel (a) of Figure Figure 2 shows that the estimated country loadings on the commodity factor are positively related to the country shares of commodity exports in total exports. Panel (b) shows that the estimated loadings on the dollar factor are negatively related to the share of dollar-denominated portfolio assets in total portfolio assets. Appendix A provides further details on the construction of these variables. Appendix Table A.5 shows the univariate regression estimates associated with both scatter graphs. The coefficient of the share of commodity exports on the commodity loadings is positive and statistically significant, and the R^2 from the regression is 29%. The coefficient of the share of dollar assets on the dollar loadings is negative and statistically significant, and the the R^2 from the regression is 29%.

Figure 2: Determinants of Factor Loadings: Baseline Regressions

(a) Commodity-loadings on commodity incidence

(b) Dollar-loadings on dollar assets



Notes: Panel (a) plots the commodity factor loadings for each currency i (y-axis), against the commodity incidence, measured by the share of commodity exports (x-axis). Panel (b) plots the dollar factor loadings for each currency i (y-axis), against the share of dollar-denominated assets (x-axes). Each scatter point represents a currency, and the red line denotes the line of best fit.

Appendix Table A.5 also shows that the empirical relationships estimated in the regressions are robust to considering various additional controls considered in the literature (e.g., Hassan, 2013; Richmond, 2019; Lustig and Richmond, 2020): trade centrality, trade openness, the economic size of the country, and a set of gravity variables relative to the US (bilateral distance, and dummies for common borders, language, and legal origins). Finally, Appendix Figure A.8 shows that we reach similar conclusions if we replace the commodity factor with the tradable-output-sensitivity factor, suggesting that both factors capture similar cross country variation.

Overall, our empirical results indicate that an important part of the variation in bilateral exchange rates can be explained by the commodity and dollar factors, with countries loading heterogeneously on them. The country-loadings on the commodity and dollar factors can be traced to fundamental country characteristics like the incidence of commodities in their exports and the incidence of dollar assets in their economy. This empirical evidence provides support for the predictions of the model.

4 Quantitative Analysis

We present our quantitative results in three steps. First, we describe the calibration procedure. Second, we describe the drivers of the relative price of local non-tradables in individual countries. Third, we show the drivers of bilateral exchange rates against the dollar in the cross-section and the factor structure of exchange rates.

4.1 Calibration

We calibrate the following five parameters internally: shock sizes $(\sigma_z, \sigma_x, \Gamma\sigma_\gamma)$, the share of non-tradables in consumption α, and the total external assets m. We pick these parameters by making the model approximate six moments. First, we include the three second moments of the factors $(\mathbb{V}[\text{comm}_t], \mathbb{V}[\text{doll}_t], \mathbb{C}[\text{comm}_t, \text{doll}_t])$ to leverage our cross-sectional identification results. Second, we make the model reproduce the average standard deviation of bilateral exchange rate depreciation $\text{std}(\Delta s_{it})$, the standard deviation of the dollar loadings $\text{std}(\beta_{i,\text{doll}})$, and the average R^2 of the factor regression. These three moments are available in closed form for any pair (e_i, m_i). Appendix C provides details on the computation. We read the pairs (e_i, m_i) from the data and take empirical averages. We also estimate the dollar asset management parameter τ using data on reserves. Appendix C provides the details. We set $\beta = 0.98$ and the inverse elasticity of intertemporal substitution $\rho = 2$ in line with the literature. Finally, we maintain assumptions on the persistence of shocks that leads to our tractable factor structure of exchange rates. Table 1 shows our choice of parameters, and Table 2 demonstrates the fit on the targeted moments.

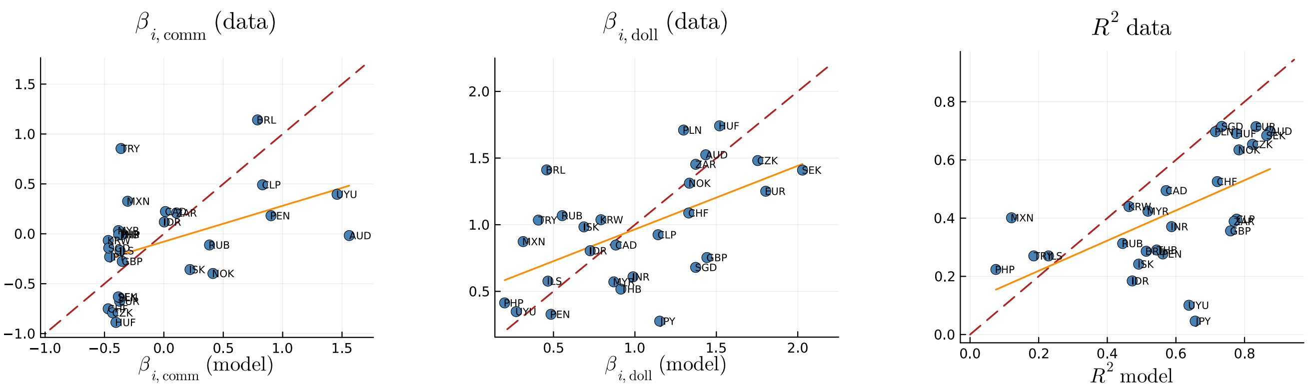
Table 1: Model parameters.

Parameter	Value	Role
		Internally calibrated
α	0.64	share of non-tradables
m	2.59	total external assets
$\Gamma\sigma_\gamma$	0.8571	standard deviation of innovations to γ_t
σ_x	2.3080	standard deviation of innovations to x_{it}
σ_z	1.9731	standard deviation of innovations to z_t
		Estimated
τ	0.83	stabilization elasticity
$\{e_i, m_i\}$		distribution of country types
		Externally set
ρ_x	1.0	persistence of shocks to x_{it} and x_{ut}
ρ_z	1.0	persistence of shocks to z_t
ρ_γ	0.0	persistence of shocks to γ_t
ρ	2.0	inverse of EIS
β	0.98	discount factor

Table 2: Model fit on targeted moments.

Moment	Data	Model
std(doll$_t$)	1.94pp	1.92pp
std(comm$_t$)	1.32pp	1.14pp
corr(doll$_t$,comm$_t$)	0.31	0.44
std(Δs_{it})	3.05pp	2.56pp
R^2 of factors	41%	59%
std($\beta_{i,\text{doll}}$)	0.43	0.50

Untargeted moments. For every country, as represented by a pair (e_i, m_i), the model predicts loadings on the dollar and commodity factors and the explanatory power of the two-factor regression. We only target the average of the dollar loading and the average R^2. Commodity factor loadings only depend on $\{e_i\}$, which we take from the data directly. The model is able to capture up to a third of cross-sectional variation in these variables. Figure 3 and Table 3 show the fit.



(a) commodity loadings (b) dollar loadings (c) explanatory power

Figure 3: Untargeted regression results in the cross-section.

Table 3: Model fit on untargeted moments.

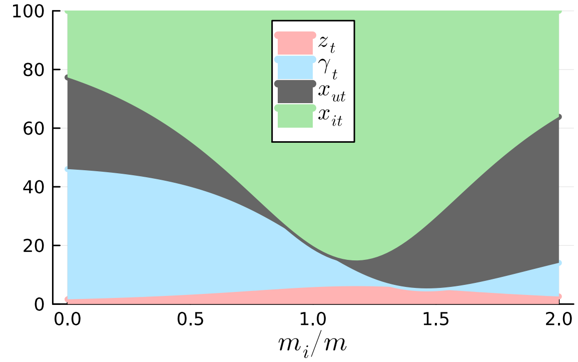
	$\beta_{i,\text{doll}}$	$\beta_{i,\text{comm}}$	R^2
const	0.47	-0.08	0.12
	(0.16)	(0.08)	(0.10)
model	0.50	0.36	0.51
	(0.14)	(0.14)	(0.16)
R^2	0.31	0.20	0.29
N	28	28	28

We view our calibration as conservative on the role of financial shocks: the model underestimates the volatility of exchange rates relative to consumption, generating the ratio of 2.5 as opposed to the standard value of 4 (see, e.g., Itskhoki and Mukhin, 2021).

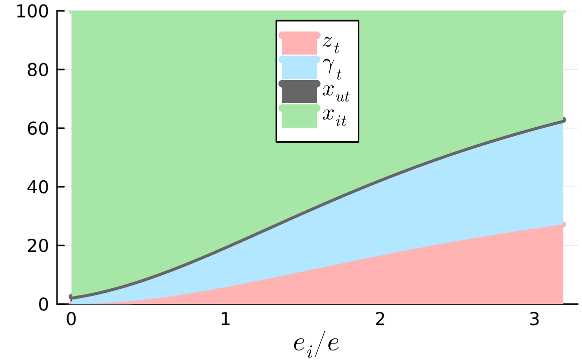
4.2 Drivers of currencies

We now turn to the drivers of individual currencies. By virtue of being relative prices, bilateral exchange rates necessarily mask information, so we start our analysis with individual consumption basket prices q_{it}. Their drivers are strongly heterogeneous across (e_i, m_i). We perform a variance decomposition of q_{it} for generic (e_i, m_i), slicing the cross-section in two dimensions. Figure 4a fixes the commodity exposure at the average level $(e_i = e)$ and varies the dollar assets m_i. Figure 4b does the opposite: it fixes $m_i = m$ and varies e_i. Appendix C provides details on the variance decompositions, which are available in closed form.

High exposure to global tradable output shocks and low dollar assets correspond to high impor-



(a) contributions of shocks to the variance of local prices, $e_i = e$



(b) contributions of shocks to the variance of local prices, $m_i = m$

tance of the financial shock γ_t, since countries in this region of the cross-section are risky. At large levels of dollar assets, US-specific shocks have high importance because they generate large accumulation flows. Financial shocks are less important because countries with highly dollarized assets are less risky. The contribution of the local shocks x_{it} is maximized in countries with $m_i = m$, where the impact of reserve accumulation is offset by the average accumulation effort, which gets recycled into uniform inflows into local currency debt.

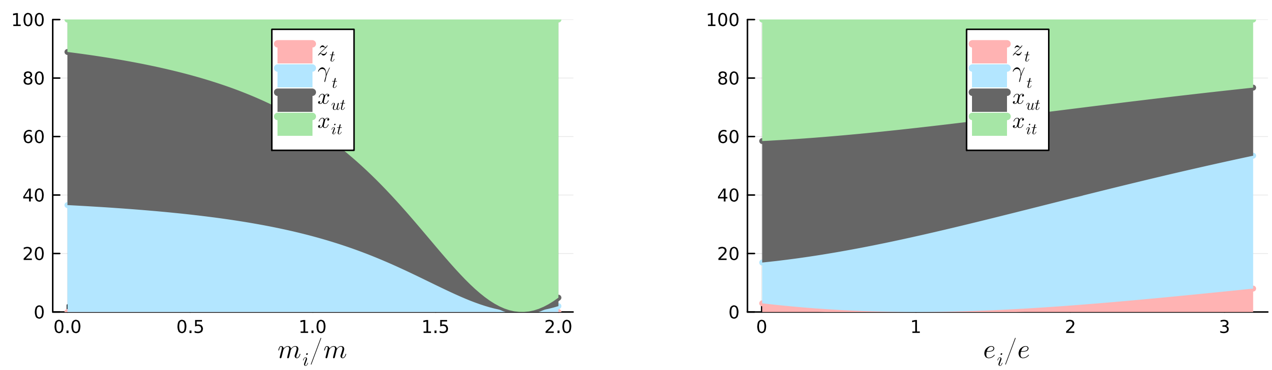
Bilateral exchange rates. We next turn to bilateral exchange rates, performing the same variance decompositions. Bilateral depreciation against the dollar incorporates the impact of the shocks on the US consumption basket as well, so the importance share of the shocks is shifted relative to Figure 4a and Figure 4b.

As a simple illustration of this point, we first show the decomposition of the US prices q_{ut} alongside the two factors. The dollar factor, in particular, measure the broad USD appreciation, and is the direct analog of the bilateral exchange rate in two-country models. The importance of financial shocks γ_t to the US prices is small relative to that of the non-tradable output shock x_{ut}. The relative importance of the financial shock to the dollar factor, which is the average bilateral exchange rate depreciation, is much larger. The reason is that financial shocks lead to a repricing of dollar risk. The US experiences inflows, and the rest of the world experiences outflows, when the intermediaries' risk aversion is elevated. Since financial shocks move local prices in the US and the rest of the world in the opposite directions, their contribution to bilateral exchange rates is larger than to local prices in isolation.

Table 4: Variance decomposition of q_{ut}, doll_t, and comm_t.

	Δq_{ut}	doll_t	comm_t
$\Delta\gamma_t$	14%	41%	48%
Δz_t	6%		52%
Δx_{ut}	80%	59%	

Figure 5a and Figure 5b show that incorporating the US prices into exchange rates increases the importance of financial and US-specific shocks in most of the cross-section. The importance of the US shock is now minimized at a higher than average level of dollar assets: bilateral exchange rates incorporate direct effects of the change in the non-tradable output, and regular countries must have large reserves to offset the exchange rate movements with their accumulation or decumulation responses. The importance share of local shocks is maximized in countries with large dollar assets, who are well insulated from US shocks and are hence less risky from the intermediaries' perspective.



(a) contributions of shocks to the variance of exchange rates, $e_i = e$

(b) contributions of shocks to the variance of exchange rates, $m_i = m$

We now illustrate our decomposition using three specific examples from the data in Table 5. We pick three countries that illustrate extremes. First, Hungary is a country with both low commodity exposure and low dollar assets. It is a European country, whose main trade partners mostly use the Euro. According to our model, Hungarian forint should have a volatile exchange rate against the US, with the US shock and the global financial shock both making important contributions to its variability. A comparison is Thailand, a country with a similar exposure to commodities but much higher dollar assets. Its currency, the baht, is considerably less volatile, and global factors contribute a substantially lower share to its variability compared to local shocks: the R^2 of the factor regression for Thailand is low. The third example is Australia, a commodity-intensive country with relatively low USD assets. Its exchange rate is also volatile, and global factors have a high explanatory power, similar to the case of the Hungarian forint. However, since Australia is exposed to both commodity and dollar risk, global financial shocks dominate Australian dollar's exchange rate, surpassing both the global and US-specific output shocks, although they also contribute non-trivial shares.

Table 5: Examples of countries.

	Hungary	Thailand	Australia
e_i	low	low	high
m_i	low	high	low
$\Delta\gamma_t$	32%	26%	59%
Δz_t	1%	4%	11%
Δx_{ut}	67%	70%	30%
R^2 (data)	69%	29%	70%
R^2 (model)	78%	54%	87%
std (data)	3.9%	2.0%	3.6%
std (model)	3.1%	2.2%	4.2%

These three countries constitute our opening example, where we interpret the Global Financial Crisis of 2008 as an episode with a large financial shock γ_t.

5 Conclusion

We develop a model of the global economy to study the drivers of exchange rate in cross-section. The model fulfills two goals. First, it provides a mapping that allows us to recover the parameters of fundamental shocks from the empirical factor structure of exchange rates. Second, it provides an equilibrium interpretation to this factor structure, mapping fundamental shocks to factors and fundamental properties of countries to their factor loadings.

We find that financial shocks are a consistently important driver of individual countries' price levels and a much more important driver of their dollar exchange rates, since these shocks move non-tradable good prices in the US and other countries in the opposite directions. High-commodity currencies are driven by a combination of financial and global output shocks. Countries with low dollar assets are affected by financial shocks and US-specific output shocks. Low-dollar and high-commodity countries are the ones with most volatile price levels and dollar exchange rates, and the contribution of financial shocks to their exchange rates is the largest. They also maximize explanatory power of global factors. High-dollar and low-commodity countries, in contrast, are the ones with the lowest volatility and lowest explanatory power of global shocks: their currencies are mostly driven by local shocks.

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A Empirical Appendix

A.1 Data description

- *Exchange rate*: The dependent variable in our empirical model equation (9) is constructed based on exchange rate contracts data from the London Stock Exchange (LSEG). We construct end-of-month bilateral exchange rates vis-a-vis the US dollar from daily to-USD spot exchange rate contracts, and then take the first difference of log exchange rate of local currency i per dollar.

- *Commodity factor*: We define high and low commodity producers based on countries' time-averaged annual share of raw materials in merchandise exports over 2015-2019, using data from the World Integrated Trade Solution (WITS). A country is categorized as a high commodity producer $i \in \mathcal{I}_{h,i}$, if its time-averaged share of raw materials in merchandise exports is above the median across countries in our sample, excluding the country itself; and as a low commodity producer $i \in \mathcal{I}_{l,i}$ if it is below the median. We then construct the country-specific commodity factor by differencing the average change in the log exchange rate of high commodity producers and that of low commodity producers as in equation (10).

- *Dollar factor*: We construct the currency-specific dollar factor as the cross-sectional-average of monthly change in the log exchange rates of all currencies against the dollar, excluding the currency itself as in equation (11).

- *Carry factor*: Following Verdelhan (2018), we first sort currencies in our sample, excluding the currency itself, into six portfolios based on their local-currency interest rates, which are approximated using the one-month forward discount rate. We then construct the currency-specific carry factor by taking the difference between the average change in the log exchange rate of the 6th portfolio (the one with the highest interest rates), and that of the first portfolio (the one with the lowest interest rates).

- *Trade-centrality factor*: We first construct the Richmond (2019) trade centrality measure as in equation (A.1), using bilateral trade data from the International Monetary Fund's (IMF) International Trade in Goods dataset (IMTS).

$$v_{it} = \sum_{j=1}^{N} \left(\frac{\widetilde{X}_{ijt} + \widetilde{X}_{jit}}{GDP_{it} + GDP_{jt}} \right) \tilde{s}_{jt} \tag{A.1}$$

  where v_{it} is the trade centrality of country i at time t; $\frac{\widetilde{X}_{ijt}+\widetilde{X}_{jit}}{GDP_{it}+GDP_{jt}}$ is the bilateral trade intensity between country i and j at time t, measured as the total bilateral exports scaled by the combined GDP; and $\tilde{s}_{jt}$ is country j's exports as a share of total trade across our sample countries at time t. We then construct the country-specific trade-centrality factor in a manner similar to the commodity factor, categorizing countries into high- or low-trade-centrality groups based on whether a country's 2015–2019 average trade centrality is above or below the median of the sample (excluding the country itself).

- *Tradable-output-sensitivity factor*: The dependent variable in the regression model equation (12) is based on data from the Organization for Economic Co-operation and Development's (OECD) Inter-Country Input-Output tables (ICIO) (Note that the ICIO database does not include data for Albania or Uruguay from our sample). We categorize agriculture, mining, and manufacturing as the tradable sectors and deflate each country's annual tradable output (measured in USD) by U.S. CPI index. The independent variable is based on monthly commodity prices from the World Bank Commodity Price Data and we use the end-of-year total index. We estimate the empirical model in equation (12) over the 2000-2019 period to obtain $\beta_{i,\text{tos}}$ for each country i, representing its tradable output sensitivity. Finally, we construct the tradable-output-sensitivity factor in a manner similar to the commodity factor, categorizing countries into high- or low-sensitivity groups based on whether a country's β_{tos} is above or below the median of the sample (excluding the country itself).

- *Share of dollar-denominated assets*: We primarily use data from the IMF's Portfolio Investment Positions by Counterpart Economy (formerly CPIS) to construct each country's share of dollar-denominated assets. We calculate this share by dividing a country's USD-denominated portfolio assets by its total portfolio assets and using the 2015-2019 average. For countries not covered by the CPIS, we impute the share using the ratio of a country's holding of US Treasuries (from the Treasury International Capital (TIC) System) to its total international investment assets (from IMF's International Investment Position (IIP)).

- *Commodity incidence (or share of commodity exports)*: We use 2015-2019 average of a country's annual share of raw materials in merchandise exports.

- *U.S. non-tradable employment*: For the regressions in panel A of Table A.3, we use the log difference of U.S. monthly non-tradable sector employment. We construct this series by subtracting total employees in goods-producing sectors from total private-sector nonfarm employees, downloaded from the FRED.

- *Commodity price index*: For the regressions in panel B of Table A.3, we use log difference of the month commodity price index (total) from the World Bank Commodity Price Data.

- *VIX*: We use the end-of-month Cboe Volatility Index in Table A.3, constructed from daily closing values of the index from the Cboe.

- *Excess bond premium*: Following Gilchrist and Zakrajšek (2012), we use growth rate of the monthly excess bond premium in Table A.3, provided by the Board of Governors of the Federal Reserve System.

- *Intermediary capital risk factor*: Following He et al. (2017), we use monthly intermediary capital risk factor in Table A.3, downloaded from Zhiguo He's website.

- *Trade centrality*: Following Richmond (2019), we use 2015-2019 average of a country's trade centrality measure as an additional control variable for specification (2) of Table A.5.

- *Trade openness*: We use 2015-2019 average of a country's total trade (imports and exports) scaled by its GDP as an additional control variable for specification (3) of Table A.5.

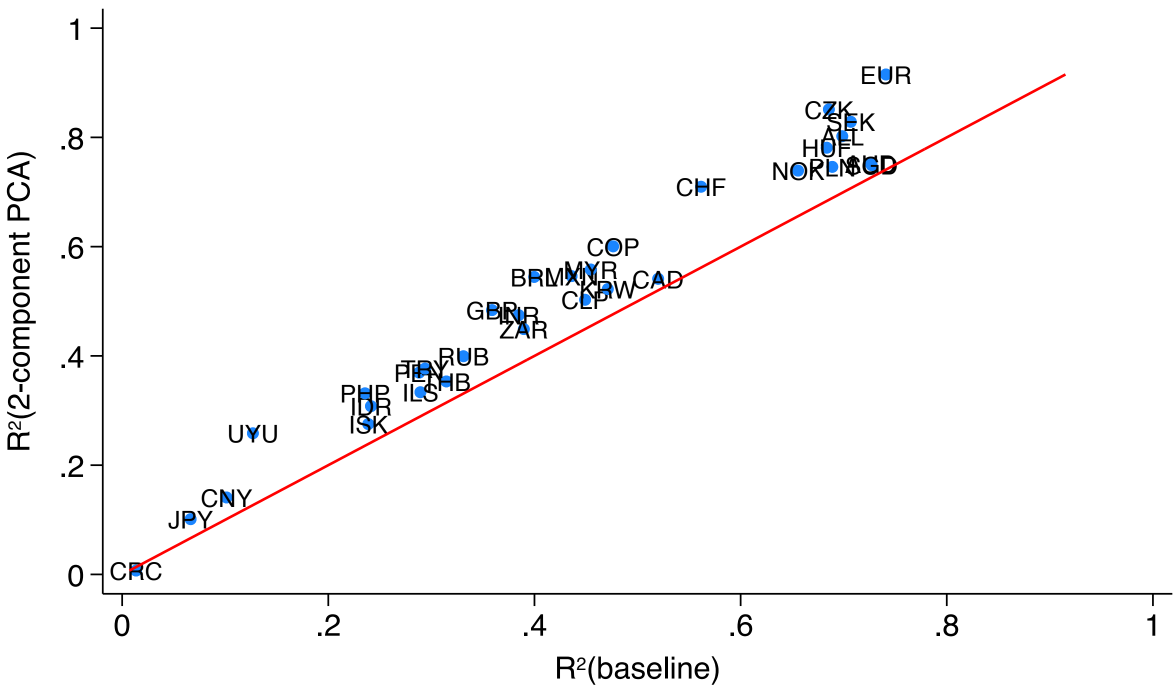
- *Country size*: Following Hassan (2013), we use 2015-2019 average of a country's log share of GDP in our sample as an additional control variable for specification (4) of Table A.5.

- *Distance*: we include log harmonic geographic distance to U.S. from the World Bank's World Development Index (WDI) as an additional control variable for specification (5) of Table A.5.

- *Gravity variables*: Following Lustig and Richmond (2020), we include a set of other gravity variables relative to U.S. as additional control variables in specification (6) of Table A.5 using data from the Research and Expertise on the World Economy (CEPII) Gravity dataset. These include: a dummy variable indicating whether country i shares a common language with the U.S.; a dummy variable for whether it shares a common legal origin; and a dummy variable for whether country i is contiguous with the U.S.

A.2 Additional figures and tables

Figure A.1: Explanatory Power of Factor Regression and Principal Components



Notes: This figure plots the R^2 from a two-principal-component model (y-axis) against the R^2 from the empirical model in equation (9) (x-axis) for each currency i. Both measures represent the fraction of variation in log exchange rate changes $\Delta s_{i,t+1}$ explained by the respective models. Each scatter point represents a currency, and the red line denotes the 45-degree line.

Table A.1: Factor Structure of Exchange Rates: Regression Estimates

Country	α	β_{comm}	β_{doll}	R^2	N
Albania	-0.001	-1.601***	1.100***	0.699	240
	(0.001)	(0.183)	(0.069)		
Australia	-0.002*	0.038	1.639***	0.726	240
	(0.001)	(0.250)	(0.070)		
Brazil	0.000	2.480***	1.546***	0.400	240
	(0.002)	(0.606)	(0.136)		
Canada	-0.002*	0.586**	0.956***	0.520	240
	(0.001)	(0.293)	(0.074)		
Switzerland	-0.002*	-2.020***	1.093***	0.562	240
	(0.001)	(0.249)	(0.086)		
Chile	-0.001	1.698***	1.072***	0.449	240
	(0.002)	(0.355)	(0.109)		
China	-0.001*	0.001	0.126***	0.101	240
	(0.000)	(0.099)	(0.027)		
Colombia	-0.000	1.686***	1.258***	0.476	240
	(0.002)	(0.399)	(0.117)		
Costa Rica	0.003***	-0.264	0.003	0.014	240
	(0.001)	(0.167)	(0.041)		
Czech Republic	-0.002**	-2.079***	1.496***	0.686	240
	(0.001)	(0.299)	(0.066)		
Euro Area	-0.001	-1.688***	1.273***	0.741	240
	(0.001)	(0.206)	(0.058)		
United Kingdom	0.000	-0.570	0.776***	0.359	240
	(0.001)	(0.361)	(0.095)		
Hungary	-0.000	-1.772***	1.768***	0.684	240
	(0.001)	(0.386)	(0.110)		
Indonesia	0.002	-0.078	0.817***	0.241	240
	(0.002)	(0.312)	(0.111)		
Israel	-0.001	-0.279	0.617***	0.289	240
	(0.001)	(0.261)	(0.066)		
India	0.001	0.221	0.655***	0.385	240
	(0.001)	(0.193)	(0.063)		

Notes: This table reports regression estimates for constant α, the commodity factor loading β_{comm}, and the dollar factor loading β_{doll} for each currency i (listed in the first column), as specified in equation (9). Newly-West adjusted standard errors are reported in brackets. The table also includes the R^2, representing the fraction of variation in Δs_{t+1} explained by our empirical model for each currency's regression.

Table A.2: Factor Structure of Exchange Rates: Regression Estimates

Country	α	β_{comm}	β_{doll}	R^2	N
Iceland	0.002	-0.695	0.995***	0.239	240
	(0.002)	(0.621)	(0.170)		
Japan	0.001	-0.892**	0.276**	0.066	240
	(0.002)	(0.402)	(0.124)		
South Korea	-0.001	-0.071	1.119***	0.471	240
	(0.001)	(0.340)	(0.109)		
Mexico	0.001	0.965***	0.996***	0.436	240
	(0.001)	(0.310)	(0.102)		
Malaysia	-0.001	0.302*	0.627***	0.455	240
	(0.001)	(0.169)	(0.062)		
Norway	-0.000	-0.964***	1.350***	0.656	240
	(0.001)	(0.331)	(0.083)		
Peru	-0.001	0.461***	0.363***	0.288	240
	(0.001)	(0.152)	(0.049)		
Philippines	0.000	0.052	0.441***	0.236	240
	(0.001)	(0.204)	(0.052)		
Poland	-0.002	-1.167***	1.729***	0.689	240
	(0.001)	(0.345)	(0.097)		
Russia	0.002	0.290	1.131***	0.331	240
	(0.002)	(0.505)	(0.152)		
Sweden	-0.000	-1.268***	1.460***	0.707	240
	(0.001)	(0.242)	(0.063)		
Singapore	-0.001***	-0.417***	0.703***	0.727	240
	(0.001)	(0.118)	(0.045)		
Thailand	-0.001	-0.184	0.543***	0.315	240
	(0.001)	(0.203)	(0.049)		
Turkey	0.007**	2.380***	1.159***	0.294	240
	(0.003)	(0.546)	(0.117)		
Uruguay	0.004*	1.341***	0.441***	0.127	240
	(0.002)	(0.497)	(0.101)		
South Africa	0.001	0.585	1.575***	0.390	240
	(0.002)	(0.460)	(0.105)		

Notes: This table reports regression estimates for constant α, the commodity factor loading β_{comm}, and the dollar factor loading β_{doll} for each currency i (listed in the first column), as specified in equation (9). Newly-West adjusted standard errors are reported in brackets. The table also includes the R^2, representing the fraction of variation in Δs_{t+1} explained by our empirical model for each currency's regression.

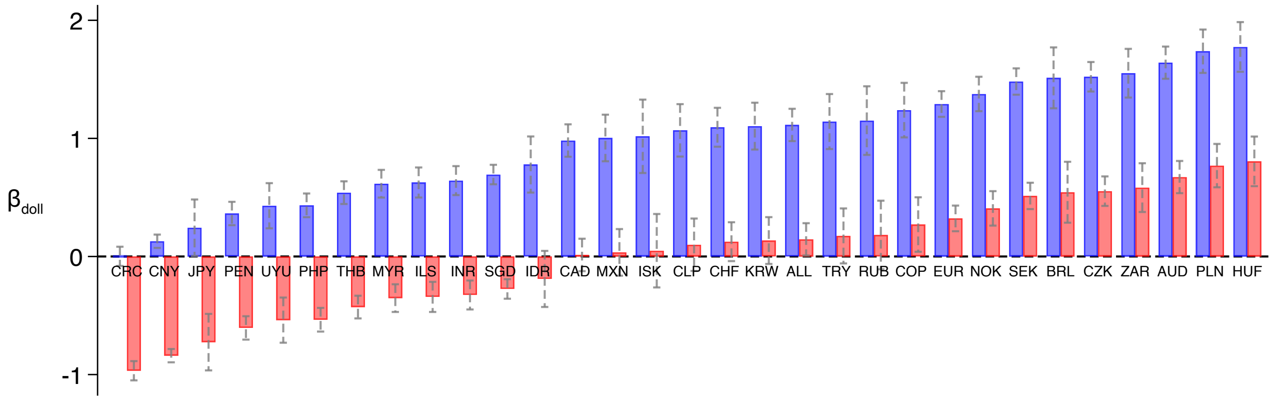
Table A.3: Macroeconomic Determinants of the Dollar and Commodity Factor

	(1)	(2)	(3)
Panel A: Dollar Factor			
US Nontradable Employment	-3.207**	-3.557**	-3.044**
	(1.513)	(1.721)	(1.455)
VIX	0.038***		
	(0.007)		
EBP		0.000	
		(0.000)	
Intermediary Capital			-0.136***
			(0.022)
R^2	0.275	0.132	0.299
Observations	240	240	240
Panel B: Commodity Factor			
Commodity Price Index	-0.013*	-0.017*	-0.011
	(0.008)	(0.008)	(0.008)
VIX	0.009***		
	(0.002)		
EBP		0.000	
		(0.000)	
Intermediary Capital			-0.023***
			(0.006)
R^2	0.118	0.028	0.092
Observations	240	240	240

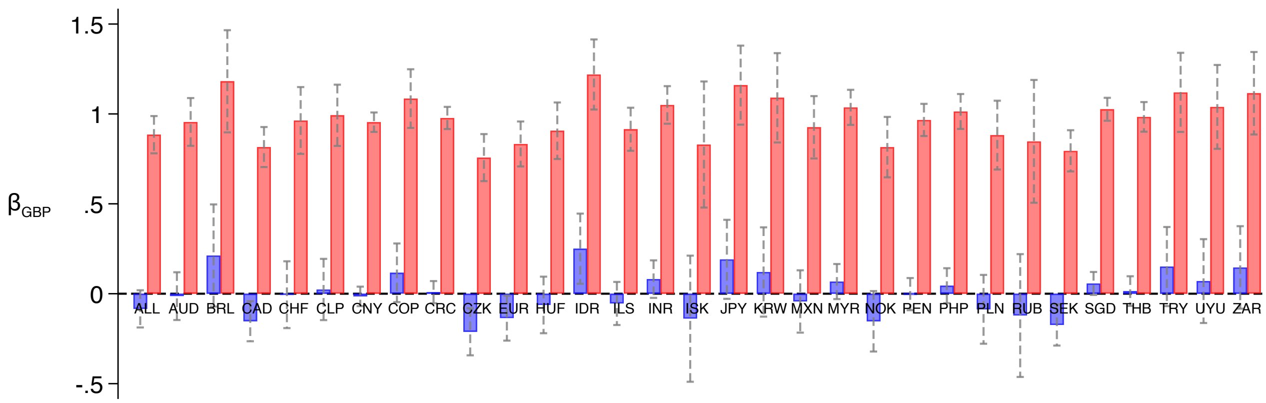
Notes: Panel A reports time-series regressions of the monthly dollar factor (constructed similarly to equation (11), but including all currencies) on the log difference of U.S. non-tradable sector employment, various global risk measures, and year fixed effects. Column (1) uses the log difference of the VIX, Column (2) uses the growth rate of the excess bond premium from Gilchrist and Zakrajšek (2012), and Column (3) uses the intermediary capital risk factor from He et al. (2017). Panel B reports time-series regressions of the monthly commodity factor (similar to equation (10), but including all currencies) on the log difference of commodity price index (total) and the same measures of global risk used in Panel A. Appendix A provides more details on variables used in this table.

Figure A.2: Factor Structure Regressions: Comparing Dollar and British Pound Bases

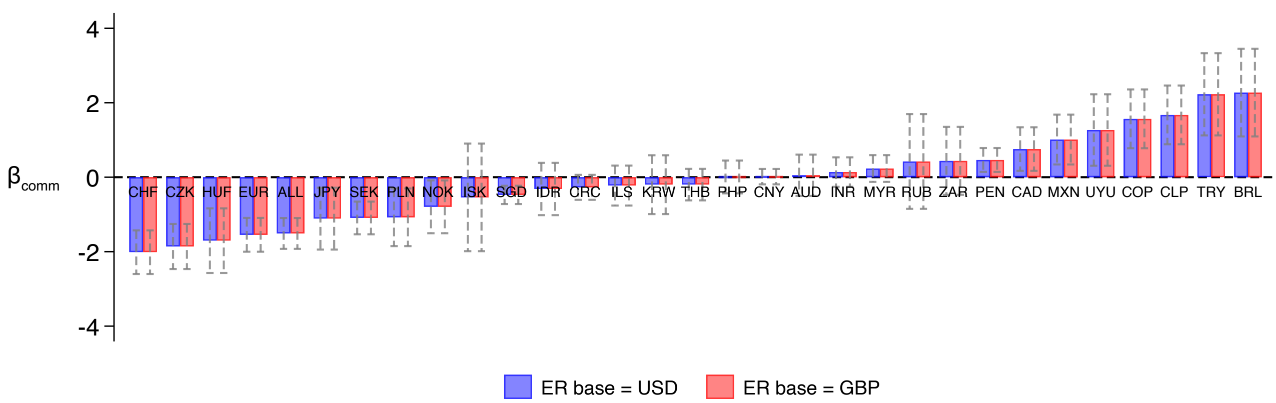
(a) Dollar-factor loadings $\beta_{i,\text{doll}}$



(g) British pound-factor loadings $\beta_{i,\text{gbp}}$



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



Notes: Each panel shows the estimated loadings in two regressions that only differ on the base of the exchange rates used as dependent variables. These regressions include the dollar factor, the British pound factor and the commodity factor as independent variables. Panel (a) shows estimated loading of the dollar factor $\text{doll}_{i,t+1}$ for each currency i. Panel (b) shows estimated loading of the British pound factor $\text{doll}_{i,t+1}$ for each currency i. Panel (c) shows the estimated loading of the commodity factor $\text{comm}_{i,t+1}$ for each currency i. Each bar includes 90% confidence interval in gray, calculated using Newey-West standard errors.

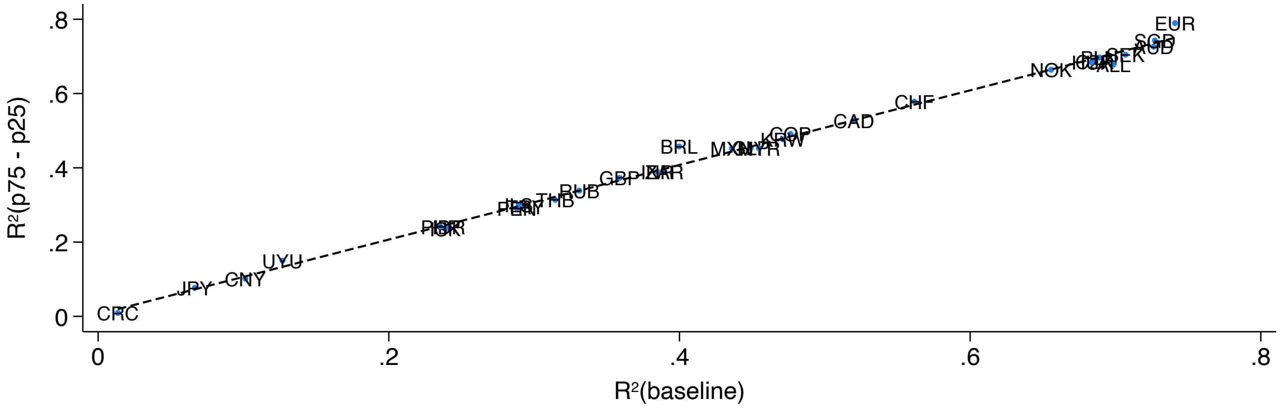
Table A.4: Factor Structure of Exchange Rates: The Role of the Dollar Basis

	# of Regressions with significant	
Currency Base	Dollar Factor	Currency-Base Factor
Albania	22	14
Australia	23	5
Brazil	19	4
Canada	22	6
Switzerland	23	9
Chile	22	1
China	17	6
Colombia	20	9
Costa Rica	19	3
Czech Republic	22	15
Euro Area	23	17
United Kingdom	22	5
Hungary	20	10
Indonesia	22	5
Israel	20	2
India	22	10
Iceland	22	5
Japan	22	9
South Korea	22	9
Mexico	22	7
Malaysia	22	14
Norway	22	12
Peru	21	6
Philippines	23	13
Poland	22	9
Russia	22	5
Sweden	19	14
Singapore	23	14
Thailand	22	11
Turkey	23	15
Uruguay	21	1
South Africa	21	3
Dependent variable relative to	currency base	USD
All currency bases	687	268

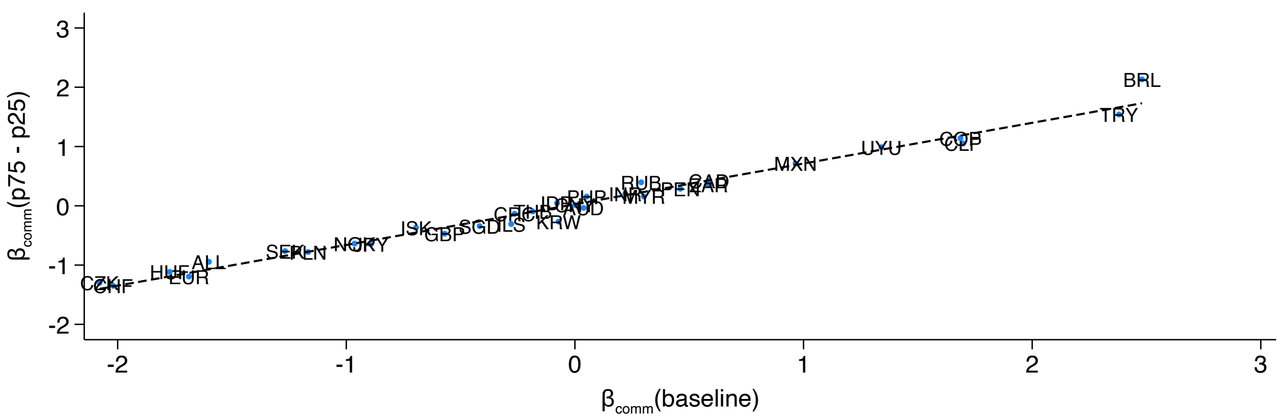
Notes: This table reports the counts of statistically significant dollar and currency-base factor loadings for each base currency i, evaluated at the 5% significance level. Appendix A provides details on the construction of the currency-base factor. The second column reports the number of significant dollar factor loadings obtained from three-factor regressions, where the dependent variable is the change in log exchange rate vis-à-vis base currency i, and the independent variables are dollar, commodity, and base-currency factors. For each base currency i, the count is out of 32; in total, 687 out of 992 regressions yield a statistically significant dollar factor loading. The last column reports the number of significant base-currency factor loadings obtained from the same three-factor regressions, but the dependent variable is relative to USD.

Figure A.3: Exchange Rate Regressions: Baseline and Alternative Commodity Factor

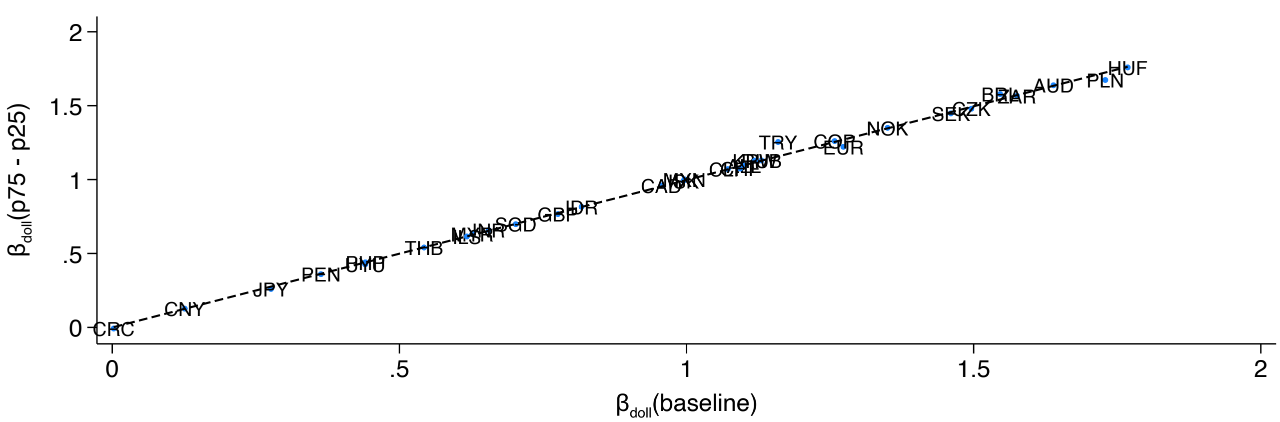
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



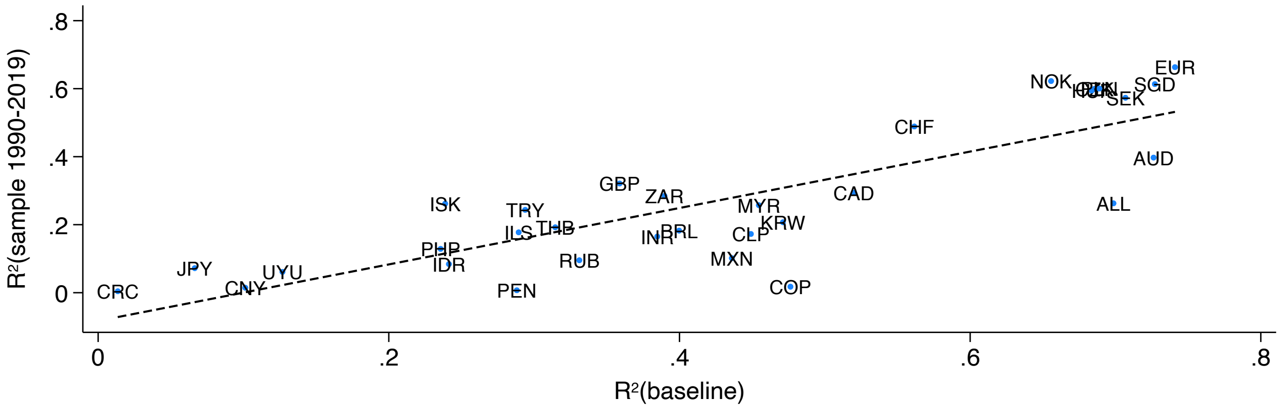
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



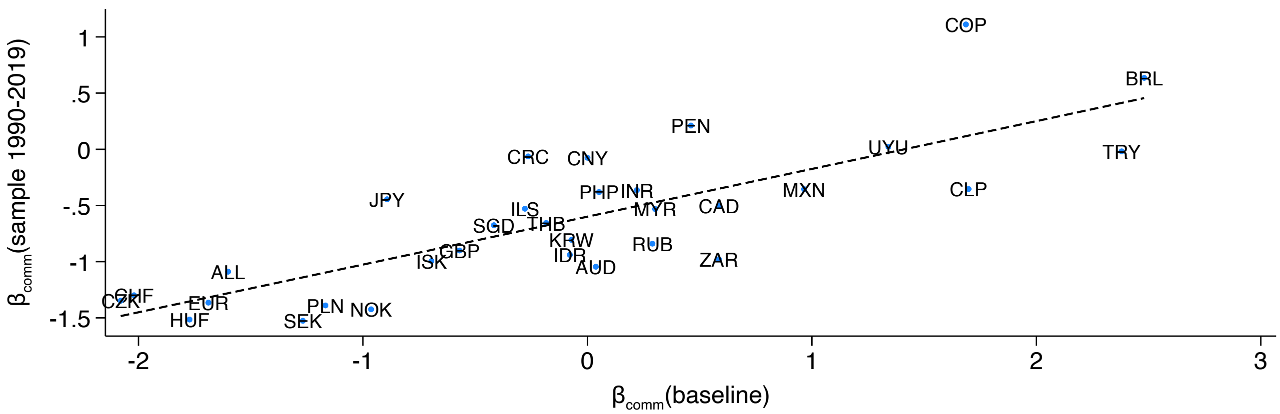
*Notes:*Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is constructed using the 75th and 25 percentiles of the commodity export share - against the R^2 from the baseline model in equation (9) (x-axis). Panel (b) plots the estimated commodity factor loadings from the modified empirical model against those from the baseline model in equation (9). Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model in equation (9). Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.4: Exchange Rate Regressions: Baseline and 1990-2019 Sample

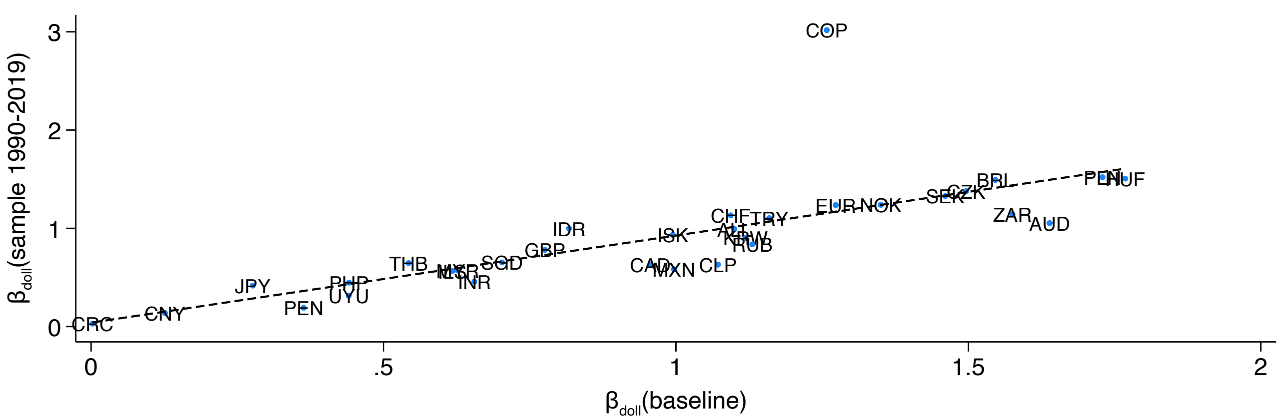
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



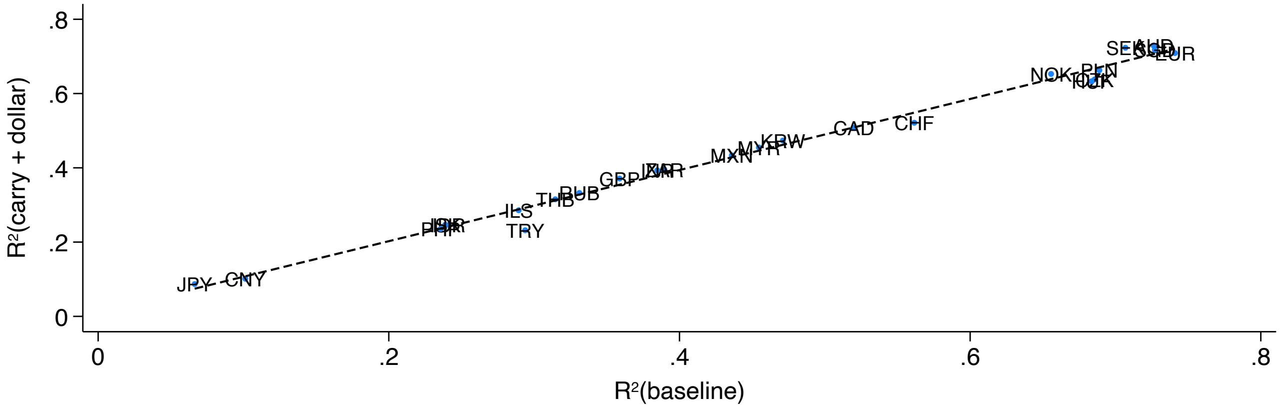
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



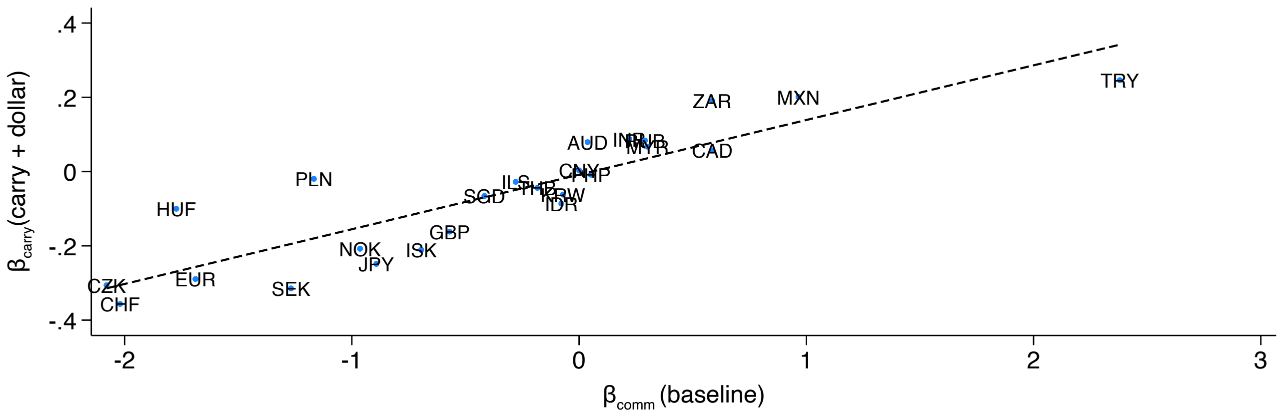
*Notes:*Panel (a) plots the R^2 for each currency i, from the 1990-2019 extended sample (y-axis), against the R^2 from the baseline model in equation (9) (x-axis), which uses the 2000-2019 sample. Panel (b) plots the estimated commodity factor loadings from the 1990-2019 extended sample against those from the baseline. Panel (c) plots the estimated dollar factor loadings from the 1990-2019 extended sample against those from the baseline. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.5: Exchange Rate Regressions: Baseline and Carry-Factor

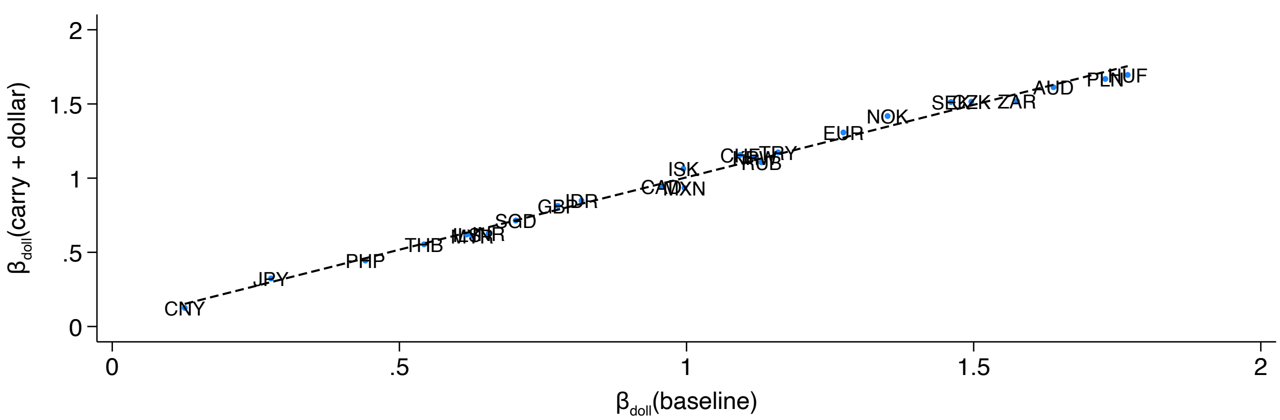
(a) Explanatory power R_i^2



(b) Commodity- and carry-factor loadings $\beta_{i,\text{comm}}$



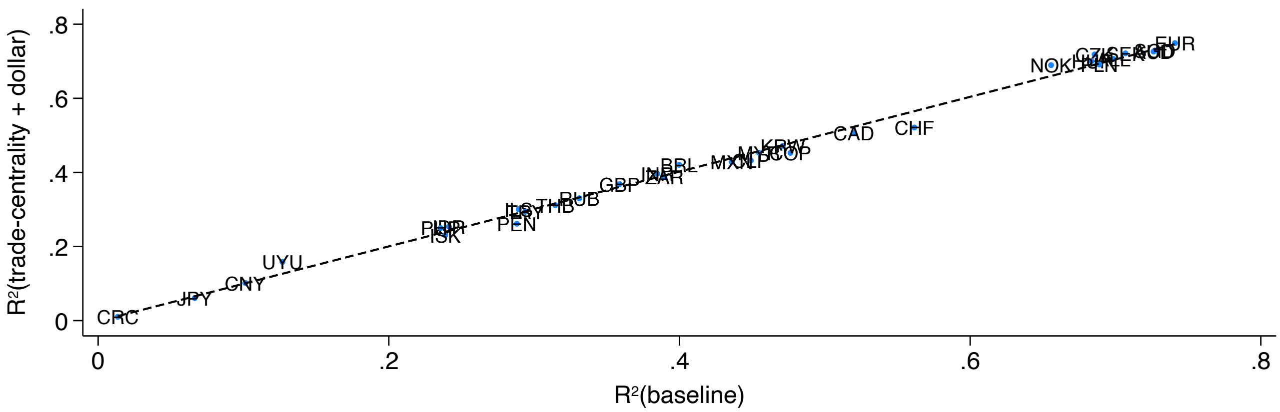
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



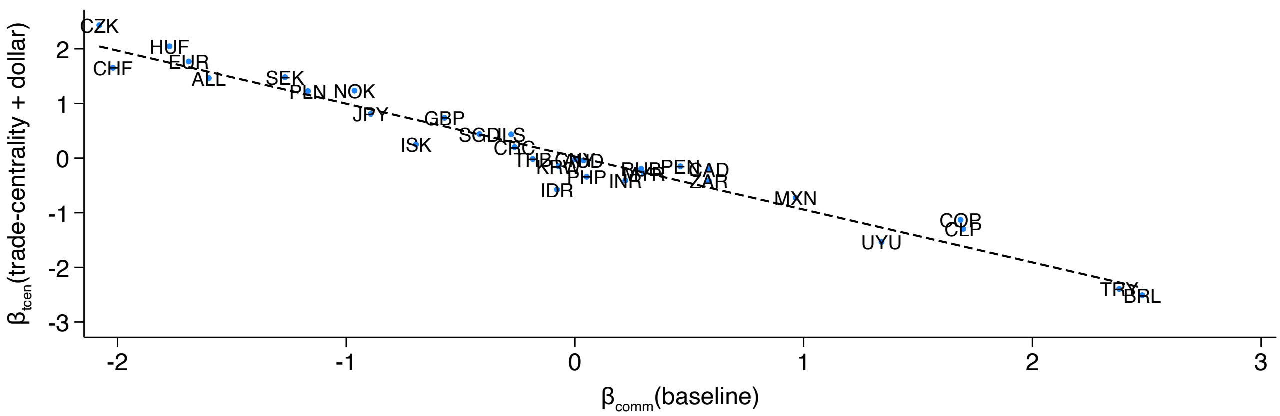
Notes: Appendix A provides details on the construction of carry factor proposed by Verdelhan (2018). Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the carry factor - against the R^2 from the baseline model in equation (9) (x-axis). Panel (b) plots the estimated carry factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure A.6: Exchange Rate Regressions: Baseline and Trade-Centrality Factor

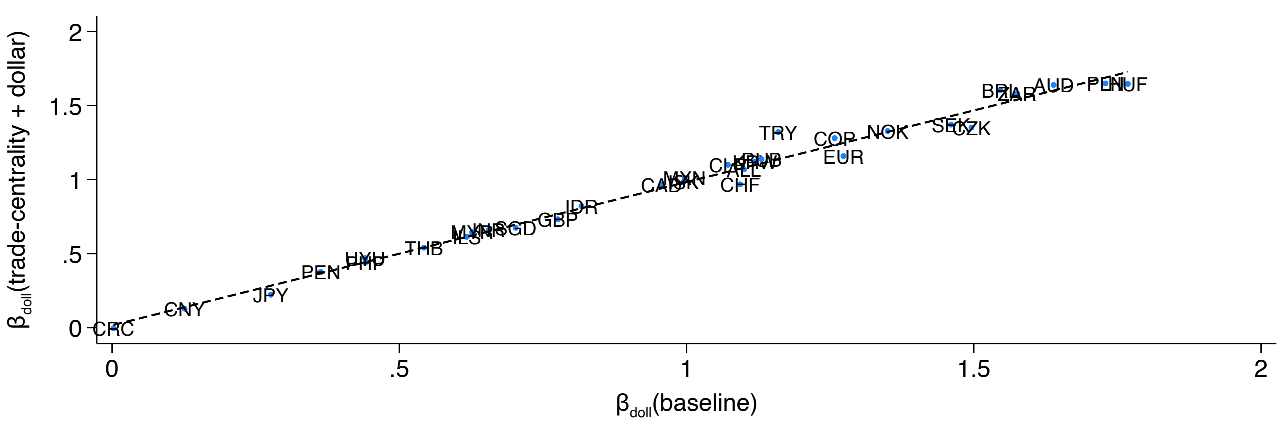
(a) Explanatory power R_i^2



(b) Commodity- and trade-centrality-factor loadings $\beta_{i,\text{comm}}$



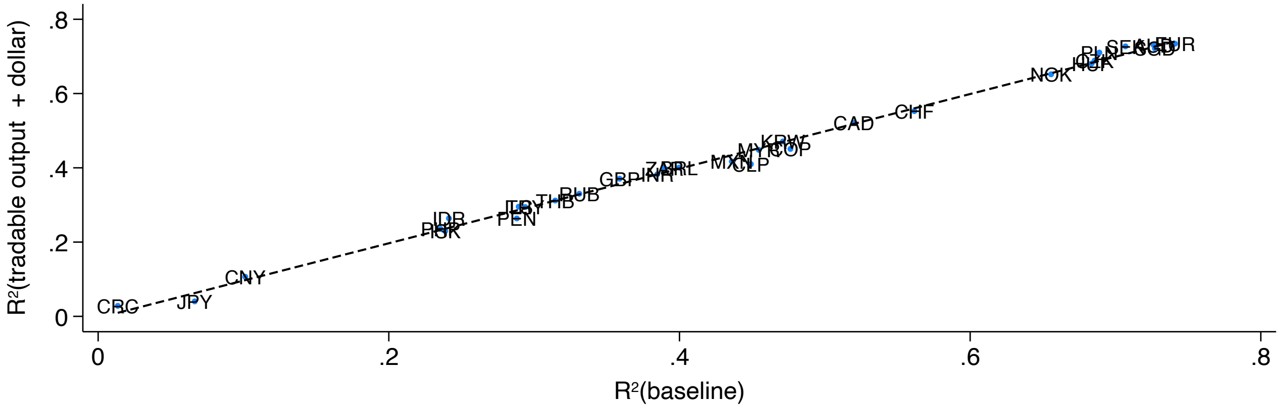
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



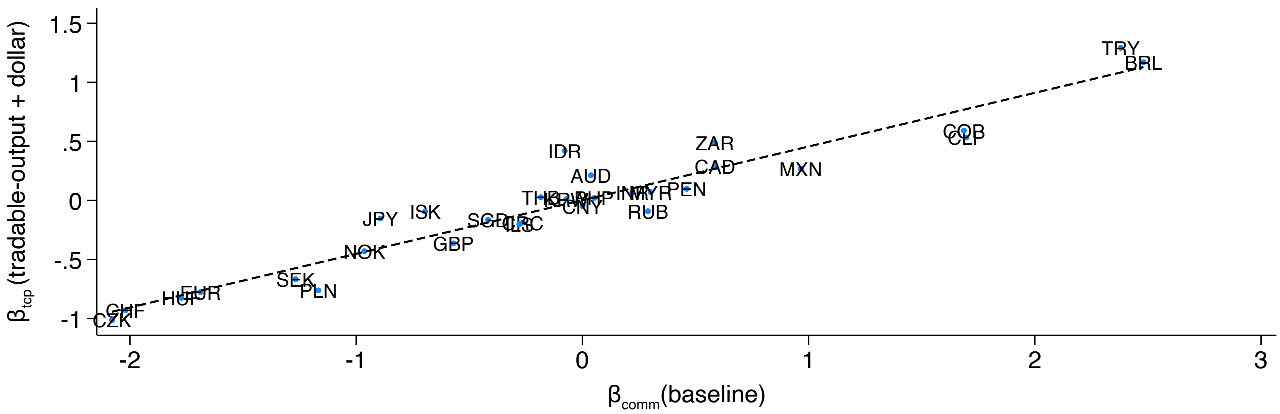
Notes: Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the trade-centrality factor - against the R^2 from the baseline model in equation (9) (x-axis). Panel (b) plots the estimated trade-centrality factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit. Appendix A provides details on the construction of the trade-centrality factor proposed by Richmond (2019).

Figure A.7: Exchange Rate Regressions: Baseline and Tradable-Output Sensitivity Factor

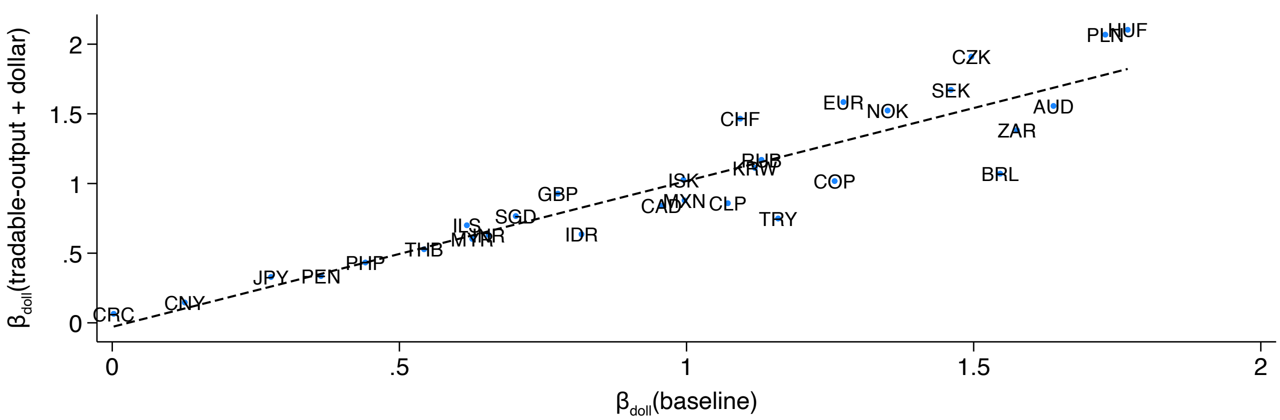
(a) Explanatory power R_i^2



(b) Commodity- and tradable-output-sensitivity-factor loadings $\beta_{i,\text{comm}}$



(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



Notes: Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the tradable-output-sensitivity factor - against the R^2 from the baseline model in equation (9) (x-axis). Panel (b) plots the estimated tradable-output-sensitivity factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit. Appendix A provides details on the construction of the tradable-output-sensitivity factor.

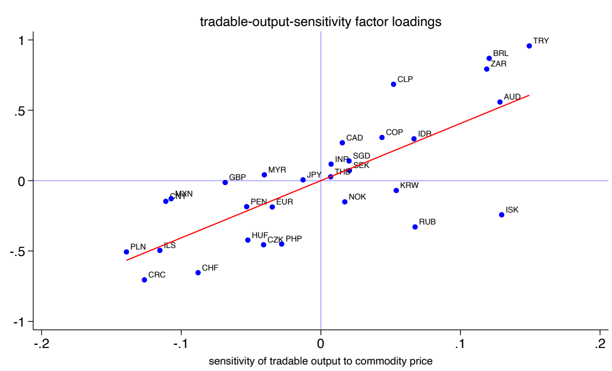
Table A.5: Cross-Sectional Regressions of Factor Loadings on USD Assets and Commodity Exports Shares

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: Commodity factor loading $\beta_{\mathbf{comm}}$							
Comm share	3.371***	2.252***	2.797***	2.649***	2.441***	2.442**	2.687**
	(0.904)	(0.794)	(0.887)	(0.942)	(0.878)	(0.932)	(1.211)
Controls		(1) + USD assets	(2) + trade cent	(3) + trade open	(4) + country size	(5) + distance	(6) + gravity
R^2	0.291	0.558	0.574	0.577	0.639	0.650	0.680
Observations	32	32	32	32	32	32	32
Panel B: Dollar factor loading $\beta_{\mathbf{doll}}$							
USD assets	-1.030***	-1.234***	-1.265***	-1.266***	-1.271***	-1.282***	-1.277***
	(0.265)	(0.265)	(0.289)	(0.290)	(0.290)	(0.295)	(0.328)
Controls		(1) + Comm share	(2) + trade cent	(3) + trade open	(4) + country size	(5) + distance	(6) + gravity
R^2	0.285	0.378	0.380	0.380	0.380	0.387	0.461
Observations	32	32	32	32	32	32	32

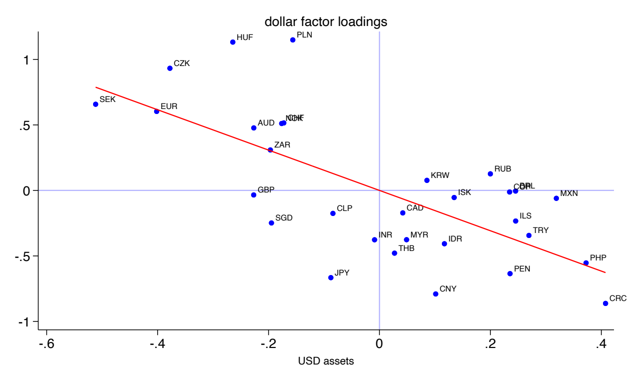
Notes: This table reports the results of cross-sectional regressions of factor loadings $\beta_{i,\text{comm}}$ and $\beta_{i,\text{doll}}$ on the share of dollar-denominated assets, the share of commodity exports, and various controls. Appendix A provides further details on the construction of these variables. Panel A presents results for the commodity factor loading. Column (1) reports the estimated coefficient from the univariate regression, with Newly-West adjusted standard errors reported in brackets. Columns (2) through (6) sequentially add the share of dollar-denominated assets, trade centrality, trade openness, log GDP share, log harmonic geographical distance to the U.S., and a set of gravity variables as controls. Panel B estimates similar specifications for the dollar factor loading.

Figure A.8: Determinants of Factor Loadings: Tradable-Output Sensitivity Specification

(a) TOS-loadings on tradable-output sensitivity



(d) Dollar-loadings on dollar assets



Notes: Panel (a) plots the tradable-output sensitivity factor loadings for each currency i (y-axis), against the commodity incidence, measured by the share of commodity exports (x-axis). Panel (d) plots the dollar factor loadings for each currency i (y-axis), against the share of dollar-denominated assets (x-axes). Each scatter point represents a currency, and the red line denotes the line of best fit.

B Proofs

We start with a lemma that characterizes the linear equilibrium:

$$q_{ut} = \theta_u z_t + \xi_u x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} + \zeta_u m_t \tag{B.1}$$
$$q_{it} = \theta_i z_t + \xi_i x_{ut} + \mu_i \gamma_t + \omega_i l_{it} + \zeta_i m_t + \eta_i x_{it} + \delta_i l_{ut} \tag{B.2}$$
$$l_{u,t+1} = D_{l,z} z_t + D_{l,x} x_{ut} + D_{l,l} l_{ut} + D_{l,m} m_t \tag{B.3}$$
$$m_{t+1} = D_{m,z} z_t + D_{m,x} x_{ut} + D_{m,l} l_{ut} + D_{m,m} m_t \tag{B.4}$$
$$l_{i,t+1} = D_{l,i} l_{it} + D_{l,x,i} x_{it} + D_{l,z,i} z_t + D_{l,u,i} x_{ut} + D_{l,l,i} l_{ut} + D_{l,m,i} m_t \tag{B.5}$$

We start with the globally important variables: the US price level, the evolution of the US debt, and the evolution of reserves.

Lemma 2. *The US price level coefficients in the linear equilibrium are*

$$\theta_u = \alpha \cdot \frac{e_u - \alpha\chi\varphi\tilde{\omega}(1-\rho_z)(e+e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.6}$$
$$\xi_u = -\alpha \cdot \frac{1 - \chi\tilde{\omega}(1+\alpha\varphi)(1-\rho_x)}{1 - 2\chi\tilde{\omega}\alpha\varphi(1-\rho_x)} \tag{B.7}$$
$$\mu_u = -\frac{\alpha L_u \tilde{\omega}}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_\gamma)} \tag{B.8}$$
$$\omega_u = \alpha\tilde{\omega} \tag{B.9}$$
$$\zeta_u = 0 \tag{B.10}$$

Here $\tilde{\omega}$ is the negative root of $2\alpha\chi\varphi\tilde{\omega}^2 - \tilde{\omega} - 1 = 0$. The coefficients in the evolution of the US debt are

$$D_{l,z} = (e_u - e) \cdot \frac{\alpha\chi\varphi\tilde{\omega}(1-\rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.11}$$
$$D_{l,x} = \frac{\chi\tilde{\omega}(1-\alpha\varphi)(1-\rho_x)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_x)} \tag{B.12}$$
$$D_{l,l} = \tilde{\omega} + 1 \tag{B.13}$$
$$D_{l,\gamma} = \frac{\mu_u}{\alpha} \tag{B.14}$$
$$D_{l,m} = 0 \tag{B.15}$$

The coefficients in the evolution of reserves are

$$D_{m,z} = \frac{\alpha\tau(e - e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.16}$$
$$D_{m,x} = \alpha\tau \cdot \frac{1 - 2\chi\tilde{\omega}(1-\rho_x)}{1 - 2\alpha\varphi\chi\tilde{\omega}(1-\rho_x)} \tag{B.17}$$
$$D_{m,l} = -2\tau\omega_u \tag{B.18}$$
$$D_{m,\gamma} = -2\tau\mu_u \tag{B.19}$$
$$D_{m,m} = 0 \tag{B.20}$$

Proof of Lemma 2. First, observe that first-order deviations of reserves are the same in all countries:

$$m_{t+1} = \tau(q_t - q_{ut}) \tag{B.21}$$

Log-linearizing the resource constraints (linearizing with respect to L_{it} nad L_{ut}),

$$q_{it} = \alpha e_i z_t - \alpha x_{it} + \alpha \Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} \tag{B.22}$$
$$q_{ut} = \alpha e_u z_t - \alpha x_{ut} + \alpha \Delta l_{u,t+1} \tag{B.23}$$

The first equation is the first part of the statement of Lemma 1. Integrating and adding up,

$$q_t + q_{ut} = (e + e_u)\alpha z_t - \alpha x_{ut} \tag{B.24}$$

The following approximation works for profits, where $L_{i,t+1} = Q_{it} A_{i,t+1}$:

$$\int X_{i,t+1} L_{i,t+1} di \approx \int (\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}) L_i di \tag{B.25}$$

Let $\Lambda^{\text{ave}}_{t+1} = \int \Lambda_{i,t+1} \mathrm{d}i + \Lambda_{u,t+1}$. The intermediaries' optimal portfolio is

$$L_{i,t+1} = L_{t+1} + \frac{\chi}{\beta}(\mathbb{E}_t[\Lambda^{\text{ave}}_{t+1}]\mathbb{E}_t[X_{i,t+1}] + \Gamma_t \mathbb{C}_t[\Lambda^{\text{ave}}_{t+1}, X_{i,t+1}]) \tag{B.26}$$

Taking the double limit $(\sigma_z, \sigma_x, \sigma_\gamma) \longrightarrow (0,0,0)$ and $(\Gamma\sigma_z^2, \Gamma\sigma_x^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_x, \Gamma_\gamma)$, rewrite this as

$$\begin{aligned} L_i + l_{i,t+1} &\approx L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] \\ &\quad - (1+\gamma_t)\chi \cdot \lim_{\Gamma\to\infty,\ (\sigma_x,\sigma_z,\sigma_\gamma)\to 0} \Gamma \mathbb{C}_t[\lambda^{\text{ave}}_{t+1}, \Delta q_{i,t+1} - \Delta q_{u,t+1}] \end{aligned} \tag{B.27}$$

In the zeroth order we have

$$L_i = L + \chi \mathcal{C}_i \tag{B.28}$$

Here

$$\mathcal{C}_i \equiv \lim_{\Gamma\to\infty,\ (\sigma_x,\sigma_z,\sigma_\gamma)\to 0} \Gamma \mathbb{C}_t[\lambda^{\text{ave}}_{t+1}, \Delta q_{i,t+1} - \Delta q_{u,t+1}] \tag{B.29}$$

In the first order, we have

$$l_{i,t+1} = L m_{t+1} + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] + \gamma_t \chi \mathcal{C}_i \tag{B.30}$$

This is the second part of the statement of Lemma 1. To eliminate the interest rates, observe that Euler equations linearize into

$$r_{it} = \rho \mathbb{E}_t[\Delta c_{i,t+1}] \tag{B.31}$$

From the condition that $\alpha Q_{it}C_{it} = P_{it}N_{it}$ and $P_{it} = Q_{it}^{\frac{1}{\alpha}}$, we get

$$c_{it} = \frac{1-\alpha}{\alpha}q_{it} + x_{it} \tag{B.32}$$

This implies

$$l_{i,t+1} = Lm_{t+1} + \chi\varphi\mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \chi\mathbb{E}_t[\Delta x_{i,t+1} - \Delta x_{u,t+1}] + \chi\mathcal{C}_i\gamma_t \tag{B.33}$$

Here the composite parameter φ is

$$\varphi = 1 + \frac{\rho(1-\alpha)}{\alpha} \tag{B.34}$$

Integrating equation (B.33) and using equation (B.24) and the fact that integrating equation (B.28) leads to $L_u = \chi\int\mathcal{C}_i\mathrm{d}i$,

$$\begin{aligned}
l_{u,t+1} &= \chi\varphi\mathbb{E}_t[\Delta q_{u,t+1} - \Delta q_{t+1}] + \chi\mathbb{E}_t[\Delta x_{u,t+1}] + L_u\gamma_t \\
&= \chi\varphi\mathbb{E}_t[2\Delta q_{u,t+1} - (e+e_u)\alpha\Delta z_{t+1}] + \chi(1+\alpha\varphi)\mathbb{E}_t[\Delta x_{u,t+1}] + L_u\gamma_t \\
&= \chi\varphi(2\theta_u - \alpha(e+e_u))\mathbb{E}_t[\Delta z_{t+1}] + \chi(2\varphi\xi_u + 1 + \alpha\varphi)\mathbb{E}_t[\Delta x_{u,t+1}] + 2\chi\varphi\mu_u\mathbb{E}_t[\Delta\gamma_{t+1}] \\
&\quad + L_u\gamma_t + 2\chi\varphi\omega_u\Delta l_{u,t+1} + 2\chi\varphi\zeta_u\Delta m_{t+1}
\end{aligned} \tag{B.35}$$

Reorganizing equation (B.35),

$$\begin{aligned}
(\alpha - 2\alpha\chi\varphi\omega_u)\Delta l_{u,t+1} = 2\alpha\chi\varphi\zeta_u\Delta m_{t+1} - \alpha l_{ut} &+ (\alpha L_u + 2\chi\varphi\alpha\mu_u(\rho_\gamma - 1))\gamma_t \\
&+ \alpha\chi\varphi(\rho_z - 1)(2\theta_u - \alpha(e+e_u))z_t \\
&+ \alpha\chi(\rho_x - 1)(2\varphi\xi_u + 1 + \alpha\varphi)x_{ut}
\end{aligned} \tag{B.36}$$

Plugging this into equation (B.23), we can see that $\zeta_u = 0$, and ω_u satisfies

$$\omega_u = -\frac{\alpha^2}{\alpha - 2\alpha\chi\varphi\omega_u} \tag{B.37}$$

Define $\tilde{\omega}$ by $\omega_u = \alpha L_u\tilde{\omega}$. This solves

$$2\alpha\chi\varphi\tilde{\omega}^2 - \tilde{\omega} - 1 = 0 \tag{B.38}$$

This implies

$$\alpha - 2\alpha\chi\varphi\omega_u = -\frac{\alpha}{\tilde{\omega}} \tag{B.39}$$

Using this to to reorganize equation (B.36),

$$\begin{aligned}
\alpha\Delta l_{u,t+1} = \omega_u l_{ut} &- (\alpha L_u + 2\chi\varphi\alpha\mu_u(\rho_\gamma - 1))\tilde{\omega}\gamma_t \\
&- \alpha\chi\varphi\tilde{\omega}(\rho_z - 1)(2\theta_u - \alpha(e+e_u))z_t - \alpha\chi\tilde{\omega}(\rho_x - 1)(2\varphi\xi_u + 1 + \alpha\varphi)x_{ut}
\end{aligned} \tag{B.40}$$

This implies

$$\theta_u = \alpha \cdot \frac{e_u - \alpha\chi\varphi\tilde{\omega}(1-\rho_z)(e+e_u)}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.41}$$

$$\xi_u = -\alpha \cdot \frac{1-\chi\tilde{\omega}(1+\alpha\varphi)(1-\rho_x)}{1-2\chi\tilde{\omega}\alpha\varphi(1-\rho_x)} \tag{B.42}$$

$$\mu_u = -\frac{\omega_u}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_\gamma)} \tag{B.43}$$

For $\alpha\Delta l_{u,t+1}$, we now have

$$\alpha\Delta l_{u,t+1} = \omega_u l_{ut} + \mu_u\gamma_t + \frac{\alpha^2\chi\varphi\tilde{\omega}(e_u-e)(1-\rho_z)}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)}z_t + \frac{\alpha\chi\tilde{\omega}(1-\alpha\varphi)(1-\rho_x)}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_x)}x_{ut} \tag{B.44}$$

This implies

$$D_{l,z} = (e_u - e)\cdot\frac{\alpha\chi\varphi\tilde{\omega}(1-\rho_z)}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.45}$$

$$D_{l,x} = \frac{\chi\tilde{\omega}(1-\alpha\varphi)(1-\rho_x)}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_x)} \tag{B.46}$$

$$D_{l,l} = 1+\tilde{\omega} \tag{B.47}$$

$$D_{l,\gamma} = \frac{\mu_u}{\alpha} \tag{B.48}$$

Recall that

$$m_{t+1} = \tau(q_t - q_{ut}) = (\alpha(e+e_u) - 2\theta_u)\tau z_t - (\alpha+2\xi_u)\tau x_{ut} - 2\tau\mu_u\gamma_t - 2\tau\omega_u l_{ut} \tag{B.49}$$

This uses equation (B.24). The implication is that

$$D_{m,z} = \frac{\alpha\tau(e-e_u)}{1-2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{B.50}$$

$$D_{m,x} = \alpha\tau\cdot\frac{1-2\chi\tilde{\omega}(1-\rho_x)}{1-2\alpha\varphi\chi\tilde{\omega}(1-\rho_x)} \tag{B.51}$$

$$D_{m,l} = -2\tau\omega_u \tag{B.52}$$

$$D_{m,\gamma} = -2\tau\mu_u \tag{B.53}$$

This completes the proof. □

Proof of Lemma 1. The statement of the lemma was derived in the proof of Lemma 2. □

We next characterize the dynamics of country-specific price levels q_{it} and external debt l_{it}.

Lemma 3. *Country-specific price level coefficients in the linear equilibrium are*

$$\theta_i = \frac{\alpha e_i - \alpha\varphi\chi\omega\theta_u(1-\rho_z) + \alpha\omega D_{m,z}(M_i - L) - \alpha\varphi\chi\omega D_{l,z}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_z)} \tag{B.54}$$

$$\xi_i = \frac{\alpha\omega D_{m,x}(M_i - L) - \alpha\varphi\chi\omega D_{l,x}(\delta_i - \omega_u) - \alpha\chi\omega(1+\varphi\xi_u)(1-\rho_x)}{1 - \alpha\varphi\chi\omega(1-\rho_x)} \tag{B.55}$$

$$\mu_i = -\frac{\alpha\omega\chi\mathcal{C}_i + \mu_u\alpha\varphi\chi\omega(1-\rho_\gamma) + 2\alpha\omega\tau\mu_u(M_i - L) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \tag{B.56}$$

$$\omega_i = \alpha\omega \tag{B.57}$$

$$\zeta_i = -\alpha\omega M_i \tag{B.58}$$

$$\eta_i = -\frac{\alpha}{1 - \alpha\chi\varphi\omega(1-\rho_x)} \tag{B.59}$$

$$\delta_i = \omega_u \cdot \frac{\alpha\varphi\chi\omega\tilde{\omega} - 2\alpha\omega\tau(M_i - L)}{\alpha\varphi\chi\omega\tilde{\omega} + 1} \tag{B.60}$$

Here ω is the negative root of $\alpha\chi\varphi\omega^2 - \omega - 1 = 0$. The coefficients in the evolution of country-specific debt are

$$D_{l,i} = 1 + \omega \tag{B.61}$$

$$D_{l,x,i} = -\frac{\alpha\varphi\chi\omega(1-\rho_x)}{1 + \alpha\varphi\chi\omega(1-\rho_x)} \tag{B.62}$$

$$D_{l,z,i} = \frac{\theta_i - \alpha e_i + \alpha M_i D_{m,z}}{\alpha L_i} \tag{B.63}$$

$$D_{l,u,i} = \frac{\xi_i + \alpha M_i D_{m,x}}{\alpha} \tag{B.64}$$

$$D_{l,\gamma,i} = \frac{\mu_i + \alpha M_i D_{m,\gamma}}{\alpha} \tag{B.65}$$

$$D_{l,l,i} = \frac{\delta_i + \alpha M_i D_{m,l}}{\alpha} \tag{B.66}$$

$$D_{l,m,i} = -(\omega + 1)M_i \tag{B.67}$$

Proof of Lemma 3. Plugging the linear conjecture into equation (B.33),

$$\begin{aligned}(\alpha - \alpha\chi\varphi\omega_i)\Delta l_{i,t+1} = {} & \alpha L m_{t+1} + \alpha\chi\varphi(\theta_i - \theta_u)(\rho_z - 1)z_t - \alpha\chi(1 - \varphi(\xi_i - \xi_u))(\rho_x - 1)x_{ut} \\ & + \alpha\chi\varphi\eta_i(\rho_x - 1)x_{it} + \alpha\chi\varphi\zeta_i\Delta m_{t+1} + \alpha\chi\varphi(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + (\alpha\chi\varphi(\mu_i - \mu_u)(\rho_\gamma - 1) + \alpha(L_i - L))\gamma_t - \alpha l_{it}\end{aligned} \tag{B.68}$$

Plugging this back into the linear conjecture, we see that ω_i satisfies

$$\omega_i = -\frac{\alpha^2}{\alpha - \alpha\chi\varphi\omega_i} \tag{B.69}$$

Define ω by $\omega_i = \alpha L_i \omega$. This parameter solves

$$\alpha\chi\varphi\omega^2 - \omega - 1 = 0 \tag{B.70}$$

Using this, we can reorganize equation (B.68):

$$
\begin{aligned}
\alpha \Delta l_{i,t+1} = \omega_i l_{it} - \alpha\omega L m_{t+1} - \alpha\chi\varphi\omega\zeta_i \Delta m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\
+ \alpha\chi\varphi\omega(\theta_i - \theta_u)(1-\rho_z)z_t + \alpha\chi\omega(\varphi(\xi_i - \xi_u) - 1)(1-\rho_x)x_{ut} + \alpha\chi\varphi\omega\eta_i(1-\rho_x)x_{it} \\
+ (\alpha\chi\varphi\omega(1-\rho_\gamma)(\mu_i - \mu_u) - \alpha\omega\chi\mathcal{C}_i)\gamma_t
\end{aligned} \tag{B.71}
$$

Collecting the coefficients on m_t in the linear conjecture,

$$
\zeta_i = \alpha\chi\varphi\omega\zeta_i + \alpha M_i \tag{B.72}
$$

This means $\zeta_i = -\alpha\omega M_i$. Collecting the coefficients on m_{t+1} in the linear conjecture,

$$
-\alpha\omega L - \alpha\chi\varphi\omega\zeta_i - \alpha M_i = \alpha\omega(M_i - L) \tag{B.73}
$$

Collecting the terms on x_{it},

$$
\eta_i = -\frac{\alpha}{1 - \alpha\chi\varphi\omega(1-\rho_x)} \tag{B.74}
$$

This means

$$
\begin{aligned}
q_{it} = \omega_i l_{it} + \zeta_i m_t + \eta_i x_{it} + \alpha\omega(M_i - L)m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\
+ [\alpha e_i + \alpha\chi\varphi\omega(\theta_i - \theta_u)(1-\rho_z)]z_t + \alpha\chi\omega(\varphi(\xi_i - \xi_u) - 1)(1-\rho_x)x_{ut} \\
+ (\alpha\chi\varphi\omega(1-\rho_\gamma)(\mu_i - \mu_u) - \alpha\omega\chi\mathcal{C}_i)\gamma_t
\end{aligned} \tag{B.75}
$$

For δ_i, we have

$$
\delta_i = \omega_u \cdot \frac{\alpha\varphi\chi\omega\tilde{\omega} - 2\alpha\omega\tau(M_i - L)}{\alpha\varphi\chi\omega\tilde{\omega} + 1} \tag{B.76}
$$

For θ_i and ξ_i, we have

$$
\theta_i = \frac{\alpha e_i - \alpha\varphi\chi\omega\theta_u(1-\rho_z) + \alpha\omega D_{m,z}(M_i - L) - \alpha\varphi\chi\omega D_{l,z}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_z)} \tag{B.77}
$$

$$
\xi_i = \frac{\alpha\omega D_{m,x}(M_i - L) - \alpha\varphi\chi\omega D_{l,x}(\delta_i - \omega_u) - \alpha\chi\omega(1 + \varphi\xi_u)(1-\rho_x)}{1 - \alpha\varphi\chi\omega(1-\rho_x)} \tag{B.78}
$$

Finally, for μ_i, we have

$$
\begin{aligned}
\mu_i &= -\frac{\alpha\omega\chi\mathcal{C}_i + \mu_u\alpha\varphi\chi\omega(1-\rho_\gamma) + 2\alpha\omega\tau\mu_u(M_i - L) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \\
&= \mu_u - \frac{\alpha\omega\chi\mathcal{C}_i + \mu_u(1 + 2\alpha\omega\tau(M_i - L)) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \\
&= \mu_u - \frac{\alpha\omega\chi\mathcal{C}_i}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} - \frac{\mu_u}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\omega\chi\tilde{\omega}}
\end{aligned} \tag{B.79}
$$

To characterize the evolution of l_{it}, use equation (B.22):

$$\alpha l_{i,t+1} = \alpha l_{it} + q_{it} - \alpha e_i z_t + \alpha x_{it} + \alpha M_i m_{t+1} - \alpha M_i m_t \tag{B.80}$$

This implies

$$D_{l,z,i} = \frac{\theta_i - \alpha e_i + \alpha M_i D_{m,z}}{\alpha} \tag{B.81}$$
$$D_{l,u,i} = \frac{\xi_i + \alpha M_i D_{m,x}}{\alpha} \tag{B.82}$$
$$D_{l,\gamma,i} = \frac{\mu_i + \alpha M_i D_{m,\gamma}}{\alpha} \tag{B.83}$$
$$D_{l,i} = \omega + 1 \tag{B.84}$$
$$D_{l,m,i} = -(\omega + 1)M_i \tag{B.85}$$
$$D_{l,l,i} = \frac{\delta_i + \alpha M_i D_{m,l}}{\alpha} \tag{B.86}$$
$$D_{l,x,i} = \frac{\alpha + \eta_i}{\alpha} = -\frac{\alpha\varphi\chi\omega(1-\rho_x)}{1 + \alpha\varphi\chi\omega(1-\rho_x)} \tag{B.87}$$

This completes the proof. □

Proof of Proposition 1. Consider the within-period optimality conditions:

$$C_{it}^T = (1-\alpha)Q_{it}C_{it} \tag{B.88}$$
$$P_{it}^N C_{it}^N = \alpha Q_{it} C_{it} \tag{B.89}$$

Using $Q_{it} = (P_{it}^N)^\alpha$ and log-linearizing,

$$c_{it}^T = q_{it} + c_{it} \tag{B.90}$$
$$c_{it}^N = \frac{\alpha - 1}{\alpha} q_{it} + c_{it} \tag{B.91}$$

From this and $c_{it}^N = x_{it}$, it follows that

$$c_{it} = \frac{1-\alpha}{\alpha} q_{it} + x_{it} \tag{B.92}$$
$$c_{it}^T = \frac{1}{\alpha} q_{it} + x_{it} \tag{B.93}$$

Now consider the log-linearization of $\Lambda_{i,t+1}$ and $\Lambda_{u,t+1}$:

$$\lambda_{i,t+1} = (1-\rho)\Delta c_{i,t+1} - \Delta c_{i,t+1}^T = -\left(1 + \frac{\rho(1-\alpha)}{\alpha}\right)\Delta q_{i,t+1} - \rho \Delta x_{i,t+1} \tag{B.94}$$
$$\lambda_{u,t+1} = (1-\rho)\Delta c_{u,t+1} - \Delta c_{u,t+1}^T = -\left(1 + \frac{\rho(1-\alpha)}{\alpha}\right)\Delta q_{u,t+1} - \rho \Delta x_{u,t+1} \tag{B.95}$$

Denote $\varphi = 1 + (1-\alpha)\rho/\alpha$. Integrating,

$$\begin{aligned}\lambda^{\text{ave}}_{t+1} &= -\varphi\left(\Delta q_{u,t+1} + \int \Delta q_{i,t+1}\mathrm{d}i\right) - \rho\Delta x_{u,t+1} - \rho\int \Delta x_{i,t+1}\mathrm{d}i \\ &= -\varphi((e+e_u)\alpha\Delta z_{t+1} - \alpha\Delta x_{u,t+1}) - \rho\Delta x_{u,t+1} = -(e+e_u)\alpha\varphi\Delta z_{t+1} - (\rho-1)\alpha\Delta x_{u,t+1}\end{aligned} \tag{B.96}$$

This uses the fact that $q_{ut} + \int q_{it}\mathrm{d}i = (e+e_u)\alpha z_t - \alpha x_{ut}$. This completes the proof. □

We next show the stationarity properties of exchange rates.

Proposition 5. *Endogenous states* (l_{ut}, l_{it}, m_t) *evolve as*

$$\begin{aligned} l_{u,t+1} &= (1+\tilde{\omega})l_{ut} + F_u(z_t, \gamma_t, x_{ut}) \\ l_{i,t+1} &= (1+\omega)l_{it} + F_i(z_t, \gamma_t, x_{ut}, x_{it}, l_{ut}, m_t) \\ m_{t+1} &= F_m(z_t, \gamma_t, x_{ut}, l_{ut}) \end{aligned}$$

Here the coefficients $\tilde{\omega}$ *and* ω *satisfy* $-1 < \omega < \tilde{\omega} < 0$ *and both converge to zero if* $\chi \longrightarrow \infty$.

If exogenous processes are stationary, endogenous processes, including exchange rates, are stationary too. The reason is the portfolio management cost. The intermediary's portfolio gravitates towards a fixed benchmark, which makes the economy return to a stable distribution of investment positions in the long run. The US position has a persistence coefficient $1+\tilde{\omega} \in (0,1)$, every other country's position has persistence $1+\omega \in (0,1)$, and the global reserve position does not depend on its own past values. Inflows into the US are more persistent than inflows into other countries due to their feedback: they affect the US dollar exchange rate, which further drives financial flows. Both persistence coefficients become a unit root if portfolio frictions disappear. This relates our portfolio costs to stationarity-inducing devices in the spirit of Schmitt-Grohé and Uribe (2003).

Proof of Proposition 5. Directly follows from the statements of Lemma 2 and Lemma 3. □

B.1 Small portfolio management costs, transitory risk aversion shocks

We next characterize the limit of small portfolio costs and large reserve assets under $e_u = e$ and a different assumption on the persistence of risk aversion shocks: $\rho_\gamma = 0$. We still assume $\rho_z = \rho_x = 1$ for tractability and keep the notation for our auxiliary variables:

$$\phi \equiv \sqrt{\frac{\alpha}{\varphi}} = \alpha\sqrt{\frac{1}{\alpha + \rho(1-\alpha)}} \tag{B.97}$$

$$\psi_i \equiv \phi\tau(m_i - l) - 1 \tag{B.98}$$

We characterize bilateral exchange rate depreciation against the US for all countries, our currency factors, and loadings.

Lemma 4. *Suppose* $\rho_x = \rho_z = 1$, $\rho_\gamma = 0$, *and* $M_i = \sqrt{\chi}m_i$. *Then, in the limit* $\chi \longrightarrow \infty$, *bilateral exchange rate depreciation against the dollar is*

$$\Delta s_{i,t+1} = \bar{\theta}_i\Delta z_{t+1} + \bar{\xi}_i\Delta x_{u,t+1} + \bar{\mu}_i\Delta\gamma_{t+1} - \alpha\Delta x_{i,t+1} \tag{B.99}$$

Here the coefficients are

$$\bar{\theta}_i = \alpha(e_u - e_i) \tag{B.100}$$
$$\bar{\xi}_i = \alpha\psi_i \tag{B.101}$$
$$\bar{\mu}_i = \Gamma_z\alpha^2(e_u + e)(e - e_i) + \Gamma_x\phi^2(\rho - 1)\psi_i \tag{B.102}$$

Proof. US coefficients are

$$\theta_u = \alpha e_u \tag{B.103}$$
$$\xi_u = -\alpha \tag{B.104}$$
$$\mu_u = \tilde{\omega}\omega_u L_u \tag{B.105}$$

Coefficients of individual countries are

$$\theta_i = \alpha e_i \tag{B.106}$$
$$\xi_i = \alpha^2\omega\tau(M_i - L) \tag{B.107}$$
$$\eta_i = -\alpha \tag{B.108}$$
$$\zeta_i = -\alpha\omega M_i \tag{B.109}$$
$$\delta_i = \omega_u - \omega_u \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\chi\omega\tilde{\omega}} \tag{B.110}$$
$$\mu_i = \mu_u - \frac{\mu_u}{1 - \alpha\varphi\chi\omega} \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\chi\omega\tilde{\omega}} - \frac{\alpha\omega\chi\mathcal{C}_i}{1 - \alpha\varphi\chi\omega} \tag{B.111}$$

US debt coefficients are

$$D_{l,z} = 0 \tag{B.112}$$
$$D_{l,x} = 0 \tag{B.113}$$
$$D_{l,l} = 1 + \tilde{\omega} \tag{B.114}$$
$$D_{l,\gamma} = \tilde{\omega}^2 \tag{B.115}$$

Total reserve demand coefficients are

$$D_{m,z} = 0 \tag{B.116}$$
$$D_{m,x} = \alpha\tau \tag{B.117}$$
$$D_{m,l} = -2\tau\omega_u \tag{B.118}$$
$$D_{m,\gamma} = -2\tau\mu_u \tag{B.119}$$

Assume $\chi \longrightarrow \infty$ and $\chi^{-\frac{1}{2}}M_i \longrightarrow m_i$ for all i. In this limit,

$$\tilde{\omega} \sim -\frac{1}{\sqrt{2\alpha\varphi\chi}} \tag{B.120}$$
$$\omega \sim -\frac{1}{\sqrt{\alpha\varphi\chi}} \tag{B.121}$$

According to Proposition 1,

$$\mathcal{C}_i \longrightarrow \Gamma_z \alpha^2 \varphi(e+e_u)(e_u-e_i) + \Gamma_x \alpha^2(\rho-1)\psi_i \equiv c_i \tag{B.122}$$

$$\frac{L_u}{\chi} \longrightarrow \Gamma_x \alpha^2(\rho-1) \equiv l_u \tag{B.123}$$

The limits of the coefficients are

$$\chi^{-\frac{1}{2}}\omega_u \longrightarrow -\frac{\phi}{\sqrt{2}} \tag{B.124}$$

$$\mu_u \longrightarrow \frac{l_u}{2\varphi} \tag{B.125}$$

$$\theta_i \longrightarrow \alpha e_i \tag{B.126}$$

$$\xi_i \longrightarrow -\alpha\tau\phi(m_i-l) \tag{B.127}$$

$$\zeta_i \longrightarrow \phi m_i \tag{B.128}$$

$$\chi^{-\frac{1}{2}}\omega_i \longrightarrow -\phi \tag{B.129}$$

$$\chi^{-\frac{1}{2}}\delta_i \longrightarrow -\frac{\phi}{\sqrt{2}} \cdot \frac{1+2\sqrt{2}\tau\phi(m_i-l)}{1+\sqrt{2}} \tag{B.130}$$

$$\mu_i \longrightarrow \mu_u + \frac{\mathcal{C}_i}{\varphi} \tag{B.131}$$

It follows that $\Delta l_{u,t+1} = O(1)$. Now consider $\alpha \Delta l_{t+1}$ and $\alpha M_i \Delta m_{t+1}$:

$$\begin{aligned} \alpha l_{i,t+1} &= \alpha\omega l_{it} + (\alpha^2\tau M_i + \alpha^2\omega\tau(M_i-L))x_{ut} - \alpha(\omega+1)M_i m_t + (\mu_i - 2\alpha\tau\mu_u M_i)\gamma_t \\ &\quad + (\delta_i - 2\tau\omega_u M_i)l_{ut} && \text{(B.132)} \\ \alpha M_i \Delta m_{t+1} &= \alpha^2\tau M_i x_{ut} - 2\alpha\tau\mu_u M_i\gamma_t - 2\alpha\tau\omega_u M_i l_{ut} - \alpha M_i m_t && \text{(B.133)} \end{aligned}$$

From this, it follows that

$$\alpha\Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} = \alpha^2\omega\tau(M_i-L)x_{ut} + \mu_i\gamma_t + \delta_i l_{ut} + \omega(\alpha l_{it} - \alpha M_i m_t) = O(1) \tag{B.134}$$

Plug this into q_{it}:

$$\begin{aligned} q_{it} &= \alpha e_i z_t - \alpha x_{it} + \alpha\Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} \\ &= \alpha e_i z_t - \alpha x_{it} + \alpha^2\omega\tau(M_i-L)x_{ut} + \mu_i\gamma_t + \delta_i l_{ut} + \omega(\alpha l_{it} - \alpha M_i m_t) && \text{(B.135)} \\ &= \alpha e_i z_t - \alpha x_{it} - \alpha\phi\tau(m_i-l)x_{ut} + \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)\gamma_t + O(\chi^{-\frac{1}{2}}) && \text{(B.136)} \end{aligned}$$

Now consider q_{ut}:

$$q_{ut} = \alpha e z_t - \alpha x_{ut} + \mu_u\gamma_t + \omega_u l_{ut} = \alpha e z_t - \alpha x_{ut} + \frac{l_u}{2\varphi}\gamma_t + O(\chi^{-\frac{1}{2}}) \tag{B.137}$$

It follows that

$$q_{it} - q_{ut} = (e_i - e_u)\alpha z_t - \alpha\psi_i x_{ut} - \alpha x_{it} + \frac{c_i}{\varphi}\gamma_t + O(\chi^{-\frac{1}{2}}) \tag{B.138}$$

Defining the depreciation of currency i against the dollar as $\Delta s_{i,t+1} = \Delta q_{u,t+1} - \Delta q_{i,t+1}$,

$$\Delta s_{i,t+1} \longrightarrow (e_u - e_i)\alpha\Delta z_{t+1} + \alpha\psi_i\Delta x_{u,t+1} + \alpha\Delta x_{i,t+1} + \kappa_i\Delta\gamma_{t+1} \tag{B.139}$$

Here κ_i is

$$\kappa_i \equiv \frac{c_i}{\varphi} = \Gamma_z\alpha^2(e + e_u)(e_i - e_u) + \Gamma_x\alpha\phi^2(\rho - 1)(1 - \tau\phi(m_i - l)) \tag{B.140}$$

This completes the proof. □

Proof of Proposition 2. Follows from Lemma 4.

Proof of Proposition 3. Assume that $e_u = e$ and that (e_i, m_i) are independent in the cross-section. Then, integrating $\Delta s_{i,t+1}$,

$$\text{doll}_{t+1} = -\alpha\Delta x_{u,t+1} + \Gamma_x\alpha\phi^2(\rho - 1)\Delta\gamma_{t+1} \tag{B.141}$$

Integrating over $e > \text{med}\{e\}$ and $e < \text{med}\{e\}$ and subtracting,

$$\text{comm}_{t+1} = (e_L - e_H)\alpha\Delta z_{t+1} - 2\alpha^2\Gamma_z(e_L - e_H)\Delta\gamma_{t+1} \tag{B.142}$$

The expression for $\Delta s_{i,t+1}$ then follows from plugging these into Proposition 2. □

Proof of Proposition 4. This functional form of the factor moments follows directly from Proposition 3 that expresses the factors in terms of the shocks. The mapping from $(\sigma_x^2, \sigma_z^2, (\Gamma\sigma_\gamma)^2)$ to the second moments is not trivially invertible because it is not linear. However, we can still express these parameters:

$$\sigma_z^2 = \frac{V_c V_d - C_{f,d}^2}{\alpha^2(e_H - e_L)^2 V_d + 2\alpha^3 e(e_H - e_L)(\rho - 1)^{-1}C_{f,d}} \tag{B.143}$$

$$\sigma_x^2 = \frac{V_c V_d - C_{f,d}^2}{\alpha^2 V_c + \alpha\phi^2(e_H - e_L)(\rho - 1)(2e)^{-1}C_{f,d}} \tag{B.144}$$

$$(\Gamma\sigma_\gamma)^2 = C_{fd} \cdot \frac{(\alpha V_c + \phi^2(e_H - e_L)(\rho - 1)(2e)^{-1}C_{f,d}) \cdot ((e_H - e_L)V_d + 2e\alpha(\rho - 1)^{-1}C_{f,d})}{\phi^2 e(\rho - 1)(V_c V_d - C_{f,d}^2)^2} \tag{B.145}$$

Here the notation is $V_d = \mathbb{V}[\text{doll}_t]$, $V_f = \mathbb{V}[\text{comm}_t]$, and $C_{fd} = \mathbb{C}[\text{doll}_t, \text{comm}_t]$. □

We next formulate the factor structure of exchange rates relative to an arbitrary base j. Fix a base currency j and let $\Delta s_{ij,t+1}$ be the exchange rate depreciation of currency i relative to j. Define the base currency factor $\text{base}_{j,t+1}$ as

$$\text{base}_{j,t+1} = \int \Delta s_{ij,t+1} \text{d}i \tag{B.146}$$

We have the following result.

Proposition 6. *Exchange rates relative to a currency j have the following factor structure:*

$$\Delta s_{ij,t+1} = \frac{e_i - e}{e_H - e_L} \cdot comm_{t+1} - \tau\phi(m_i - m) \cdot doll_{t+1} + 1 \cdot base_{j,t+1} + \alpha\Delta x_{i,t+1} \tag{B.147}$$

The base currency factor $base_{j,t+1}$ is given by

$$base_{j,t+1} = \frac{e - e_j}{e_H - e_L} \cdot comm_{t+1} - \tau\phi(m_j - m) \cdot doll_{t+1} - \alpha\Delta x_{j,t+1} \tag{B.148}$$

Proof. Take $\Delta s_{ij,t+1} \equiv \Delta s_{i,t+1} - \Delta s_{j,t+1}$. According to Proposition 3,

$$\Delta s_{ij,t+1} = \frac{e_i - e_j}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m_j - m_i) \cdot \text{doll}_{t+1} + \alpha\Delta x_{i,t+1} - \alpha\Delta x_{j,t+1} \tag{B.149}$$

Integrating this over i,

$$\text{base}_{j,t+1} = \frac{e - e_j}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m_j - m) \cdot \text{doll}_{t+1} - \alpha\Delta x_{j,t+1} \tag{B.150}$$

Plugging this back,

$$\Delta s_{ij,t+1} = \text{base}_{j,t+1} + \frac{e_i - e}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m - m_i) \cdot \text{doll}_{t+1} + \alpha\Delta x_{i,t+1} \tag{B.151}$$

This completes the proof. □

C Details of the quantitative analysis

Estimation of τ. We estimate τ by running the following regression implied by our model:

$$\Delta m_{t+1} = \alpha_{\text{reserves}} + \tau_{\text{reserves}}(\Delta q_t - \Delta q_{ut}) + \epsilon_{t+1}$$

We use the logarithm of the broad dollar index DXY as a measure of $q_{ut} - q_t$ and the logarithm of total reserves expressed in dollars as m_t. After estimating $\tau_{\text{reserves}} = 0.9$ (standard error of 0.15), we adjust it for the share of USD in reserves reported by IMF (equal to 0.6 on average). For simplicity, we assume that the non-dollar part of reserves is denominated in the broad basket of currencies and is not actively managed in response to the change in the broad dollar. This means that the USD value of the non-dollar part of reserves changes one-for-one with the change in DXY. If the average share of the dollar in reserves is S_{USD}, under this assumption the relationship between the recovered τ_{reserves} and τ in our model is

$$\tau_{\text{reserves}} = (1 - S_{\text{USD}}) \cdot 1 + S_{USD} \cdot \tau$$

This leads to $\tau = 0.83$.

Estimating (e_i, m_i). We use the model of a commodity-based global economy presented in Appendix D to inform the mapping from the share of commodities in exports to e_i. Specifically, if S_i is country i's share of primary inputs in exports, we recover its e_i as $e_i = S_i/(1 - S_i)$. We read

m_i from the CPIS, where it corresponds to the USD share of portfolio assets. Where the data in CPIS are unavailable, we impute the USD share using predicted values based on treasury holdings in TIC. We then re-center m_i around m, which is a parameter set in internal calibration.

Variance decompositions. For local price changes in i and u and for the bilateral depreciation of i's currency relative to u, we have

$$\Delta q_{i,t+1} = \alpha e_i \sigma_z \epsilon_{z,t+1} - \alpha\phi\tau(m_i - l)\sigma_x \epsilon_{x,t+1} + \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) - \alpha\sigma_x\epsilon_{i,t+1} \tag{C.1}$$

$$\Delta q_{u,t+1} = \alpha e \sigma_z \epsilon_{z,t+1} - \alpha\sigma_x\epsilon_{x,t+1} + \frac{l_u}{2\varphi}\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) \tag{C.2}$$

$$\Delta s_{i,t+1} = (e - e_i)\alpha\sigma_z\epsilon_{z,t+1} + \alpha\psi_i\sigma_x\epsilon_{x,t+1} - \frac{c_i}{\varphi}\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) + \alpha\sigma_x\epsilon_{i,t+1} \tag{C.3}$$

Here

$$\psi_i = \phi\tau(m_i - l) - 1 \tag{C.4}$$
$$c_i = 2\sigma_z^2\alpha^2\varphi e(e - e_i) + \sigma_x^2\alpha^2(\rho - 1)\psi_i \tag{C.5}$$
$$l_u = \sigma_x^2\alpha^2(\rho - 1) \tag{C.6}$$

Consumption growth is

$$\begin{aligned}\Delta c_{i,t+1} &= \frac{1-\alpha}{\alpha}\Delta q_{i,t+1} + \Delta x_{i,t+1} \\ &= (1-\alpha)e_i\sigma_z\epsilon_{z,t+1} - (1-\alpha)\phi\tau(m_i - l)\sigma_x\epsilon_{x,t+1} + \frac{1-\alpha}{\alpha}\left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) \\ &\quad - \alpha\sigma_x\epsilon_{i,t+1}\end{aligned} \tag{C.7}$$

For the relative consumption growth, we have

$$\Delta c_{i,t+1} - \Delta c_{u,t+1} = \Delta x_{i,t+1} - \Delta x_{u,t+1} - \frac{1-\alpha}{\alpha}\Delta s_{i,t+1} \tag{C.8}$$

$$\begin{aligned}&= (1-\alpha)(e_i - e)\sigma_z\epsilon_{z,t+1} - (1 + (1-\alpha)\psi_i)\sigma_x\epsilon_{x,t+1} \\ &\quad + \frac{(1-\alpha)c_i}{\alpha\varphi}\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) + \alpha\sigma_x\epsilon_{i,t+1}\end{aligned} \tag{C.9}$$

Compute the risk-sharing covariance $C_i \equiv \mathbb{C}[\Delta c_{i,t+1} - \Delta c_{u,t+1}, \Delta s_{i,t+1}]$:

$$C_i = \alpha^2\sigma_x^2 - \alpha\psi_i(1 + (1-\alpha)\psi_i)\sigma_x^2 - \alpha(1-\alpha)(e_i - e)^2\sigma_z^2 - \frac{2(1-\alpha)c_i^2}{\alpha\varphi^2}\sigma_\gamma^2 \tag{C.10}$$

We can then compute the correlation $\rho_i \equiv C_i/\text{std}(\Delta s_{i,t+1})/\text{std}(\Delta c_{i,t+1} - \Delta c_{u,t+1})$.

Another moment of interest is the relative volatility of exchange rate appreciation and consumption growth. The variance of consumption growth and exchange rate appreciation are

$$\mathbb{V}[\Delta c_{i,t+1}] = (1-\alpha)^2 e_i^2 \sigma_z^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)^2 \cdot 2\sigma_\gamma^2 + \alpha^2 \sigma_x^2 + (1-\alpha)^2 \phi^2 \tau^2 (m_i - l)^2 \sigma_x^2 \tag{C.11}$$

$$\mathbb{V}[\Delta s_{i,t+1}] = \alpha^2 e_i^2 \sigma_z^2 + \left(\frac{c_i}{\varphi}\right)^2 \cdot 2\sigma_\gamma^2 + \alpha^2 \sigma_x^2 + \alpha^2 \psi_i^2 \sigma_x^2 \tag{C.12}$$

The moment of interest is $rat_i \equiv \mathbb{V}[\Delta s_{i,t+1}]/\mathbb{V}[\Delta c_{i,t+1}]$.

The variance contributions to $\Delta q_{i,t+1}$ are

$$V_{q,i,z} = \alpha^2 e_i^2 \sigma_z^2 \tag{C.13}$$

$$V_{q,i,x} = \alpha^2 \phi^2 \tau^2 (m_i - l)^2 \sigma_x^2 \tag{C.14}$$

$$V_{q,i,\gamma} = 2 \cdot \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)^2 \sigma_\gamma^2 \tag{C.15}$$

$$V_{q,i,\epsilon} = \alpha^2 \sigma_x^2 \tag{C.16}$$

The variance contributions to $\Delta q_{u,t+1}$ are

$$V_{q,u,z} = \alpha^2 e^2 \sigma_z^2 \tag{C.17}$$

$$V_{q,u,x} = \alpha^2 \sigma_x^2 \tag{C.18}$$

$$V_{q,u,\gamma} = 2 \cdot \frac{l_u^2}{4\varphi^2} \sigma_\gamma^2 \tag{C.19}$$

$$\tag{C.20}$$

The variance contributions to $\Delta s_{i,t+1}$ are

$$V_{s,i,z} = \alpha^2 (e_i - e)^2 \sigma_z^2 \tag{C.21}$$

$$V_{s,i,x} = \alpha^2 \psi_i^2 \sigma_x^2 \tag{C.22}$$

$$V_{s,i,\gamma} = 2 \cdot \left(\frac{c_i}{\varphi}\right)^2 \sigma_\gamma^2 \tag{C.23}$$

$$V_{s,i,\epsilon} = \alpha^2 \sigma_x^2 \tag{C.24}$$

The variance shares are the contributions scaled by their sum. The standard deviation of currency i's depreciation is

$$std_i = \sqrt{V_{s,i,z} + V_{s,i,x} + V_{s,i,\gamma} + V_{s,i,\epsilon}} \tag{C.25}$$

The factor R-squared for currency i is

$$rsq_i = \frac{V_{s,i,z} + V_{s,i,x} + V_{s,i,\gamma}}{V_{s,i,z} + V_{s,i,x} + V_{s,i,\gamma} + V_{s,i,\epsilon}} \tag{C.26}$$

D A commodity-based economy

In this section, we present a simple model of a production economy that underlies the endowment economy of our main exposition. In this version of the mode, commodity intensity is well defined, and we map it into the tradable output exposure parameter e_i in our baseline model. We also show the mapping between e_i and the steady-state export share of commodities.

Each country produces two final goods: tradable and non-tradable. It also exports non-differentiated commodities and uses labor. Non-tradable goods are simply endowed at the level Y_{it}^N. Tradable goods are produced by a representative firm using labor H_{it} and commodities X_{it}:

$$Y_{it}^T = z_i H_{it} + Z_t X_{it} \tag{D.1}$$

Here Z_t is the global productivity shock with $\mathbb{E}[Z_t] = 1$, and z_i is the country-specific labor productivity. The price of the commodity input is M_t, the wage is W_{it}. The firm solves

$$\max_{H_{it}, X_{it}} \; P_{it} Y_{it}^T - W_{it} H_{it} - M_t X_{it} \tag{D.2}$$

An interior solution implies that, for all i,

$$P_{it} Z_t = M_t \tag{D.3}$$

$$P_{it} z_i = W_{it} \tag{D.4}$$

Countries consume a Cobb-Douglas aggregator of their own non-tradable good and all tradable goods, putting a mass point on their own good:

$$\ln(C_{it}) = \theta \log(C_{it}^N) + (1-\theta)\alpha \log(C_{iit}^T) + (1-\theta)(1-\alpha) \int \log(C_{ijt}^T) dj - \nu \tag{D.5}$$

Here ν is a normalization constant:

$$\nu = \theta \log(\theta) + (1-\theta)\alpha \log((1-\theta)\alpha) + (1-\theta)(1-\alpha) \log((1-\theta)(1-\alpha)) \tag{D.6}$$

This leads to the following demand:

$$C_{it}^N = \theta \frac{Q_{it} C_{it}}{P_{it}^N} \tag{D.7}$$

$$C_{iit}^T = (1-\theta)\alpha \frac{Q_{it} C_{it}}{P_{it}^T} \tag{D.8}$$

$$C_{ijt}^T = (1-\theta)(1-\alpha) \frac{Q_{it} C_{it}}{P_{jt}^T} \tag{D.9}$$

Here the price index is

$$Q_{it} = \theta \log(P_{it}^N) + (1-\theta)\alpha \log(P_{it}^T) + (1-\theta)(1-\alpha) \int \log(P_{jt}^T) dj \tag{D.10}$$

Normalize

$$\int \log(P_{jt}^T)dj = 0 \tag{D.11}$$

Since we have $P_{it}Z_t = M_t$ for all i, which means P_{it} are the same for all i, we have $P_{it} = 1$ for all i. The individual country's price index is then $Q_{it} = (P_{it}^N)^\theta$. Moreover, $M_t = Z_t$.

Country i has a commodity endowment e_i. We parameterize it so that $e_i + z_i = 1$ for all i: natural advantage in commodities in some countries is balanced out by a natural advantage in producing final goods in others. This will allow us to arrive at a symmetric steady state. Country i's consolidated budget constraint is

$$\begin{aligned} Q_{it}C_{it} = P_{it}^N Y_{it}^N + P_{it}^T Y_{it}^T + M_t(e_i - X_{it}) \\ + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \end{aligned} \tag{D.12}$$

Here $B_{i,t+1}$ is borrowing in local currency bonds for the next period, B_{it} is the debt at the start of this period, T_{it} are net transfers from the government, and Π_{it} are the profit rebates from the intermediaries. Using the fact that $P_{it}^N Y_{it}^N = \theta Q_{it}C_{it}$, the fact that $Q_{it} = (P_{it}^N)^\theta$, assuming $Y_{it}^N = \theta/(1-\theta)$, the fact that $P_{it}^T Y_{it}^T = z_i H_{it} + Z_t X_{it}$, and the fact that $M_t = Z_t$,

$$(Q_{it})^\theta = z_i + Z_t e_i + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \tag{D.13}$$

In the steady state, $Z_t = 1$. The profit rebate rule and reserve management rules from the baseline model imply $R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} = 0$. We also have $z_i + e_i = 1$. All of this implies $Q_i = 1$. To arrive at the steady-state export shares in commodities, use the market clearing for traded goods:

$$Y_{it}^T = C_{iit}^T + \int C_{jit}^T dj \tag{D.14}$$

This leads to the following expression:

$$Y_{it}^T = \alpha(1-\theta)Q_{it}C_{it} + (1-\alpha)(1-\theta)\int Q_{jt}C_{jt}dj \tag{D.15}$$

Since $Q_i = P_i^N = 1$ in the steady state, using $P_{it}^N Y_{it}^N = \theta Q_{it}C_{it}$ leads to $C_i = 1/(1-\theta)$, so $Y_i^T = 1$ for all i. The steady-state export share of commodities is then

$$S_i = \frac{e_i}{1+e_i} \tag{D.16}$$

Denoting $z_t = Z_t - 1$ and using $e_i + z_i = 1$, we can transform the budget constraint in equation (D.12) into

$$C_{it}^T + P_{it}^N C_{it}^N = 1 + e_i z_t + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \tag{D.17}$$