A Model Solution and Proofs

This appendix describes the solution of the problem of the final good producers, the worker's problem, and contains proofs of the propositions in the text.

A.1 Details of the retailer and worker problems

Retailers. The solution to the problem in equation (11) is a standard set of factor demands:

$$q_{jt}^N = (1-\eta) q_{jt} \left(\frac{m_{jt}}{p_t^N} \right)^{\theta} \tag{43}$$

$$q_{jt}^T = \eta q_{jt} \left(\frac{m_{jt}}{p_t^T} \right)^{\theta} \tag{44}$$

$$q_{jt}^F = (1-\alpha) q_{jt}^T \left(\frac{p_t^T}{e_t \tilde{p}^F} \right)^{\theta_g} \tag{45}$$

$$q_{jt}^H = \alpha q_{jt}^T \left(\frac{p_t^T}{p_t^H} \right)^{\theta_g} \tag{46}$$

The real marginal cost m_{jt} of a retailer j is given by

$$m_{jt} q_{jt} = p_t^N q_{jt}^N + p_t^H q_{jt}^H + e_t \tilde{p}^F q_{jt}^F \tag{47}$$

After optimization, it satisfies equation (12) and hence is the same for all retailers, $m_{jt} = m_t$.

Retailers sell their final bundles to workers and the government. The elasticity of substitution between retailers is θ_r. The bundle that each worker or the government demands is a CES aggregate of her demand $\{c_{jt}\}_j$ or $\{g_{jt}\}_j$ for each variety j:

$$(c_t)^{1-\frac{1}{\theta_r}} = \int_0^1 (c_{jt})^{1-\frac{1}{\theta_r}} dj \tag{48}$$

$$(g_t)^{1-\frac{1}{\theta_r}} = \int_0^1 (g_{jt})^{1-\frac{1}{\theta_r}} dj \tag{49}$$

This leads to standard demand functions:

$$c_{jt} = c_t \left(\frac{P_t}{P_{jt}} \right)^{\theta_r} \tag{50}$$

$$g_{jt} = g_t \left(\frac{P_t}{P_{jt}} \right)^{\theta_r} \tag{51}$$

Aggregating this leads to equation (16).

I next describe the dynamic problem of the retailer. A retailer j has quadratic adjustment cost that is proportional to the total sales of the final bundles $P_t q_t$:

$$C_t(\pi_{jt}) = \frac{\hat{\phi} P_t q_t}{2} (\pi_{jt})^2 \tag{52}$$

where the individual inflation is $\pi_{jt} = \dot{P}_{jt}/P_{jt}$. I assume these costs are virtual and do not enter the resource constraint of the economy. Retailers receive a revenue subsidy $\hat{\tau}$ setting the steady-state markup to one. The physical profits are rebated to the government. The value of a retailer j at time t is

$$J(P_{jt}, t) = \max_{\pi_{js}} \int_t^{\infty} e^{-\hat{\rho}(s-t)} \left[\frac{(1+\hat{\tau})P_{js} - M_s}{P_s} q_{js} - \frac{\hat{\phi} q_s}{2} (\pi_{js})^2 \right] ds \tag{53}$$

subject to equation (16) and $\dot{P}_{jt} = \pi_{jt} P_{jt}$ for all t. They maximize the present discounted value of the stream of real profits net of adjustment cost.

To make the time horizon of the retail managers infinitely short, I impose $\hat{\phi} = \phi\Delta$ and $\hat{\rho} = \rho/\Delta$, where Δ is small. I first solve the problem for finite Δ and then take the limit $\Delta \to 0$.

The HJB equation for this problem is

$$\hat{\rho} J(P_{jt}, t) - \partial_t J(P_{jt}, t) = \frac{(1+\hat{\tau})P_{jt} - M_t}{P_t} q_{jt} + \max_{\pi} \left(-\frac{\hat{\phi} q_t}{2} \pi^2 + \pi P_{jt} \partial_p J(P_{jt}, t) \right) \tag{54}$$

The first-order condition and the envelope theorem lead to

$$P_{jt} \partial_p J(P_{jt}, t) = \pi_{jt} \hat{\phi} q_t \tag{55}$$

$$\hat{\rho} \partial_p J(P_{jt}, t) - \partial^2_{tp} J(P_{jt}, t) = \theta_r \frac{q_{jt}}{P_t} \left(\frac{M_t}{P_{jt}} - (1+\hat{\tau}) \frac{\theta_r - 1}{\theta_r} \right) + \pi_{jt} (\partial_p J(P_{jt}, t) + P_{jt} \partial^2_{pp} J(P_{jt}, t)) \tag{56}$$

Taking the time derivative of equation (55) and plugging into equation (56),

$$\hat{\rho} \hat{\phi} \pi_{jt} q_t = \theta_r q_{jt} \frac{P_{jt}}{P_t} \left(\frac{M_t}{P_{jt}} - (1+\hat{\tau}) \frac{\theta_r - 1}{\theta_r} \right) + \hat{\phi} (\dot{\pi}_{jt} q_t + \pi_{jt} \dot{q}_t) \tag{57}$$

Invoking symmetry across j,

$$\hat{\rho} \hat{\phi} \pi_t = \theta_r (m_t - \bar{m}) + \hat{\phi} \left(\dot{\pi}_t + \frac{\dot{q}_t}{q_t} \right) \tag{58}$$

Here $\bar{m} = (1+\hat{\tau})(\theta_r - 1)/\theta_r$ is the real marginal cost in the static optimum and hence in the steady state. With the subsidy set at $\hat{\tau} = 1/(\theta_r - 1)$, it is equal to 1.

I now take the limit $\Delta \to 0$, so $\hat{\rho} \to \infty$, $\hat{\phi} \to 0$, and $\hat{\rho}\hat{\phi} \to \rho\phi$. In this limit, the managers of the retail firms have extremely short horizon but the cost of price adjustment is very low, so they still have incentives to change prices. In effect, they trade off adjustment costs against losses from suboptimal pricing in the next instant. The Phillips curve takes the following form:

$$\rho \pi_t = \kappa (m_t - 1) \tag{59}$$

with $\kappa = \theta_r/\phi$. Compared to the Phillips curves in Kaplan, Moll, and Violante (2018) and Alves (2019), this Phillips curve is missing the forward-looking term $\dot{\pi}_t$, making it a first-degree differential equation, which improves stability properties of the solution algorithm.

Workers. The solution of worker's problem in equation (4) generates a value function $v_t(a, z)$ and a distribution of agents $g_t(a, z)$ that satisfy Kolmogorov equations. Asset holdings a and labor productivity z are two individual state variables, and all aggregate states are suppressed in the subindex t. The aggregate sequences that the workers have to know are the after-tax wage and the interest rate $\{w_t, r_t(a)\}_t$. Here the dependence of $r_t(a)$ on a reflects that the interest on deposits is not the same as that on loans. The control variables that they choose are consumption $c_t(a,z)$ and labor supply $l_t(a,z)$, which also maps into a choice of the saving rate $s_t(a,z)$. The following lemma characterizes the value function, the distribution of agents, and the choice of control variables. Define the functions $h(\cdot)$ and $\xi(\cdot)$ by $h(\cdot)^{-1} = u'(\cdot)$ and $\xi(\cdot)^{-1} = \chi'(\cdot)$. Denote the switching intensity of the labor productivity by λ_z and the transition probabilities by $p_{zz'}$.

Lemma 1. The labor supply and consumption of the workers satisfy

$$l_t(a,z) = \xi(w_t \cdot \partial_a v_t(a,z)) \tag{60}$$

$$c_t(a,z) = h(\partial_a v_t(a,z)) \tag{61}$$

The value function $v_t(a,z)$ solves the following Kolmogorov backward equation on $(\bar{a},\infty)\times Z$ on the time scale $(0,\infty)$:

$$\begin{aligned}\rho v_t(a,z) - \dot{v}_t(a,z) = {} & u(h(\partial_a v_t(a,z))) - z\chi(\xi(w_t \cdot \partial_a v_t(a,z))) \\ & + \partial_a v_t(a,z) \cdot (r_t(a)a + zw_t\xi(w_t \cdot \partial_a v_t(a,z)) - h(\partial_a v_t(a,z))) \\ & + \lambda_z \sum_{z'} p_{zz'}(v_t(a,z') - v_t(a,z))\end{aligned} \tag{62}$$

The density $g_t(a,z)$ solves the following Kolmogorov forward equation on $((\bar{a},0)\cup(0,\infty))\times Z$ on the time scale $(0,\infty)$:

$$\dot{g}_t(a,z) = -\partial_a[g_t(a,z)\cdot(r_t(a)a + zw_t\xi(w_t\cdot\partial_a v_t(a,z)) - h(\partial_a v_t(a,z)))] \tag{63}$$

$$+ \sum_{z'} \lambda_{z'} p_{z'z} g_t(a,z') - \lambda_z g_t(a,z) \tag{64}$$

Proof. (of Lemma 1) The HJB equation for the problem in equation (4) is

$$\begin{aligned}\rho v_t(a,z) - \dot{v}_t(a,z) = {} & \max_{c,l}\Big\{u(c) - z\chi(l) + \partial_a v_t(a,z)\cdot(r_t(a)a + zw_t l - c_t)\Big\} \\ & + \lambda_z \sum_{z'} p_{zz'}(v_t(a,z') - v_t(a,z))\end{aligned} \tag{65}$$

The first order conditions for the control variables c and l are

$$u'(c) = \partial_a v_t(a,z) \tag{66}$$

$$z\chi'(l) = zw_t u'(c) \tag{67}$$

They immediately imply $\chi'(l) = w_t u'(c)$, which translates into $l_t(a,z) = \xi(w_t u'(c_t(z,a)))$. The first one can be rewritten as $c_t(z,a) = h(\partial_a v_t(a,z))$. Plugging this into equation (65) leads to the differential equation (62). The Kolmogorov forward equation for the problem in equation (4) is

$$\dot{g}_t(a,z) + \partial_a[g_t(a,z)\cdot(r_t(a)a + zw_t l_t(a,z) - c_t(a,z))] = \sum_{z'} \lambda_{z'} p_{z'z} g_t(a,z') - \lambda_z g_t(a,z) \tag{68}$$

Plugging the expressions for the optimal $c_t(a,z)$ and $l_t(a,z)$ leads to equation (64). □

Proof. (of Proposition 1) Consider the change in variance of log consumption:

$$\mathbb{V}\left[\ln\left(C_1\right)\right] - \mathbb{V}\left[\ln\left(C\right)\right] = \mathbb{V}\left[\ln\left(C\right) + \Delta_1\right] - \mathbb{V}\left[\ln\left(C\right)\right] = \mathbb{V}\left[\Delta_1\right] + 2\mathbb{C}\left[\ln\left(C\right), \Delta_1\right] \tag{69}$$

The first term can be decomposed as

$$\begin{aligned}\mathbb{V}[\Delta_1] &= \mathbb{E}[\Delta_1^2] - \mathbb{E}[\Delta_1]^2 = \zeta\mathbb{E}[(\Delta_1^T)^2] + (1-\zeta)\mathbb{E}[(\Delta_1^N)^2] - (\zeta\mathbb{E}[\Delta_1^T] + (1-\zeta)\mathbb{E}[\Delta_1^N])^2 \\ &= \zeta\mathbb{V}[\Delta_1^T] + (1-\zeta)\mathbb{V}[\Delta_1^N] + \zeta(1-\zeta)(\mathbb{E}[\Delta_1^T] - \mathbb{E}[\Delta_1^N])^2 \end{aligned} \tag{70}$$

The second term can be decomposed as

$$\begin{aligned}2\mathbb{C}[\ln(C), \Delta_1] &= 2\mathbb{E}[\ln(C)\Delta_1] - 2\mathbb{E}[\ln(C)]\mathbb{E}[\Delta_1] = 2\zeta\mathbb{E}[\ln(C^T)\Delta_1^T] + 2(1-\zeta)\mathbb{E}[\ln(C^N)\Delta_1^N] \\ &\quad - 2(\zeta\mathbb{E}[\ln(C^T)] + (1-\zeta)\mathbb{E}[\ln(C^N)])(\zeta\mathbb{E}[\Delta_1^T] + (1-\zeta)\mathbb{E}[\Delta_1^N]) \\ &= 2\zeta\mathbb{C}[\ln(C_1^T), \Delta_1^T] + 2(1-\zeta)\mathbb{C}[\ln(C_1^N), \Delta_1^N] \\ &\quad + 2\zeta(1-\zeta)(\mathbb{E}[\ln(C^T)] - \mathbb{E}[\ln(C^N)])(\mathbb{E}[\Delta_1^T] - \mathbb{E}[\Delta_1^N]) \end{aligned} \tag{71}$$

Adding everything up,

$$\begin{aligned}\mathbb{V}[\Delta_1] + 2\mathbb{C}[\ln(C), \Delta_1] &= \zeta\mathbb{V}[\Delta_1^T] + (1-\zeta)\mathbb{V}[\Delta_1^N] + 2\zeta\mathbb{C}[\ln(C_1^T), \Delta_1^T] + 2(1-\zeta)\mathbb{C}[\ln(C_1^N), \Delta_1^N] \\ &\quad + \zeta(1-\zeta)(\mathbb{E}[\Delta_1^T - \Delta_1^N]^2 + 2\mathbb{E}[\ln(C^T) - \ln(C^N)]\mathbb{E}[\Delta_1^T - \Delta_1^N]) \\ &= \zeta\mathbb{V}[\Delta_1^T] + (1-\zeta)\mathbb{V}[\Delta_1^N] + 2\zeta\mathbb{C}[\ln(C_1^T), \Delta_1^T] + 2(1-\zeta)\mathbb{C}[\ln(C_1^N), \Delta_1^N] \\ &\quad + \zeta(1-\zeta)(\mathbb{E}[\Delta_1^T + \ln(C^T) - \Delta_1^N - \ln(C^N)]^2 - \mathbb{E}[\ln(C^T) - \ln(C^N)]^2) \\ &= \zeta\mathbb{V}[\Delta_1^T] + (1-\zeta)\mathbb{V}[\Delta_1^N] + 2\zeta\mathbb{C}[\ln(C_1^T), \Delta_1^T] + 2(1-\zeta)\mathbb{C}[\ln(C_1^N), \Delta_1^N] \\ &\quad + \zeta(1-\zeta)(\mathbb{E}[\ln(C_1^T) - \ln(C_1^N)]^2 - \mathbb{E}[\ln(C^T) - \ln(C^N)]^2) \end{aligned} \tag{72}$$

This completes the proof. □

Proof. (of Proposition 2) Integrating, equation (40) from t to infinity,

$$\ln(e_t) - \lim_{s\to\infty}\ln(e_s) = \frac{1}{1-\phi_\varepsilon}\int_t^\infty \psi_s ds - \frac{\phi_\pi + 1 - \phi_\varepsilon}{1-\phi_\varepsilon}\int_t^\infty \pi_s ds$$

Since the economy converges to the steady state at infinity, the limit of the real exchange rate is one.

$$\ln(e_t) = \ln\left(\mathcal{E}_t\right) - \ln(P_t) = \frac{1}{1-\phi_\varepsilon}\int_t^\infty \psi_s ds - \frac{\phi_\pi + 1 - \phi_\varepsilon}{1-\phi_\varepsilon}\int_t^\infty \pi_s ds$$

This leads to equation (41). □.

B Additional Figures

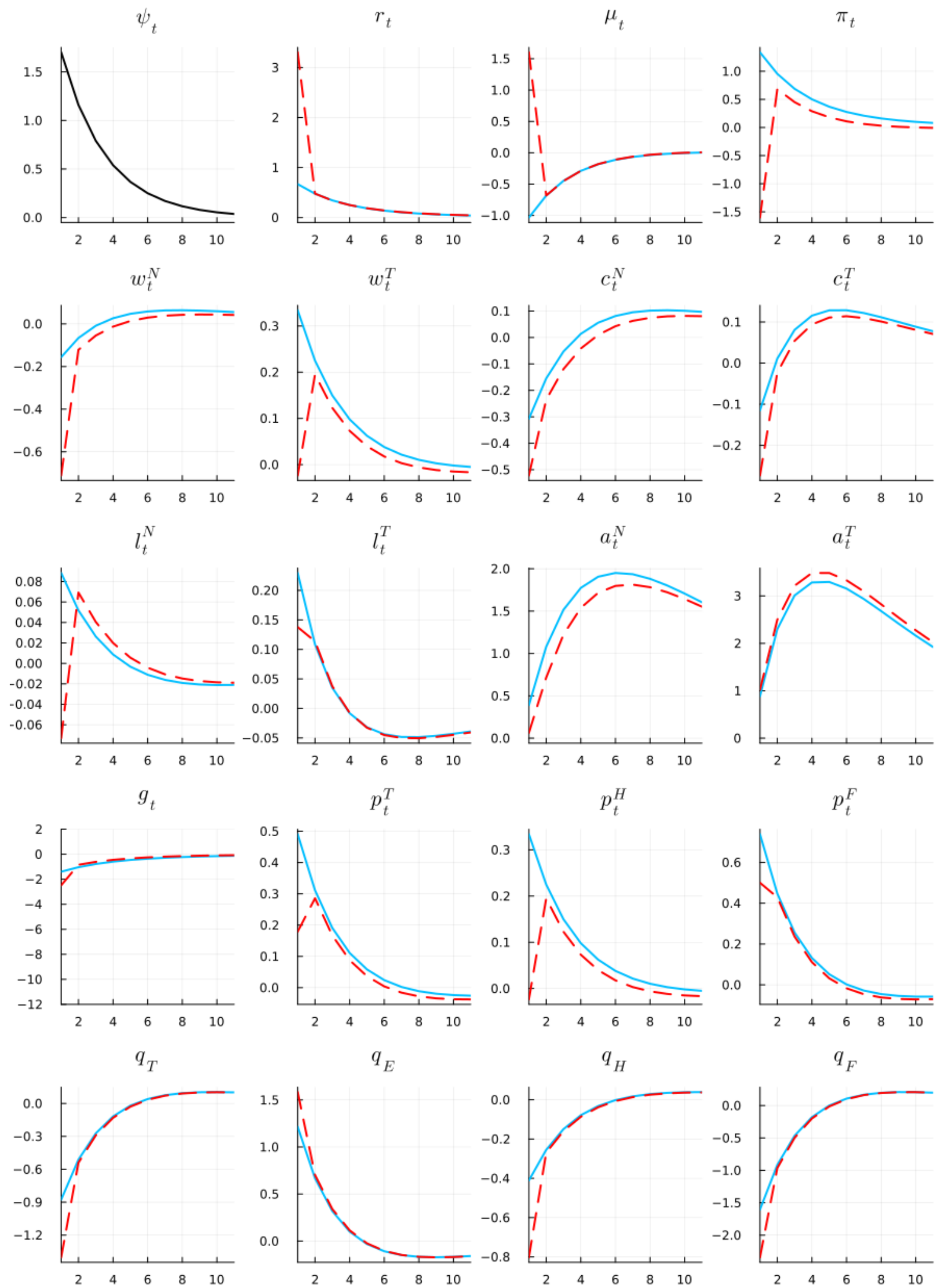

Figure A.1: Impulse responses under float and peg. Units: percentage points and percent.

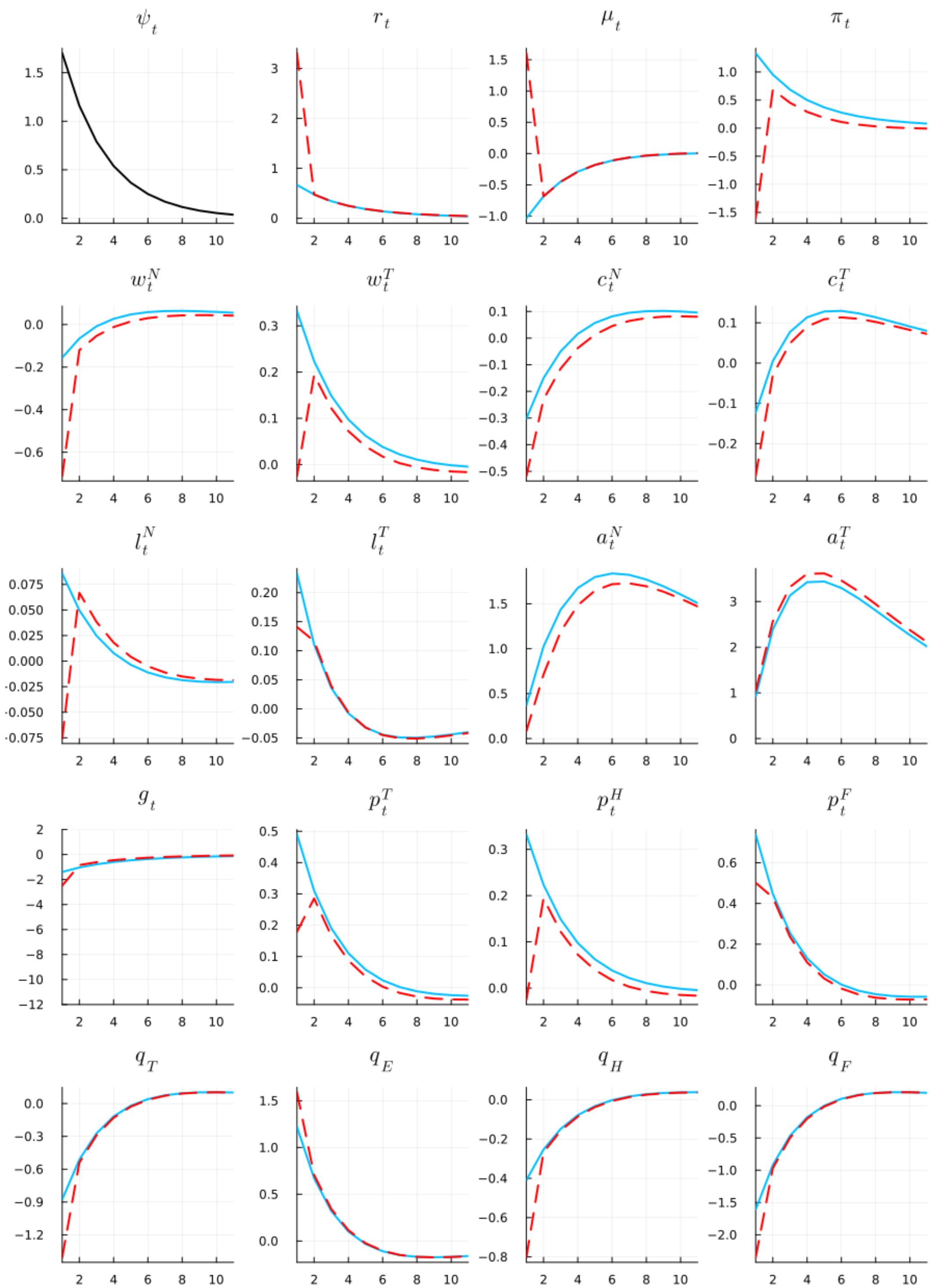

Figure A.2: Impulse responses with the same productivity in the two sectors. Units: percentage points and percent.

References

Alves, Felipe. 2019. "Job ladder and business cycles." *Manuscript, New York University. https://drive. google. com/file/d/16Rzfy_Eu28a1-XMYUTJTe1r4TMjiyYT_/view* .

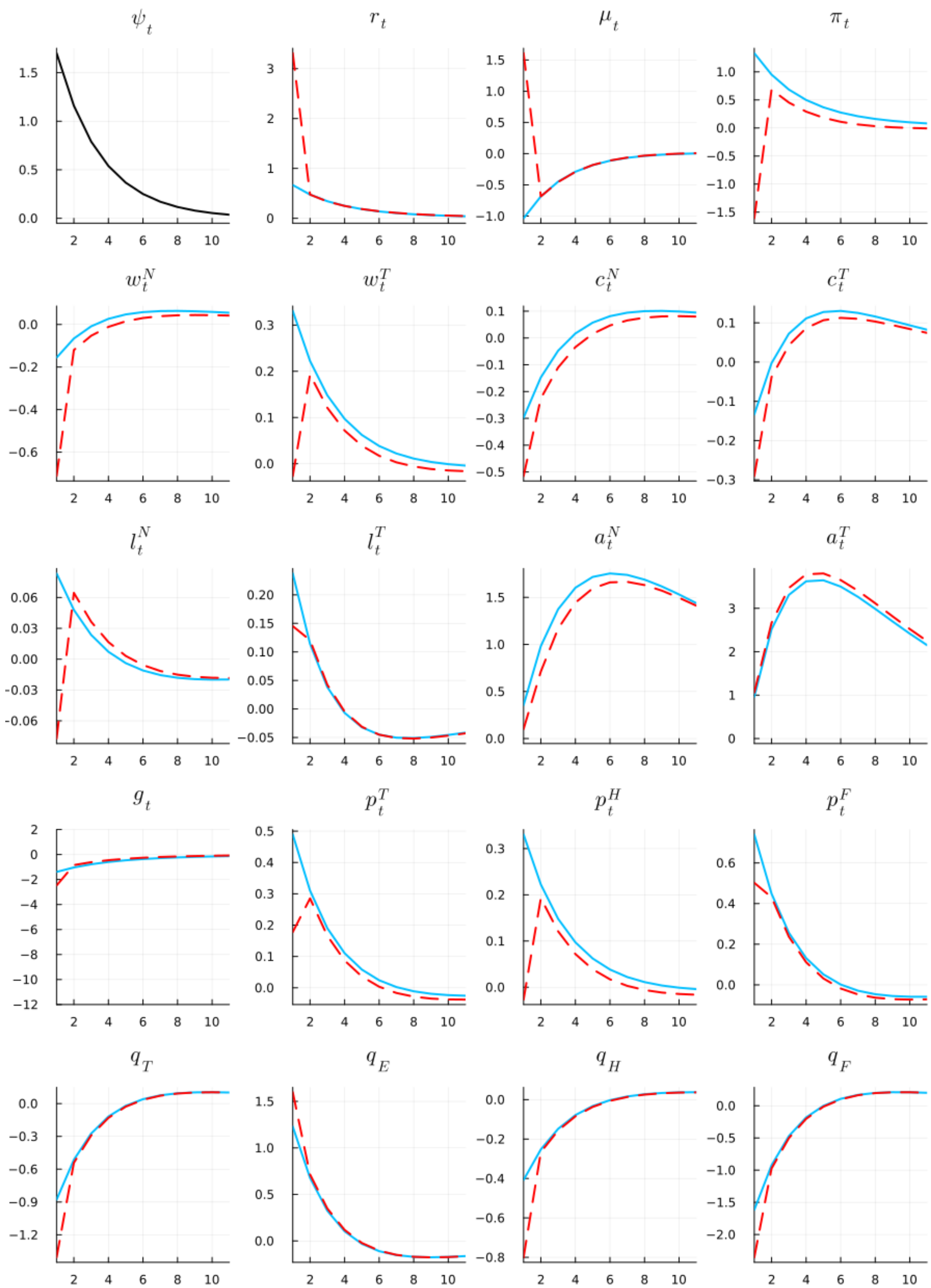

Figure A.3: Impulse responses with workers in tradables less productive. Units: percentage points and percent.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. 2018. "Monetary policy according to HANK." *American Economic Review* 108 (3):697–743.

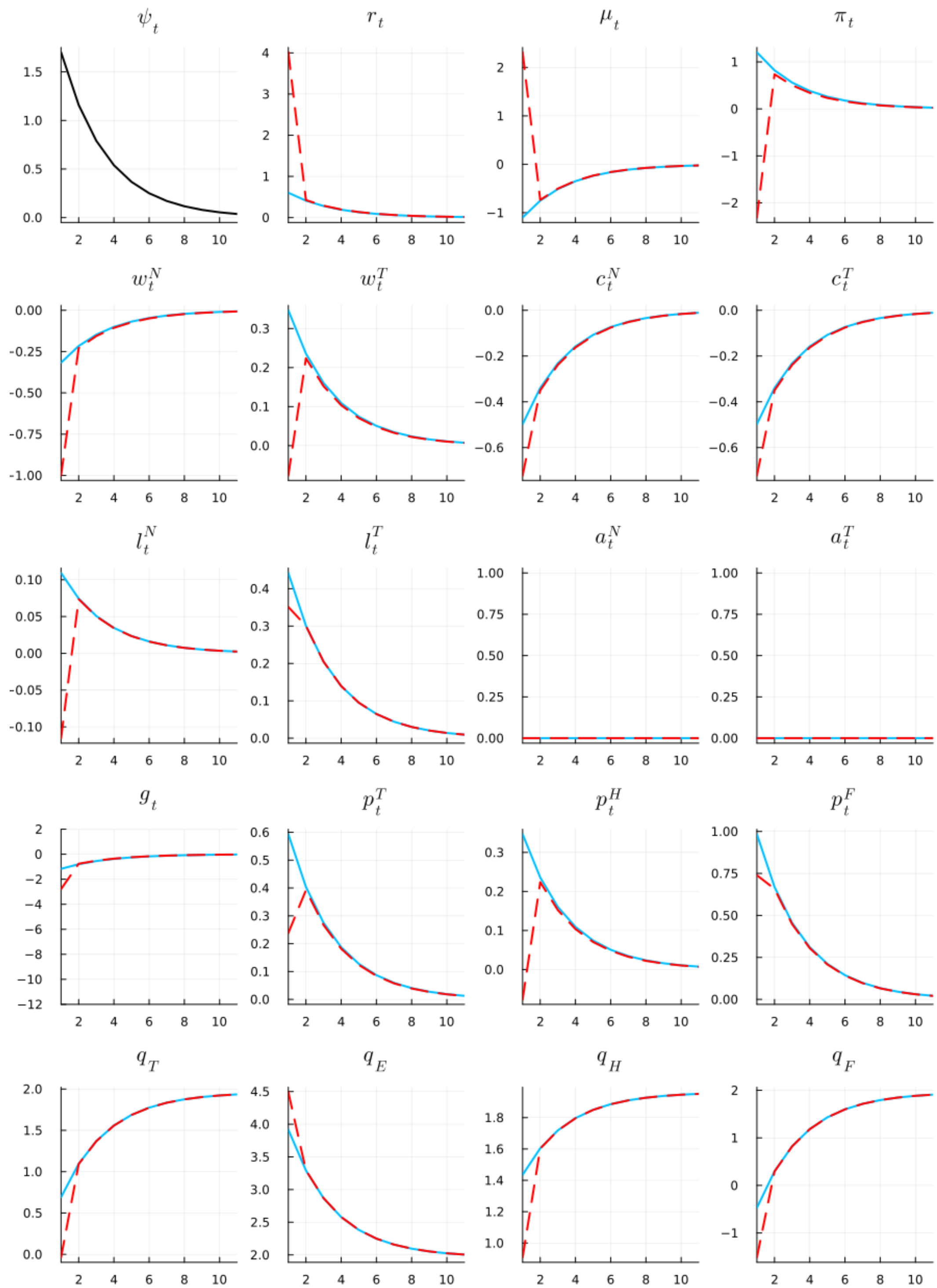

Figure A.4: Impulse responses in a two-sector RANK model. Inflation in percentage points, everything else in percent.

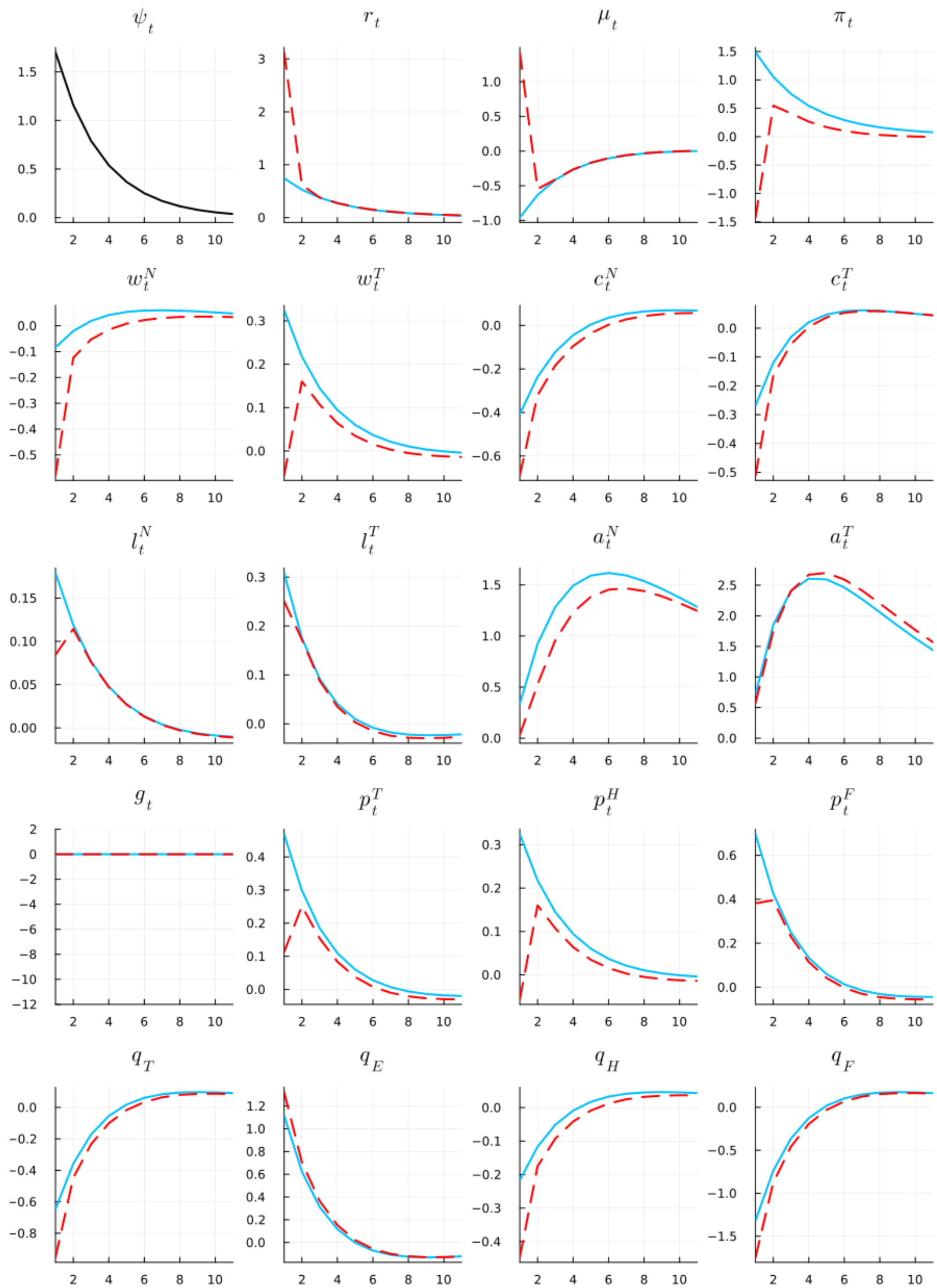

Figure A.5: Impulse responses under proportional taxes. Units: percentage points and percent.

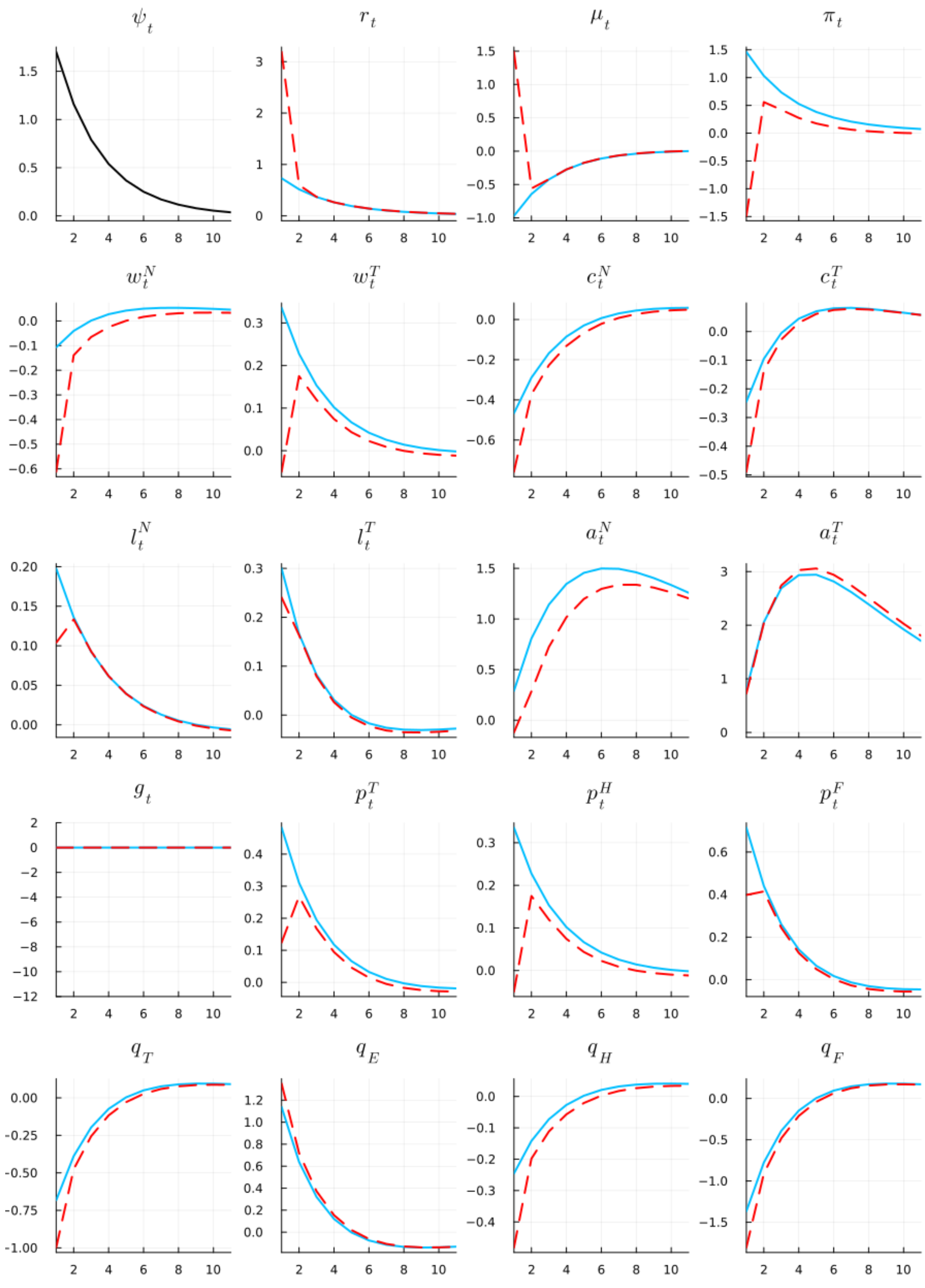

Figure A.6: Impulse responses under flat taxes. Units: percentage points and percent.

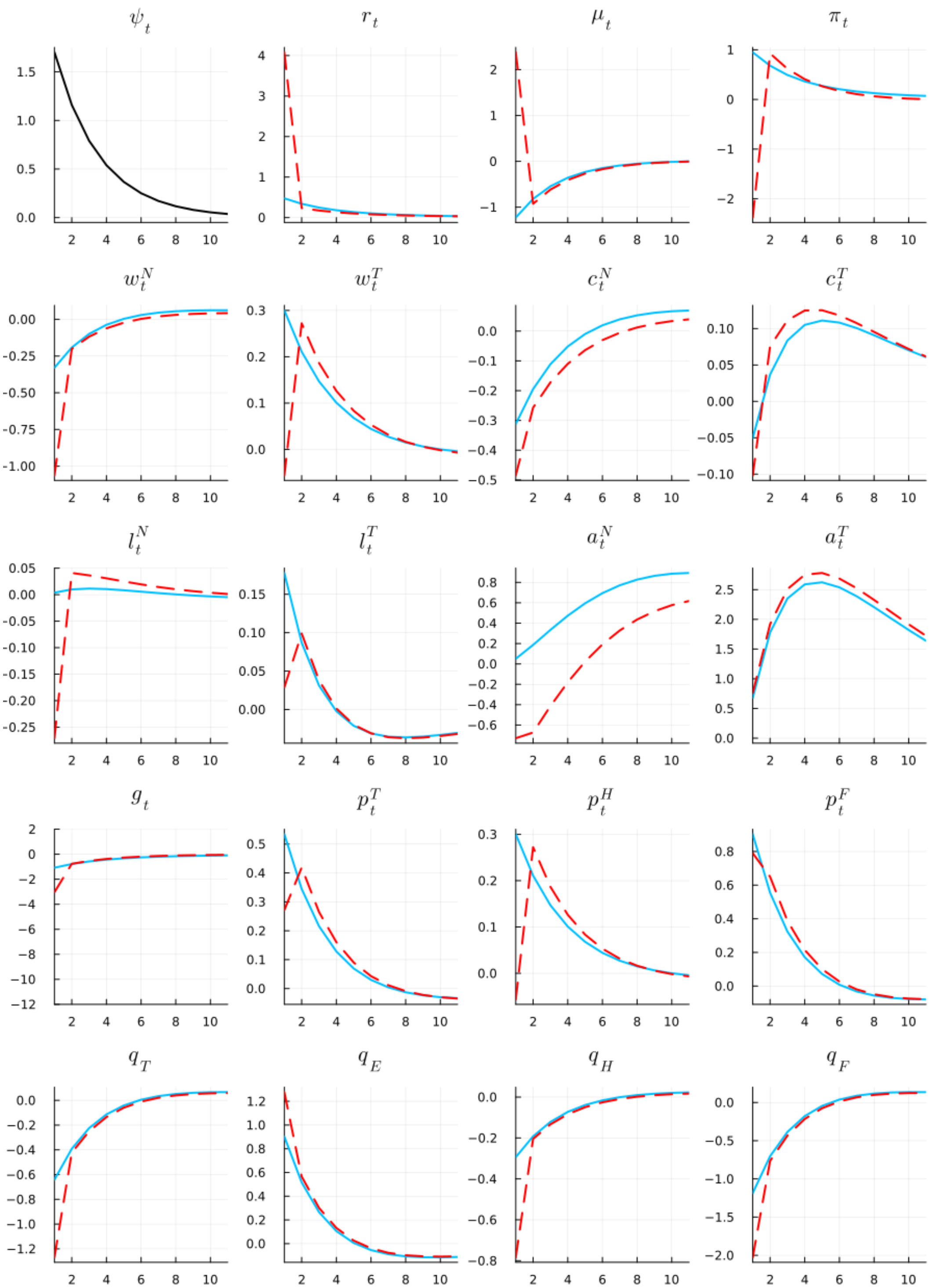

Figure A.7: Impulse responses with $(\theta, \theta_g, \theta_e) = (0.75, 1.5, 1.5)$. Units: percentage points and percent.

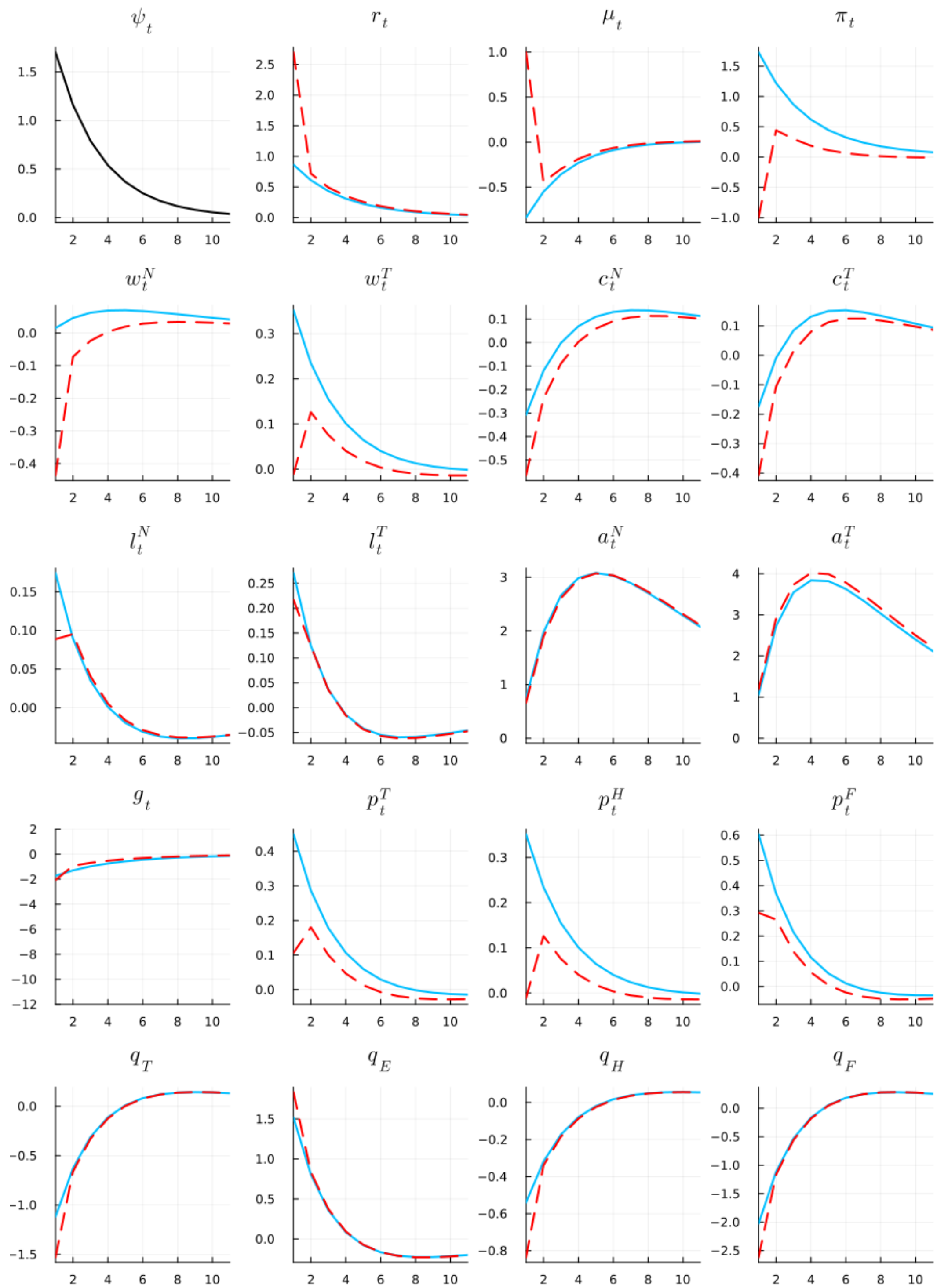

Figure A.8: Impulse responses with $(\theta, \theta_g, \theta_e) = (3, 6, 6)$. Inflation in percentage points, everything else in percent.

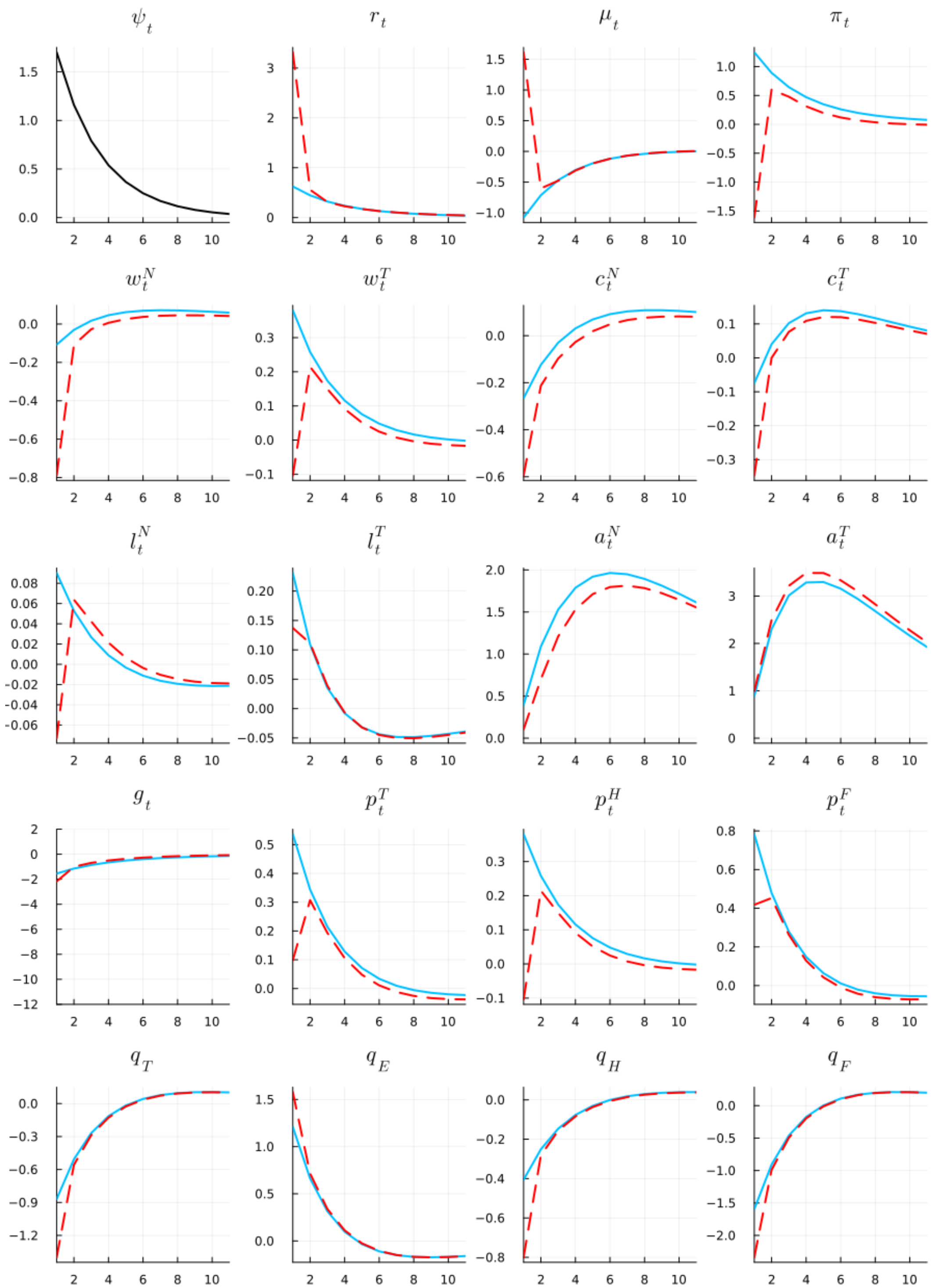

Figure A.9: Impulse responses with $\kappa = 0.005$. Inflation in percentage points, everything else in percent.

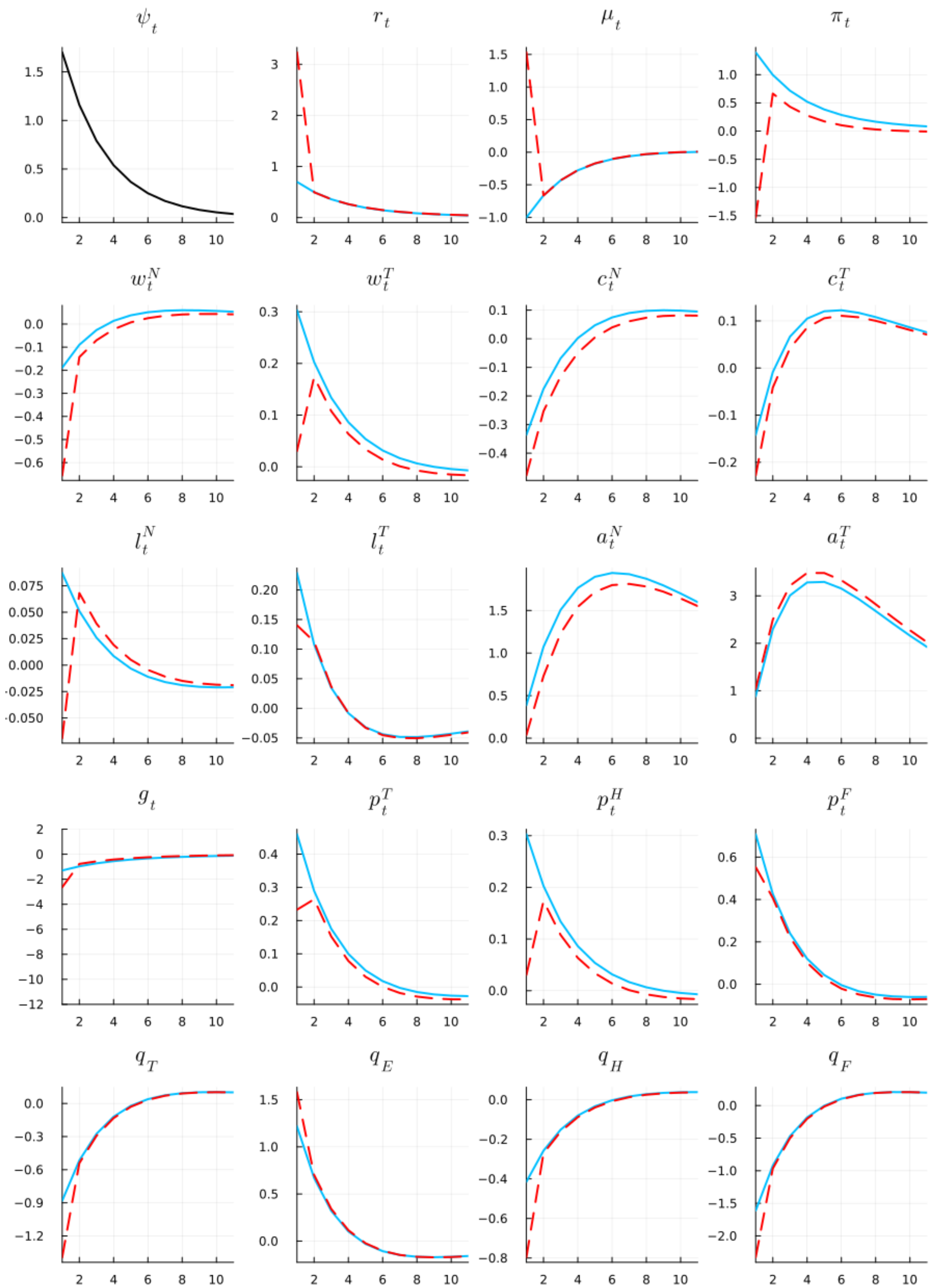

Figure A.10: Impulse responses with $\kappa = 0.0084$. Inflation in percentage points, everything else in percent.

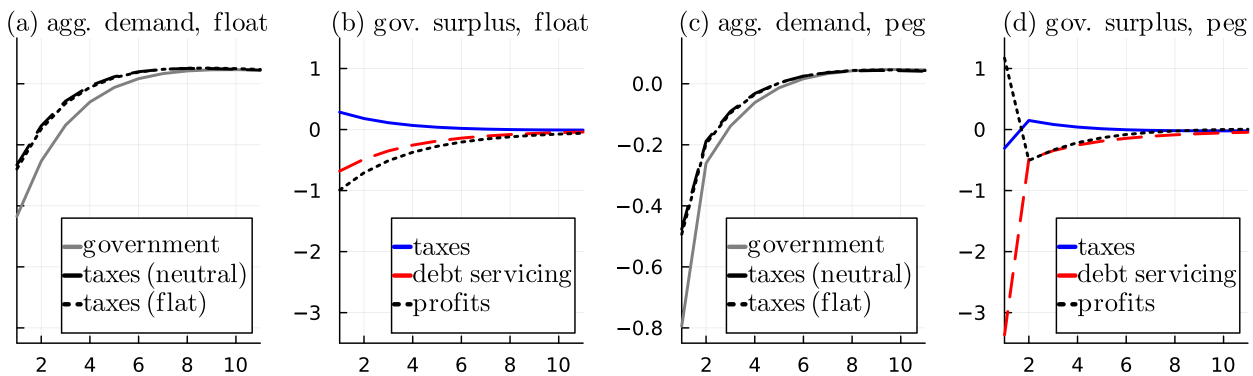

Figure A.11: Responses of aggregate demand under the three fiscal regimes (panels (a) and (c) for float and peg). Responses of components of government expenditures in percent of their steady state value (panels (b) and (d) for float and peg).