A Heterogeneous-Country Model of the Global Financial Cycle*

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Abstract

I develop a heterogeneous-country model of the global economy with financial intermediaries. Imperfect risk-sharing creates a wealth distribution across countries. Risk-off shocks make the intermediaries revise their models of returns, which leads to repricing of country risk and triggers aggregate capital flight. In these events, risk premia rise steeply in poor countries, while rich countries experience "retrenchment": local agents use their external assets to replace foreign investors and stabilize asset prices. Risky assets in rich countries endogenously become safe havens. I derive asset price responses to global shocks in closed form and show that accounting for gross capital flows is crucial for explaining the international cross-section of asset returns. The wealth gradient in the cross-section of asset price responses identifies real and financial shocks behind the global financial cycle. A simple two-factor regression implied by the model explains 57% of the variation in the global cross-section of equities.

Key Words: capital flows, risk premium, global financial cycle, heterogeneity, retrenchment

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1 Introduction

There is an aggregate cycle in capital flows and asset prices. Miranda-Agrippino and Rey (2022) show that one global factor explains more that 20% of the variation in gross capital flows across the world. This factor is strongly correlated with measures of global risk-taking capacity and the dominant component in asset prices. In booms, when asset prices are high, investors tend to accumulate foreign holdings. In global downturns, they "retrench" by selling foreign assets and shifting portfolios towards their domestic markets.

Countries are not equally exposed to the cycle. Emerging markets are especially strongly affected by changes in the global risk appetite, their asset prices are more volatile, and risk premia on their assets are higher. At the same time, capital flows are more active in advanced economies: their investors accumulate more external assets in booms and retrench more in downturns.

In this paper, I propose a model of the global financial cycle that both generates a strong international co-movement in asset prices and financial flows and accounts for heterogeneous exposures to global shocks. The model features a wealth distribution across countries and jointly determines asset prices and capital flows. In global downturns, risk premia rise in asset-poor countries, while foreign investor outflows and domestic retrenchment are the largest in asset-rich ones, which I show is consistent with the data. Global interest rates fall, and risky assets depreciate in most of the world. Importantly, risky assets in the right tail of the wealth distribution of countries appreciate in global downturns, which endogenously gives them a safe status. I derive closed-form expressions for the responses of risk premia, asset prices, and capital flows to aggregate shocks as functions of observable statistics. I then use the model to identify the shocks behind the global financial cycle from the cross-section, utilizing the wealth gradient in asset price cyclicality.

The model describes the global economy as a collection of identical countries and a financial intermediary that facilitates international capital flows. Each country has a domestic investor and a risky asset subject to country-specific risk in payoffs. Financial markets are segmented: investors have to use the intermediary's services to invest abroad. The intermediary issues riskless bonds to investors in all countries and buys shares in their risky assets instead. Its risk-taking capacity is limited: it cannot fully take over all asset markets, leaving some country-specific risk with local investors. Uninsured idiosyncratic risk generates a cross-country wealth distribution. Countries are fundamentally similar but heterogeneous ex post due to different histories of shocks.

In the steady state of the model, countries can be sorted by their investor's wealth. In rich countries, investors have accumulated large external assets and mostly rely on safe, low-returning foreign investments. Risk premia on their domestic assets are low because they enjoy large and stable consumption streams. In poor countries, local investors are smaller, and their asset markets are highly reliant on the foreign investor, the intermediary. Risk premia are elevated, since local investors are highly levered, and their consumption streams are exposed to domestic risk.

The global financial cycle in the model is driven by shocks to the intermediary's risk-taking capacity. Specifically, the intermediary is unsure it has the right model of country risk. It seeks to make its investments robust to model misspecification and entertains alternative scenarios, benchmarking them against real-time data. The main primitive shock hits the intermediary's tolerance to uncertainty across models, inviting more pessimistic scenarios and increasing its effective risk aversion. This generates capital flight.

The first source of heterogeneity in outcomes is that poor countries rely on foreign investors more than rich ones, and aggregate capital flight events hit them with a full-scale sudden stop. The second source of heterogeneity is the response of local investors. In rich countries, they use their large buffer stocks of external assets to replace the foreign investor. Risk premia in these countries do not rise substantially, since local investors have safe consumption streams and are ready to absorb additional risk. In poor countries, domestic investor base is too thin to absorb what the intermediary tries to sell, and risk premia have to rise steeply to convince it to stay. These markets adjust through movements in prices rather then quantities, while in rich countries, asset markets show much more elasticity: large movements in quantities and little in prices. As a result, rich countries face substantially larger outflows of foreign investors in equilibrium, but nevertheless outperform the poor countries in global downturns due to more active retrenchment.

Importantly, the model determines all prices in general equilibrium, including the global interest rate. When the intermediary's risk-taking capacity falls, the global interest rate falls as well. There are two reasons for this. First, precautionary motives become stronger: as uninsured idiosyncratic risk looms larger on balance sheets, the risk-free rate is depressed. Second, investors feel poorer with losses on their risky investments, which decreases their desired consumption, and the interest rate clears the goods markets by making saving less attractive. In equilibrium, of course, these two forces are different sides of the same coin.

The implication of the falling global interest rate is that all long-term assets are revalued. In poor countries, this does little against the backdrop of higher risk premia. In contrast, in rich countries, risk premia rise relatively little, and the falling interest rate dominates. Global downturns increase prices of risky assets in the right tail of the wealth distribution. This happens exclusively due to the higher retrenchment capacity of these countries: the physical properties of their risky assets are the same as everywhere else.

I then exploit this result to identify the shocks behind the global financial cycle. A challenge is that not all asset prices movements are driven by financial shocks: real shocks, both aggregate and local, contribute to asset prices too. I augment the model with global output shocks and derive the mapping from the shocks to two main time series: the global average of risky asset prices and a high-minus-low factor that compares asset returns in in rich countries to those in poor ones. The average, or level, factor is driven by both real and financial shocks. The high-minus-low, or slope, factor isolates the financial shock by using the wealth gradient in asset price sensitivity.

Total wealth is not readily observable for a large cross-section of countries, but my model generates an injective mapping from a country's total wealth to the ratio of its external assets to its external liabilities. In the model, this ratio is the most important statistic that shows retrenchment capacity: it measures how much ammunition domestic agents have to replace foreign investors in the event of capital flight. I calibrate the model to reproduce the empirical distribution of this ratio across countries, taking the data on flows and positions from the IMF. I then construct the average level and the high-minus-low slope factors in equity prices from MSCI and recover the global real and financial shocks. The recovered real shocks are strongly correlated with the GDP time series of the US and OECD. The recovered financial shocks are correlated with barometers of the global risk-taking capacity such as VIX and excess bond premium of Gilchrist and Zakrajšek (2012), as well as with the principal components in inward and outward capital flows.

I then test the model's ability to explain the international cross-section of risky asset prices. The global co-movement in equity prices is very strong: two principal components explain 70% of the variation in a panel of 39 countries in 2000-2020. The average price level factor coincides with the first principal component. The high-minus-low slope factor has a correlation of 0.53 with the second principal component. As a diagnostic, I feed the recovered aggregate shocks into the model and predict asset price responses for each country based on its external asset-liability ratio according to the mapping generated by the model. This simple level-slope factor structure from the model predicts 57% of the realized variation in asset prices, and 76% of the variation explained by the principal components, which takes out idiosyncratic shocks.

In addition to explaining the cross-sectional heterogeneity in the exposure to aggregate shocks, the model clarifies the direction of wealth redistribution in global downturns. Risk-off shocks are regressive: rich countries become richer, poor countries face falling asset prices. Despite that, rich countries provide insurance to the rest of the world: amid capital flight, the intermediary realizes capital gains on assets issued by rich countries, which keeps its net worth from falling. Since the intermediary's net worth is an important state variable for all countries, this indirectly supports asset prices across the entire wealth distribution.

The model also sheds light on the role of the US in global downturns. I take seriously the role of the US as the global insurer, as described by Gourinchas and Rey (2022): it houses the intermediary, the intermediary's profits from risk-free borrowing and risky investments ensure its "exorbitant privilege", the ability to run perpetual trade and current account deficits. In global downturns, the US suffers losses on its risky investment position and transfers wealth to the rest of the world, bearing "exorbitant duty". I show that the cyclicality of the US wealth share depends on one parameter, the size of its own domestic asset market, which must be sufficiently large for this wealth share to increase in global downturns. This operationalizes a potential resolution of the reserve currency paradox of Maggiori (2017), who points out the inconsistency between strong dollar appreciations in crises and large wealth transfers from the US to the rest of the world.

1.1 Related literature

I contribute to three broad strands of literature. The first one studies capital flows and retrenchment in equilibrium. The most closely related paper is Caballero and Simsek (2020). They show how retrenchment stabilizes domestic asset markets in a model where capital flight is driven by idiosyncratic shocks. Liquidity needs trigger fire sales by foreign investors. Local investors use their foreign holdings to pick up the unwanted asset and support its price. This mechanism is also present in Jeanne and Sandri (2023). I build a dynamic version of this model in the style of Brunnermeier and Sannikov (2014) with global intermediaries, aggregate shocks, and endogenous differences wealth, focusing on the distributional consequences of aggregate capital flight.

In the same context, Farboodi and Kondor (2022) study heterogeneous boom-bust dynamics with imperfect information about asset quality. In their model, shocks determine what investors learn about firms. In bad times, they flee from emerging markets to advanced economies. Fu (2023) models joint determination of capital flows and exchange rates, showing that currency betas are lower in countries where domestic investors have a higher propensity to retrench than foreign ones. Similar to my model, Fu (2023) maps retrenchment capacity to riskiness of assets. Zhou (2023) shows how exposure of assets to foreign shocks depends on the willingness of other investors to absorb additional supply. A similar mechanism involving domestic investors is at the heart of my model. Davis and Van Wincoop (2022) construct a multicountry model to generate gross flows after a shock to global risk aversion and show the importance of within-country heterogeneity.

The second strand of literature studies global shocks with cross-sectional heterogeneity. Closely related papers are Morelli, Ottonello, and Perez (2022) and Bai, Kehoe, and Perri (2019). Morelli, Ottonello, and Perez (2022) model a global intermediary invests in emerging markets. They account for the feedback from the cross-section of returns to the intermediary's net worth and find that shocks to the intermediary's balance sheet are an important driver of borrowing costs around the world. Bai, Kehoe, and Perri (2019) use a similar model to measure the relative importance of global and local shocks in explaining the cross-section of sovereign spreads. My main contribution relative to these papers is adding gross capital flows that are jointly determined with asset prices as the primary mechanism behind heterogeneous responses to global shocks.

A large part of the literature on the international cross-section of returns focuses on exchange rates, as opposed to asset prices, as the main outcome variable. Verdelhan (2018) documents the cross-sectional structure of currencies and offers a multi-country model that reproduces it. Hassan (2013), Ready, Roussanov, and Ward (2017), Richmond (2019), Lustig and Richmond (2020), Kekre and Lenel (2025), Oskolkov and Perez (2026) offer models that can account for the strong co-movement and cross-sectional heterogeneity in exchange rates. I contribute to this literature by offering a mechanism that relates the cross-section of asset returns to the cross-section of capital flows and generates heterogeneous outcomes in a fundamentally symmetric world.

Finally, the third large and growing strand of literature I contribute to concerns the global financial cycle. Miranda-Agrippino and Rey (2022) provide a comprehensive review. The dominant global factor in a large panel of risky asset prices has been extracted by Miranda-Agrippino, Nenova, and Rey (2020) and more recently updated by Miranda-Agrippino and Rey (2020). Similarly strong co-movement has been documented for capital flows. Forbes and Warnock (2012) and Forbes and Warnock (2021) show co-movement between gross flows. Barrot and Serven (2018) identify common components in gross flows and show that these common components are strongly related to aggregate variables such as VIX, US dollar exchange rate, and interest rates. Part of this literature deals with heterogeneity between advanced economies and emerging markets. Barrot and Serven (2018) and Cerutti, Claessens, and Puy (2019) show that flows in advanced economies are more responsive to common factors. This fact is at the heart of the model, which is built to generate more elastic asset markets in rich countries. The literature studying distributions of returns and flows includes Chari, Stedman, and Lundblad (2020), Gelos, Gornicka, Koepke, Sahay, and Sgherri (2022), and Eguren Martin, O'Neill, Sokol, and von dem Berge (2021). Kalemli-Özcan (2019) and Bräuning and Ivashina (2020) show that US monetary policy spillovers have a more pronounced effect on emerging markets. Chari, Stedman, and Lundblad (2020) show the outsized effect of risk-off episodes on the worst realizations, the left tail.

On the theoretical side, a set of papers study the propagation of global shocks through financial markets. These include, among others, Bruno and Shin (2015), Maggiori (2017), Farhi and Maggiori (2018), Jiang, Krishnamurthy, and Lustig (2020), Kekre and Lenel (2021), Sauzet (2023), Devereux, Engel, and Wu (2023), and Jiang (2024). In Jiang, Krishnamurthy, and Lustig (2020) and Kekre and Lenel (2021), the dollar carries a convenience yield. Kekre and Lenel (2021) study flight to safety caused by a shock to this convenience yield in a model with nominal frictions and investment. I contribute to this literature by offering a perspective on global financial shocks. In my model, they stem from model revisions of global financial intermediaries. Their assessment of country-specific risk occasionally becomes more pessimistic, which leads to a wave of repricing of idiosyncratic country risk and aggregate capital flight. I also relate to"exorbitant privilege" and "exorbitant duty" studied by Gourinchas and Rey (2022). Specifically, I offer way to resolve the reserve currency paradox posed by Maggiori (2017): the inconsistency between the strong appreciation of the dollar and large wealth transfers from the US to the rest of the world during global downturns. My resolution is related to that in Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022), who show that the US wealth share increases in global downturns due to large capital gains on domestic assets.

The paper is organized as follows. Section 2 presents motivating evidence, Section 3 lays out the model, Section 4 describes equilibrium, Section 5 describes the steady state, Section 6 analyzes global shocks and states the identification results, Section 7 conducts empirical analysis. Section 8 concludes.

2 Motivating Evidence

I start by presenting motivating evidence. I take four aggregates that time the global financial cycle: the principal components of financial inflows and outflows, VIX, and excess bond premium (EBP) of Gilchrist and Zakrajšek (2012). I then sort countries by one cross-sectional variable: the ratio of external assets to external liabilities. This variable shows the country's theoretical retrenchment capacity. I show that equities in economies with high external asset-liability ratios outperform those in economies with low ratios in global downturns. I then show that asset acquisition in economies with high asset-liabilities ratios is more procyclical: countries with high ratios accumulate foreign asset more actively in booms and retrench more actively in busts.

2.1 Data

To construct the data on outward flows, I take the Balance of Payments and International Investment Positions data from the IMF. For a country i in quarter t, the quantity $F_{it}^{\text{acq(raw)}}$ denotes net purchases of foreign assets measured in dollars. This is a flow variable that measures transactions, leaving out changes in positions due to valuation. Following Forbes and Warnock (2012) and Forbes and Warnock (2021), I take a smooth, deseasoned version:

$$F_{it}^{\text{acq}} = \sum_{t-3}^{t} F_{is}^{\text{acq(raw)}} - \sum_{t-7}^{t-4} F_{is}^{\text{acq(raw)}}$$

I restrict attention to portfolio debt, portfolio equity, and "other" assets, the latter corresponding to banking flows. Incurrence of liabilities F_{it}^{inc} is constructed in the same way.

The four aggregate series that time the global financial cycle are the first principal components f_t^{acq} and f_t^{inc} of $\{F_{it}^{\text{acq}}\}$ and $\{F_{it}^{\text{inc}}\}$, VIX, and EBP. I choose 2000Q1-2019Q4 as the sample period. Data on inflows and outflows are fully available for 53 and 56 countries, respectively. I use balanced these panels to exctract the principal components. The firs principal components explain 19 and 21 percent of variation in their respective panels, with $\text{corr}(f_t^{\text{acq}}, f_t^{\text{inc}}) = 0.90$. The correlations of the asset acquisition serie f_t^{acq} with VIX and EBP are -0.57 and -0.72. The correlations of the incurrence serie f_t^{inc} with VIX and EBP are -0.48 and -0.59. Figure 1 shows their time paths.

2.2 Cyclicality of financial flows and asset prices

I next document the cyclicality of asset returns and financial flows across the distribution of external assets. Normalized outward flows for country i are defined as

$$a_{it} = \frac{F_{it}^{\text{acq}}}{L_{i,t-1}},$$

Figure 1: aggregate series.

where $L_{i,t-1}$ is the dollar stock of country i's external liabilities one quarter before. In global downturns, this variable shows retrenchment: it calculates how much of external liabilities domestic investors take over by selling their foreign assets. In global booms, it measures the accumulation of a buffer stock of external assets that can substitute foreign investors in downturns to come.

For asset prices, I take an HP-filtered version of MSCI stock price indices (converted to US dollars) in logs. I denote this measure by p_{it}. I use the price indices as opposed to total return indices for a more direct correspondence to the model.

At the first stage, I compute each country's cyclicality of both outcomes p_{it} and a_{it} to the four measures of the global financial cycle:

$$o_{it} = \alpha_i + \beta_i^{(o,f)} f_t + \epsilon_{it}$$

Here $o_{it} \in \{p_{it}, a_{it}\}$ is one of the two outcomes and $f_t \in \{f_t^{\text{acq}}, f_t^{\text{inc}}, \text{VIX}_t, \text{EBP}_t\}$ is one of the four aggregates. For every country, I collect the set of coefficients $\beta_i^{(o,f)}$. At the second stage, I investigate how $\beta_i^{(o,f)}$ relate to asset-liability ratios:

$$\beta_i^{(o,f)} = \alpha^{(o,f)} + \Gamma^{(o,f)} w_i + \epsilon_i$$

Here the coefficient of interest is $\Gamma^{(o,f)}$. Appendix C shows the scatterplot versions of all second-stage regressions, provides additional details on the sample, and shows alternative versions of the second-stage regressions with observations weighted by the standard errors of the first-stage estimates. It also shows robustness to outliers.

Table 1 shows the results for asset prices as the outcome. The sample for the asset flow principal components is constrained to countries that have full time series of both stock prices and flows, so that they contribute to the factor construction.

Table 1: Cyclicality of asset prices and asset-liability ratios. Prices multiplied by 100 to show percentage changes. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	9.51	10.83	-11.51	-11.52
	(8.83)	(10.62)	(-10.81)	(-9.24)
Γ	-3.07	-3.32	2.41	3.69
	(-2.20)	(-2.43)	(1.61)	(2.11)
R^2	0.16	0.17	0.07	0.11
N	28	30	39	39
T	80	80	80	80

Across all four measures of the global financial cycle, asset prices are strongly procyclical, as shown by the constant α. The are more procyclical in countries with low ratios of external assets to external liabilities. In global downturns, countries with high ratios outperform those with low ratios. This suggests that retrenchment capacity afforded by high external assets reduces exposure to the global financial cycle. To verify that the behavior of financial flows is consistent with this, I show the results for asset acquisition as the outcome variable in Table 2.

Table 2: Cyclicality of financial flows and asset-liability ratios. Asset flows multiplied by 100 to show percentage changes. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	0.20	0.01	0.11	0.68
	(0.38)	(0.03)	(0.13)	(0.92)
Γ	2.12	2.46	-2.19	-3.69
	(3.81)	(5.50)	(-2.55)	(-4.78)
R^2	0.14	0.26	0.07	0.21
N	89	89	89	89
T	80	80	80	80

These regressions add countries that only have partially available time series for asset flows, which increases the number of countries to 90. Across all four measures of the global financial cycle, foreign asset acquisition (as a share of external liabilities) is more procyclical in countries with high asset-liability ratios. This is consistent with stronger retrenchment in asset-rich economies.

Asset prices and financial flows are, of course, both endogenous outcomes that are jointly determined in equilibrium. I next construct a model that characterizes this joint determination, construct financial shocks that leads to repricing of country risk and capital flight events, and operationalizes the dynamic retrenchment mechanism in general equilibrium. I then use the model to identify the shocks behind the global financial cycle from price data and study the impact of global shocks in the cross-section.

3 Model

Time is continuous and runs forever. The world is a unit measure of countries indexed by $i \in [0,1]$ and a large special country populated by intermediaries. Each country has a Lucas tree in fixed unit supply. Output is homogeneous across countries. Cumulative output of i's tree up to time t is denoted by y_{it}, and flow output is $dy_{it} = \nu_t dt + \sigma dZ_{it}$. Expected output ν_t is common to all countries and evolves deterministically. The volatility σ is constant, and the random increments dZ_{it} are standard Brownian, independent across countries.

Agents from these countries only invest in their domestic trees and bonds issued by global intermediaries. Bonds are riskless and short-term, paying interest $r_t dt$. This is the risk-free rate of the economy. The price p_{it} of i's tree is an endogenous stochastic process. The instantaneous excess return on trees is the dividend yield and capital gains over and above the risk-free rate:

$$dR_{it} = \frac{dy_{it} + dp_{it}}{p_{it}} - r_t dt$$

Each country i houses a continuum of identical agents. Denote the wealth of the representative agent in i by w_{it} and the aggregate wealth of her country by $\underline{w}_{it}$. These processes will coincide in equilibrium, but agents take $\underline{w}_{it}$ as given.

For stationarity, the model includes a version of perpetual youth with additional wealth-sharing across countries. Agents in countries $i \in [0,1]$ die with a Poisson intensity λ. In this event, the dying agent's wealth is sent to the special country, and she is replaced with a newborn. All survivors transfer a share of their wealth to the newborn to make her net worth the same as her predecessor's. Similarly, agents in the special country die with a Poisson intensity λ^*, and their wealth is sent uniformly to regular countries $i \in [0,1]$, where it is shared between local residents in proportion to their net worth. In country i, the evolution of individual agent's wealth w_{it} is

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} - \frac{w_{it}}{\underline{w}_{it}} \cdot \lambda \underline{w}_{it} dt + \frac{w_{it}}{\underline{w}_{it}} \cdot \lambda^* \underline{w}_t^* dt \qquad (1)$$

Here c_{it} is consumption. The second term is the return on the tree, where θ_{it} is its portfolio share. The remaining terms reflect perpetual youth in the spirit of Blanchard (1985) and Yaari (1965). The third one is the transfers to newborns: the total flow of the dying agents' wealth is $\lambda \underline{w}_{it}$, and everyone in i compensates the newborns according to her own wealth share $w_{it}/\underline{w}_{it}$. The fourth term is the inflow of wealth from the special country: $\underline{w}_t^*$ is its aggregate wealth, λ^* is the death rate, and all arriving wealth is again shared in proportion to the net worth of local residents.

The sequence problem of the agent in the country i is

$$\max_{\{c_{is}, \theta_{is}\}_{s \geq t}} \mathbb{E}_t \left[\rho \int_t^\infty e^{\rho(t-s)} \log(c_{is}) ds \right]$$

subject to equation (1). Since everyone in i is the same, in equilibrium $w_{it} = \underline{w}_{it}$, and

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} + (\lambda^* w_t^* - \lambda w_{it})dt$$

I next describe the special country. It is special for two reasons. First, its asset supply is different. Second, its local agents are global intermediaries. I explain these two properties below, using notation with stars to separate this country from regular ones.

In contrast to regular countries, the special country is large, with a finite measure q^* of trees that are pooled together in a fund. The random components of their yields wash out, so the total output over dt in the special country is $q^* \nu_t dt$. These trees can only be traded as one, in a bundle with equal weights. I refer to this fund as the special country's tree for convenience. Its price is p_t^*, and the excess return is $dR_t^* = (\nu_t dt + dp_t^*)/p_t^* - r_t dt$.

The representative global intermediary is a head office and a set of trading desks. Each country i is assigned a continuum of identical trading desks. An individual desk has capital w_{it}^* evolving as

$$dw_{it}^* = (r_t w_{it}^* - c_{it}^*)dt + \theta_{it}^* w_{it}^* dR_{it} + \eta_{it}^* w_{it}^* dR_t^* - \frac{w_{it}^*}{\underline{w}_{it}^*} d\pi_{it} \tag{2}$$

The desk manager chooses consumption c_{it}^*, the share of portfolio θ_{it}^* to invest in country i's tree, and the share of portfolio η_{it}^* to keep in the special country's tree. The desk can issue short-term bonds paying r_t and makes profit rebates $d\pi_{it} \cdot w_{it}^*/\underline{w}_{it}^*$ to the head office. This profit rebate scales with its capital relative to the total capital $\underline{w}_{it}^*$ of trading desks assigned to country i. In equilibrium, desks will be identical, and $w_{it}^* = \underline{w}_{it}^*$.

The desk manager is worried about model misspecification. She is not sure about her statistical model of country i's returns dR_{it} and fears that the true Brownian motion is $d\hat{Z}_{it} = dZ_{it} - h_{it}dt$, where h_{it} is a drift correction. She considers alternative probability measures $\mathbb{Q}_i$, indexed by $\{h_{it}\}$, under which $\{\hat{Z}_{it}\}$ is standard Brownian. These measures generate likelihood ratio processes $\{M_{it}\}$ with respect to the truth, and the manager uses this likelihood ratio to benchmark her alternative scenarios against real-time data. Importantly, she does not consider misspecified models for other states, such as country i's local investor's wealth w_{it} or total capital $\underline{w}_{it}^*$ of desks assigned to i, only distorting the mapping from shocks to returns. Her problem is a variant of the multiplier problem in Hansen and Sargent (2001):

$$\max_{\{c_{is}^*, \theta_{is}^*, \eta_{is}^*\}_{t \geq 0}} \inf_{\mathbb{Q}} \mathbb{E}_t^{\mathbb{Q}} \left[\rho \int_t^\infty e^{\rho(t-s)} \log(c_{is}^*) ds + \underbrace{\int_t^\infty e^{\rho(t-s)} \gamma_s dm_{is}}_{\text{entropy penalty}} \right]$$

subject to equation (2). She optimally chooses consumption and portfolio allocation that are robust to pessimistic scenarios. These scenarios are disciplined by the relative entropy increments:

$m_{it} = \log(M_{it})$ is the log-likelihood ratio, and γ_t is the penalty multiplier. High γ_t curbs the manager's pessimism and makes her effectively more risk tolerant. It is exogenous and common across desks. Choosing $\mathbb{Q}_{it}$ is isomorphic to choosing the drift correction h_{it}. Limiting alternative models to drift correction ensures that all measures considered are absolutely continuous with respect to the true data-generating process, meaning they share zero-probability events.

The profit rebates ensure that all desks have the same wealth levels: $w^*_{it} = w^*_t$. They are structured as follows:

$$d\pi_{it} = \left(\underline{\theta}^*_{it}\underline{w}^*_{it}dR_{it} - \int \theta^*_{jt}w^*_t dR_{jt}\right) + \left(\underline{\eta}^*_{it}\underline{w}^*_{it}dR^*_t - \int \eta^*_{jt}w^*_t dR^*_t\right) + (\lambda^* w^*_t - \lambda w_t)dt \tag{3}$$

Here $\underline{\theta}^*_{it}$ is the average portfolio share allocated to the tree by desks assigned to i, $\underline{\eta}^*_{it}$ is the average share allocated to the special country's tree, w^*_t is the intermediary's total wealth, and w_t is the average wealth of regular countries. The intermediary's total wealth evolves as

$$dw^*_t = (r_t w^*_t - c^*_t)dt + \int [\theta^*_{it}w^*_t dR_{it}]di + \int [\eta^*_{it}w^*_t dR^*_t]di + \lambda w_t dt - \lambda^* w^*_t dt$$

This completes the description of the environment. To define equilibrium, I need notation for tree and bond holdings instead of positions $\{\theta_{it}, \theta^*_{it}, \eta^*_{it}\}$. Denote bond holdings of country i by b_{it} and their holdings of domestic trees by h_{it}. By construction, i's wealth is $w_{it} = b_{it} + p_{it}h_{it}$, and the risky share θ_{it} determines the split: $\theta_{it}w_{it} = p_{it}h_{it}$ and $(1-\theta_{it})w_{it} = b_{it}$.

To track the intermediary's holdings of trees and bond issuance, let b^*_{it} be the total bonds issued by desks assigned to i, let h^*_{it} be their holdings of trees in regular countries, and let s^*_{it} be their holdings of the special country's tree. By construction,

$$w^*_{it} = p_{it}h^*_{it} + p^*_t s^*_{it} - b^*_{it}$$
$$b^*_{it} = (\theta^*_{it} + \eta^*_{it} - 1)w^*_{it}$$

Portfolio weights $\{\theta^*_{it}, \eta^*_{it}\}$ satisfy $\theta^*_{it}w^*_{it} = p_{it}h^*_{it}$ and $\eta^*_{it}w^*_{it} = p^*_t s^*_{it}$ for all i.

Definition 1. *Given the processes $\{\nu_t, \gamma_t, \{Z_{it}\}\}_{t\geq 0}$ and the associated filtrations, an equilibrium is a collection of adapted price processes $\{r_t, \{p_{it}\}, p^*_t\}_{t\geq 0}$, wealth processes $\{\{w_{it}\}, \{\underline{w}_{it}\}, w^*_t, \underline{w}^*_t\}_{t\geq 0}$, consumption processes $\{\{c_{it}\}, c^*_t\}_{t\geq 0}$, and processes for asset holdings $\{\{h_{it}\}, \{h^*_{it}\}, \{b_{it}\}, b^*_t, h^*_t\}_{t\geq 0}$ such that all agents optimize and*

- *aggregate wealth process agrees with individual wealth: $w_{it} = \underline{w}_{it}$ and $w^*_{it} = \underline{w}^*_{it}$ for all i,*
- *bond market clears: $\int b_{it}di = \int b^*_{it}di$,*
- *markets for regular country trees clear: $h_{it} + h^*_{it} = 1$ for all $i \in [0,1]$,*

- *market for the special country tree clears:* $\int s^*_{it} di = q^*$,
- *market for consumption goods clears:* $\int c_{it} di + \int c^*_{it} di = (1 + q^*)\nu$.

Markets for trees clear country by country, and the market for the intermediary's bonds clears globally. The market for consumption goods clears automatically as soon as other markets do.

Modeling choices. There are two non-standard ingredients in the model. The first is a particular form of robustness concerns in the intermediary. As will be apparent below, this specification of preferences allows for time-varying effective risk tolerance and easy aggregation in all markets with closed-form solutions for portfolios. The second non-standard element is the specific form of market segmentation. Local agents in regular countries do not have access to risky assets elsewhere, and neither do trading desks assigned to those countries. This creates a wealth distribution across countries, since idiosyncratic shocks are not fully diversified. At the same time, trading desks share wealth, which allows me to save on state variables: there is no distribution of capital across desks. A model without immediate sharing of profits would be interesting, if costly in terms of computation, since it could capture the slow-moving capital implications of global shocks described by Mitchell, Pedersen, and Pulvino (2007), Acharya, Shin, and Yorulmazer (2009), and Duffie (2010).

4 Equilibrium

I now characterize equilibrium of the model. I first state the auxiliary results and the main asset-pricing equation, still referring to countries by their index i. I next switch to characterizing country-specific variables as functions of their local wealth, the only local state variable, and time. This allows me to describe the economy as a coupled system of partial differential equations: one for asset prices and one for the wealth distribution.

4.1 Main asset pricing equation

Solving for asset prices p_{it} requires characterizing equilibrium excess returns dR_{it} and using market clearing conditions. There is only idiosyncratic uncertainty in this economy. Only agents from country i and the intermediary have access to country i's tree, so prices and excess returns dR_{it} have the form

$$dp_{it} = \mu^p_{it} dt + \sigma^p_{it} dZ_{it}$$
$$dR_{it} = \mu^R_{it} dt + \sigma^R_{it} dZ_{it}$$

These equations define the drift and volatility of prices and excess returns $(\mu^p_{it}, \mu^R_{it}, \sigma^p_{it}, \sigma^R_{it})$, equilibrium objects related to each other by

$$\mu^R_{it} = \frac{\nu_t + \mu^p_{it}}{p_{it}} - r_t$$
$$\sigma^R_{it} = \frac{\sigma + \sigma^p_{it}}{p_{it}}$$

As usual, the volatility of returns has an exogenous and an endogenous component. The first result concerns consumption and portfolio choice.

LEMMA 1. *Agents consume a constant fraction of their wealth,* $c_{it} = \rho w_{it}$ *and* $c^*_t = \rho w^*_t$*, and choose mean-variance portfolios:*

$$\theta_{it} = \frac{\mu^R_{it}}{(\sigma^R_{it})^2} \text{ and } \theta^*_{it} = \gamma_t \frac{\mu^R_{it}}{(\sigma^R_{it})^2}$$

The results for the regular countries are not surprising given the log utility. The non-trivial part of this lemma is the intermediary's consumption and portfolio choice. The specific form of robustness concerns I use does not break the constant-share consumption rule, and the only change in portfolio choice is the new effective risk-tolerance coefficient γ_t. This allows for time-varying risk aversion while keeping tractability of portfolio choice characteristic of log utility. Other ways to achieve time-varying risk aversion include, for example, recursive preferences of Duffie and Epstein (1992) or habits of Campbell and Cochrane (1999). Recursive preferences create hedging motives in portfolio choice, making portfolios less tractable and introducing additional state variables that solve partial differential equations. Habits create additional state variables, increasing the number of endogenous objects to solve for. My specification instead introduces an exogenous state γ_t and keeps portfolios tractable and easy to aggregate. Oskolkov (2024) describes the general treatment of this vintage of robustness concerns and and shows that it is equivalent to value-st-risk constraints with stochastic risk limits. Shocks to the parameter γ_t that govern attitudes to uncertainty across models become shocks to risk-taking capacity. This is the main shock of the model that drives the global financial cycle.

A useful feature of portfolio shares θ^*_{it} and θ_{it} in Lemma 1 is that their ratio is γ_t. This means that the ratio of holdings h^*_{it} and h_{it} is $\gamma_t w^*_t / w_{it}$. At the same time, holdings sum to one, implying

$$h_{it} = \frac{w_{it}}{w_{it} + \gamma_t w^*_t} \text{ and } h^*_{it} = \frac{\gamma_t w^*_t}{w_{it} + \gamma_t w^*_t}$$

There are two participants in each country's market, and they split the tree according to their wealth shares and effective risk tolerance: one for local agents and γ_t for the intermediary. Domestic ownership of the tree is monotone in local wealth, converging to one in extremely rich countries

($w_{it} \longrightarrow \infty$) and zero in extremely poor ones ($w_{it} \longrightarrow 0$). The equilibrium portfolio shares are

$$\theta_{it} = \frac{p_{it}}{w_{it} + \gamma_t w_t^*} \text{ and } \theta_{it}^* = \frac{\gamma_t p_{it}}{w_{it} + \gamma_t w_t^*}$$

Using Lemma 1 to relate θ_{it} and θ_{it}^* to the returns process leads to the main result.

PROPOSITION 1. *The asset pricing equation for i's tree is*

$$\underbrace{\mu_{it}^p + \nu_t - r_t p_{it}}_{\text{risk premium}} = \underbrace{(\sigma_{it}^p + \sigma)^2}_{\text{quantity of risk}} \cdot \frac{1}{\gamma_t w_t^* + w_{it}} \tag{4}$$

Equation (4) is the main asset pricing equation in the model. It shows that the price of risk in every country is the weighted sum of the wealth in its market, where risk tolerance coefficients are the weights. The price of risk can rise due to local factors, such as negative output shocks that make the local agent poorer. It can also rise due to global factors: a negative shock to the intermediary's wealth or risk-taking capacity γ_t. These global factors are common to all countries, inducing synchronous movements in risk premia and ultimately asset prices.

A useful benchmark is $\gamma_t = \infty$. In this case, the intermediary can take advantage of full diversification, with country-specific risk washing out. Equation (4) shows that expected excess returns $\mu_{it}^R = (\mu_{it}^p + \nu_t - r_t p_{it})/p_{it}$ have to be zero in equilibrium. Otherwise, the intermediary would demand assets in unbounded quantities. Since $\mu_{it}^R = 0$, by Lemma 1, local agents are unwilling to hold their trees because they cannot take advantage of the continuum of foreign assets, and local shocks for them continue to be aggregate. As a result, the intermediary holds the entire global supply of risky assets: $h_{it}^* = 1$ for all i. Local agents are not exposed to risk and all have the same wealth in the long run due to perpetual-youth wealth reallocation between countries.

With a finite γ_t, the intermediary demands a positive risk premium, which induces local agents to participate as well. A non-degenerate wealth distribution emerges in equilibrium due to exposure to shocks. This distribution is a global state variable that determines all prices.

4.2 Coupled system

I will characterize asset prices and other quantities as functions of two variables: local wealth and time. This is possible because aggregate dynamics are deterministic, and the evolution of aggregate states, including the wealth distribution $G(w, t)$, can be indexed by time t only. Domestic wealth w replaces the index i since the local dynamics of two countries with the same wealth are identical.

Define the drift and volatility of local wealth w and tree prices $p(w, t)$ by

$$dw = \mu_w(w,t)dt + \sigma_w(w,t)dZ$$
$$dp(w,t) = \mu_p(w,t)dt + \sigma_p(w,t)dZ$$

The density $g(w,t)$ associated with the distribution $G(w,t)$ solves

$$\partial g_t(w,t) = -\partial_w(\mu_w(w,t)g(w,t)) + \frac{1}{2}\partial^2_{ww}(\sigma_w(w,t)^2 g(w,t)) \tag{5}$$

subject to an initial condition on $g(w,0)$. The wealth distribution, as always, is a backward-looking object that solves a forward Kolmogorov equation. The corresponding Kolmogorov backward equation is usually reserved for the agents' value functions. In my model, log utility makes solving for them redundant, and the backward equation instead applies to asset prices. The following proposition characterizes $p(w,t)$ and $r(t)$ for given exogenous processes $\{\gamma(t), \nu(t)\}$.

Proposition 2. *Asset prices $p(w,t)$ and the interest rate $r(t)$ solve the following equations:*

$$r(t)p(w,t) - \partial_t p(w,t) = \nu(t) \; + \; \underbrace{\mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww}p(w,t)}_{\text{expected capital gains}} \; \underbrace{- \; \sigma^2\pi(w,t)}_{\text{risk adjustment}} \tag{6}$$

$$r(t) = \rho + \frac{\nu'(t)}{\nu(t)} - \frac{\rho\sigma^2}{(1+q^*)\nu(t)}\int \pi(w,t)dG(w,t)$$

The drift and variance of wealth are

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \lambda^* w^*(t) + \frac{\sigma_w(w,t)^2}{w}$$

$$\sigma_w(w,t) = \frac{\sigma w}{w \; + \; \gamma(t)w^*(t) \underbrace{- \; w\partial_w p(w,t)}_{\text{amplification}}}$$

The risk compensation $\pi(w,t)$ is

$$\pi(w,t) = \frac{w + \gamma(t)w^*(t)}{[w + \gamma(t)w^*(t) - w\partial_w p(w,t)]^2}$$

The backward equation for prices follows from applying Itô's lemma to equation (4). There are two additional terms relative to fair-value pricing $r(t)p(w,t) - \partial_t p(w,t) = \nu(t)$. First, prices are a function of wealth, so its drift introduces a drift in $p(w,t)$. Second, risk adjustment reflects the market compensation for country-specific shocks.

The interest rate is related to the subjective discount rate and the growth rate of aggregate consumption, which is equal to the total output. The last term in the expression for the interest rate reflects precautionary motives: uninsured risk depresses the equilibrium interest rate.

Wealth volatility $\sigma_w(w,t)$ originates in the dividend risk and is proportional to σ and wealth w. The denominator has two terms. The first is the total market wealth $w + \gamma(t)w^*(t)$, which reflects risk sharing between the two participants. The second term is $-w\partial_w p(w,t)$, which accounts for endogenous risk amplification: dividend shocks hit wealth, which translates to prices and feeds back into wealth through them. The drift in wealth has a consumption-savings part $(r(t)-\rho)w$, the perpetual youth part $\lambda^* w^*(t) - \lambda w$, and the last term is the risk premium that investors receive for exposing their wealth to the trees. The risk compensation $\pi(w,t)$ inherits the term $-w\partial_w p(w,t)$ from the wealth volatility because local wealth w is a key variable driving asset returns. Without endogenous volatility, and hence this endogenous part in the denominator, $\pi(w,t)$ would simply be equal to $(\gamma(t)w^*(t)+w)^{-1}$, the partial equilibrium price of risk as seen in equation (4).

Finally, the special country's wealth $w^*(t)$ is an endogenous aggregate state. This is the last piece of equilibrium required to solve for asset prices. Following from consumption market clearing and the fact that aggregate consumption is a share ρ of aggregate output,

$$w^*(t) = \frac{(1+q^*)\nu(t)}{\rho} - \int w dG(w,t)$$

The special country's tree satisfies the fair value pricing equation $r(t)p^*(t) - \partial_t p^*(t) = \nu(t)$ because there is no associated risk.

I study the paths of aggregate states $\{\gamma(t), \nu(t)\}$ that lead the economy to converge to a global steady state. This steady state provides terminal conditions for equation (6).

5 Steady State

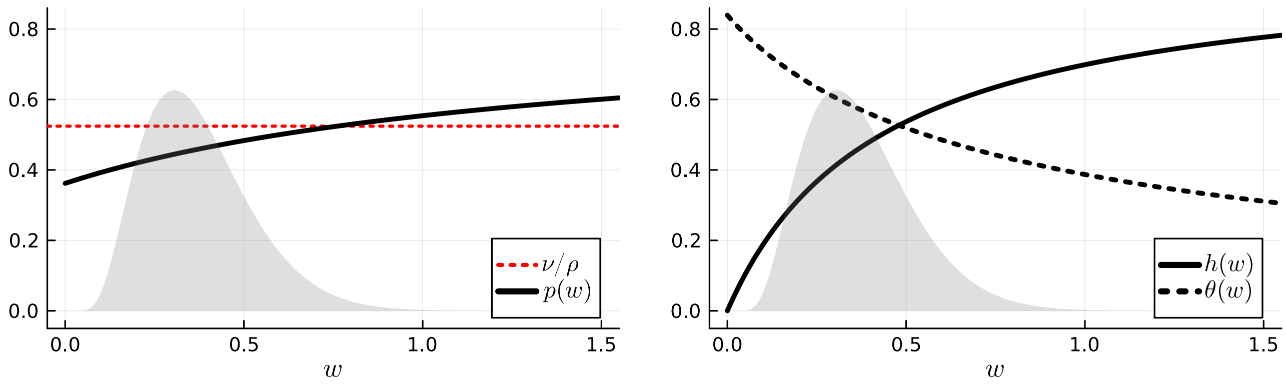
This section describes the steady state of the model. The economy features non-trivial dynamics even absent global shocks. Idiosyncratic dividends are only partially diversified, and countries with different histories of output form a cross-section, which then becomes the spectrum of responses to global shocks. To solve for the steady state, I drop the time subscripts and evaluate the model at the steady-state wealth distribution. I also choose equilibria with a finite limit of $p(w)$ as $w \longrightarrow \infty$ and $p^* = \nu/r$, ruling out bubbles in all assets.

5.1 Cross-section of countries

The left panel of Figure 2 shows local tree prices. The right panel of Figure 2 shows local tree holdings and risky portfolio shares. The steady-state distribution of wealth is in the background. Section 7 describes the parameters used for the figure.

Characterizing asset prices in general is challenging because of the non-linearity of the key coupled system. However, it is possible to characterize the dynamics in the right tail of the wealth distribution and the special country in general.

Proposition 3. *In the steady state,* $p^* = \nu/r > \nu/\rho$. *In rich regular countries, wealth volatility converges to a constant:* $\lim_{w\longrightarrow\infty} \sigma_w(w) = \sigma$. *The limiting wealth drift is negative:* $\lim_{w\longrightarrow\infty} \mu_w(w)/w = r - \rho - \lambda < 0$. *Prices converge to the special country's:* $\lim_{w\longrightarrow\infty} p(w) = p^*$.



(a) Asset prices and the fair value

(b) Holdings and risky shares of local agents

Figure 2: Panel (a): asset prices $p(w)$ and the fair value limit. Panel (b): tree holdings of local agents $h(w)$ and the risky portfolio shares $\theta(w)$.

Riskiness does not scale with wealth, and rich countries enjoy safe payoffs and consumption in relative terms: $\sigma_w(w)/w \longrightarrow 0$. The trend of their net worth goes negative, as they simply consume their accumulated wealth and drift back to the middle of the wealth distribution. Their asset prices converge to the safe asset benchmark. As a country grows richer, it increasingly looks like the special country, except it cannot get stuck in the right tail forever.

As the global economy contains uninsured risk, precautionary savings motive depress the interest rate: $r < \rho$. Safe assets are overvalued relative to fundamental cash flows: $p^* = \nu/r > \nu/\rho$. It is expected in the case of the special country that has no inherent risk. Proposition 3 shows that rich regular countries become so safe that they inherit this safety premium.

The special country enjoys its central position in the global financial system. The intermediary takes positions in trees all around the wealth distribution. With access to a continuum of uncorrelated returns, it earns excess returns with certainty. Its liabilities, on the contrary, are low-yielding risk-free bonds. This leads to "exorbitant privilege": profits allow the special country to sustain perpetual trade deficits even if it runs a net debt to the rest of the world. As shown, for example, by Gourinchas and Rey (2022), this corresponds to the empirical description of the US, whose external liabilities are dominated by safe assets, and external assets are mostly risky.

Proposition 4. *Suppose $\lambda = \lambda^* = 0$. The special country's trade deficit is*

$$\begin{aligned} c^* - q^*\nu &= r \cdot \underbrace{\left(\int p(w)h^*(w)dG(w) - b^* \right)}_{\text{net foreign assets}} + \underbrace{\int \sigma^2 \pi(w) h^*(w) dG(w)}_{\text{risk compensation } > \, 0} \\ &= r \cdot \left(\int p(w)h^*(w)dG(w) - b^* \right) + \underbrace{\int (\nu - rp(w))h^*(w)dG(w)}_{\text{risk discount } > \, 0} + \underbrace{\int \mu_p(w)h^*(w)dG(w)}_{\text{trading profits } > \, 0} \end{aligned}$$

The trade deficit $c^* - q^*\nu$ can be funded through interest payments on the net foreign asset position and the risk compensation. In the data, the US has negative net foreign assets, so the risk compensation term must be positive. This term can be decomposed into two sources. First, the risk discount reflects the fact that trees are priced at less than their fair value. The intermediary takes advantage of that, borrowing at r and buying claims to a stream of dividends ν at $p(w) < \nu/r$. Second, trading profits arise because countries move around the wealth distribution due to idiosyncratic shocks. This churn generates capital flows as the intermediary trades trees with local agents. Its trading strategy takes advantage of the drift in prices. The average drift is zero in the steady state, but the intermediary takes positions $h^*(w)$ that skew towards growing countries. As shocks reshuffle the distribution, these countries become richer, their assets appreciate, and the intermediary sells them to buy cheaper assets from countries that arrive to the left tail.

The trading profits term is also equal to the financial account surplus of the special country, while the trade deficit and factor payments (interest paid on debt less dividends received) combine into the current account deficit. The fact that trading profits are positive implies that the special country can run perpetual current account deficits as well as trade deficits.

5.2 Asset prices

I now show analytical expressions for the interest rate and asset prices in the steady state. To make progress on the non-linear coupled system, I consider the limit of small idiosyncratic shocks: $\sigma \longrightarrow 0$. This approximation generates closed-form expressions for risk premia and clarifies the magnitudes of the different moving parts in equilibrium.

Unsurprisingly, risk premia are comparable to the variance of idiosyncratic shocks σ^2. Less intuitively, it turns out that endogenous risk and wealth amplification effects only contribute terms of order $O(\sigma^4)$ to risk premia and asset prices. This has important implications for aggregate shocks and heterogeneity. Shifts in the wealth distribution created by aggregate shocks do affect risk premia, but these effects are qualitatively smaller than the direct impact of changing preferences for risk or dividends. Accounting for this difference in magnitudes substantially simplifies characterizing responses to aggregate shocks. Key impulse responses are available in closed form.

Define a price of risk function $\boldsymbol{\pi}(w)$:

$$\boldsymbol{\pi}(w) \equiv \frac{1}{w + \gamma w^*}$$

Let $\lambda = \boldsymbol{\lambda}\sigma^2$ and $\lambda^* = \boldsymbol{\lambda}^*\sigma^2$ and define also a pair of functions $\boldsymbol{m}(\cdot)$ and $\boldsymbol{s}(\cdot)$ as

$$\begin{aligned}
\boldsymbol{m}(w) &= w\left(\boldsymbol{\pi}(w)^2 - \frac{\rho}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) - \boldsymbol{\lambda}\right) + \boldsymbol{\lambda}^* w^* \\
\boldsymbol{s}(w) &= w\boldsymbol{\pi}(w)
\end{aligned}$$

Here $\mathcal{G}(\cdot)$ denotes the distribution with an associated density $\boldsymbol{g}(\cdot)$ that solves

$$(\boldsymbol{m}(w)\boldsymbol{g}(w))' = \frac{1}{2}(\boldsymbol{s}(w)^2\boldsymbol{g}(w))''$$

Finally, the special country's wealth w^* in the expression for $\boldsymbol{\pi}(\cdot)$ is

$$w^* = \frac{(1+q^*)\nu}{\rho} - \int x d\mathcal{G}(x)$$

With these ingredients, the following proposition characterizes the limiting steady state.

PROPOSITION 5. *In the limit $\sigma \longrightarrow 0$, the interest rate and asset prices are, up to $O(\sigma^4)$,*

$$r = \rho - \frac{\rho\sigma^2}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)d\mathcal{G}(x)$$
$$p^* = \frac{\nu}{\rho} + \frac{\sigma^2}{(1+q^*)\rho}\int \boldsymbol{\pi}(x)d\mathcal{G}(x)$$
$$p(w) = \frac{\nu}{\rho} + \frac{\sigma^2}{\rho}\cdot\left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) - \boldsymbol{\pi}(w)\right]$$

The second-order term in $p(w)$ is monotone in w, with $p(0) < \nu/\rho$ and $p(w) > \nu/\rho$ when $w \longrightarrow \infty$.

An important property of the small-σ approximation is that the risk compensation $\boldsymbol{\pi}(\cdot)$ is missing the amplification term $-wp'(w)$ in the denominator. This amplification channel reflects the equilibrium feedback loop from dividend shocks to wealth to prices, a source of non-linearity. It turns out that this non-linearity is of the next order of importance compared to the total wealth-weighted risk tolerance $w+\gamma w^*$ in the market. The "partial equilibrium" price of risk $(w+\gamma w^*)^{-1}$ is a good approximation for the general equilibrium risk compensation.

The strength of precautionary motives that depress the interest rate relative to the subjective discount rate is the average price of risk. Asset prices have an especially revealing functional form too. Country-specific price of risk $\boldsymbol{\pi}(w)$ depresses $p(w)$ relative to the fundamental cash flows ν/ρ. On the other hand, because of precautionary motives that lower the interest rate, all asset prices are inflated by the average price of risk. Since $\boldsymbol{\pi}(w) \longrightarrow 0$ as $w \longrightarrow \infty$, the second effect dominates for rich countries. Their risky assets trade at a premium relative to the fundamentals. At the other end, when $w \longrightarrow 0$, the first effect dominates owing to the monotonicity of $\boldsymbol{\pi}(\cdot)$. Finally, the special country's tree does not have an associated price of risk, which means it is inflated relative to ν/ρ by exactly as much as the interest rate is depressed relative to ρ.

All approximation errors are proportional to σ^2 instead of σ. This fact is due to symmetry: replacing σ with $-\sigma$ should not change anything in the economy because the underlying shocks are Brownian motions. Figure A.1 in Appendix A demonstrates the quality of the approximation by plotting the exact solutions for $\{\pi(\cdot), g(\cdot), \mu_w(\cdot), \sigma_w(\cdot)\}$ and the corresponding approximations for my preferred calibration described in Section 7.

6 Global Financial Cycle

The global cycle in assetp rices in the model is driven by a combination of aggregate financial and real shocks. I now characterize them, starting with the shock to intermediaries' risk-taking capacity $\gamma(t)$, which drives the financial component of the cycle. This financial shock generates a rich cross-section of responses in asset prices and capital flows. The interactions between gross capital flows and risk premia are at the heart of the transmission mechanism. I then characterize shocks to output $\nu(t)$ and finish with an identification result that allows me to back out financial and real shocks from asset price data.

6.1 Financial shocks

Consider a negative shock to intermediaries' risk-taking capacity $\gamma(t)$. This shock makes it less costly to entertain pessimistic worst-case scenarios for idiosyncratic shocks and leads the intermediary to back away from country risk. It does not discriminate between countries, inducing the intermediary to sell the same proportion of her position in all countries. The impact, however, is unequal for two reasons. First, since the intermediary holds a larger share of the market in poor countries, the resulting sudden stop targets the left tail of the wealth distribution. Second, the response of local agents depends on their wealth. In rich countries, local agents have large buffer stock abroad, and they can buy large quantities of domestic assets without a sharp rise in expected excess returns. In poor countries, local agents cannot absorb much more of their domestic assets. Expected excess returns have to rise more, convincing the intermediary to sell less in equilibrium. Adjustment in these markets happens through prices rather than quantities. Risk premia rise sharply in the left tail. Transaction volumes are much larger in the right tail.

In general equilibrium, the global risk-free rate changes in response to aggregate shocks. The price of risk rises because aggregate risk tolerance falls in all markets. Uninsured idiosyncratic risk depresses the risk-free rate more than before. With the global interest rate falling, asset prices are inflated relative to the steady state. This effect counteracts the rise in risk premia. It dominates in rich countries, where retrenchment effectively contains risk premia. As a result, risky asset prices in rich countries rise after a negative shock to global risk-taking capacity. These assets behave like colloquial "safe" assets because they have countercyclical returns. Importantly, their safe asset status is endogenous: the physical properties of dividends are fixed across countries. There are no fundamental differences in the cross-section, only wealth accumulated before the shock.

Formally, consider a single unanticipated shock $\tilde{\gamma}(t) = \delta e^{-\mu_\gamma t}$. I will use $\tilde{\gamma}(t) < 0$, a negative shock, as the running example. The economy approaches $t = 0$ resting at the steady state. The following proposition characterizes the deviations $\tilde{p}(w,t)$ and $\tilde{r}(t)$ from the steady-state values.

PROPOSITION 6. *The first-order deviations of the interest rate and asset prices are, up to $O(\sigma^4)$,*

$$\begin{aligned}\tilde{r}(t) &= \tilde{\gamma}(t)\sigma^2 \cdot \frac{\rho w^*}{(1+q^*)\nu} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) \\ \tilde{p}^*(t) &= -\tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{\rho+\mu_\gamma} \cdot \frac{1}{1+q^*} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) \\ \tilde{p}(w,t) &= -\tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{\rho+\mu_\gamma} \left[\frac{1}{1+q^*} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right]\end{aligned}$$

The function $\tilde{p}(w,t)$ is monotone in w, and it has the opposite signs at $w=0$ and $w \longrightarrow \infty$ for all $t \geq 0$. The change in the risk compensation is, up to $O(\sigma^2)$, given by $\tilde{\pi}(w,t) = -\tilde{\gamma}(t)w^*\boldsymbol{\pi}(w)^2$.

Proposition 6 clarifies what happens to the price of risk and asset prices across the wealth distribution. The change in the risk compensation is zeroth-order in σ. The intermediary's risk tolerance affects it directly. This depresses the global interest rate: uninsured idiosyncratic risk generates more precautionary motives if $\tilde{\gamma}(t) < 0$, and the interest rate falls. The price of the special country's tree rises, reacting to a lower discount rate.

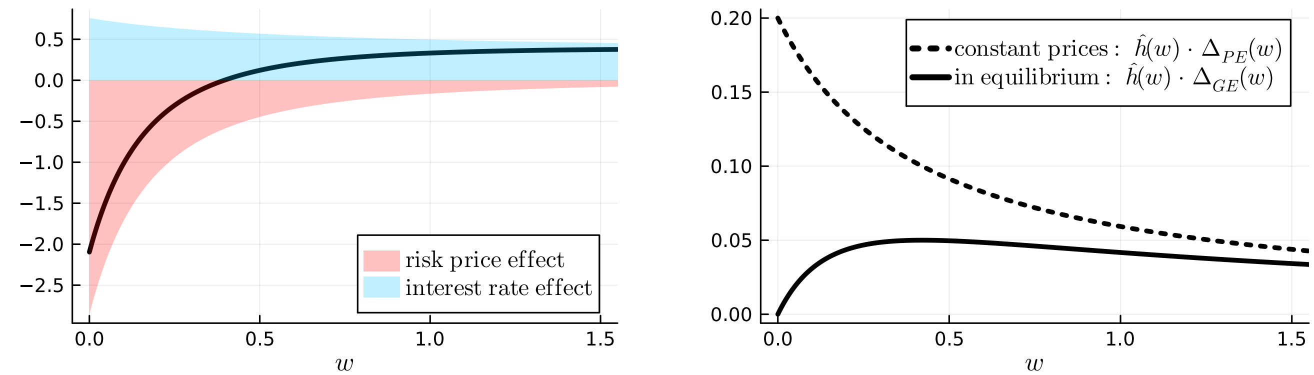
Prices of risky assets in regular countries are subject to two effects: they fall due to a rising local price of risk, but the global increase in the price of risk depresses the risk-free rate, and this increases all prices. The left panel on Figure 3 plots the change in $p(w)$ on impact as a function of the country's wealth before the shock, along with the two components.

Because of monotonicity in $\boldsymbol{\pi}(\cdot)$, the first effect surely dominates around $w=0$, and risky asset prices fall on impact in poor countries. The second effect comes out on top in rich countries, where $\boldsymbol{\pi}(w)^2 \longrightarrow 0$. Steady-state risk premia are small in those countries due to large domestic investors saturated with safe assets: they do not require a high premium to hold an additional unit of risk. The same large domestic investors pour in when there is capital flight, replacing foreign investors without asking for much higher expected excess returns.

To see this more clearly, consider the following quantity: how much the intermediary tries to sell in a given country at constant prices, as a fraction of its position before the shock. This is simply how much $\gamma(t)$ falls: $\Delta_{PE}(w) = -\tilde{\gamma}(t)/\gamma$. On the other hand, the general equilibrium change in its position is

$$\Delta_{GE}(w) = \frac{h^*(w,t) - h^*(w)}{h^*(w)} = -\frac{\tilde{\gamma}(t)}{\gamma} \cdot \frac{w}{w+\gamma w^*}.$$

This is how much domestic investors actually buy in equilibrium. This function converges to zero in poor countries, since local investors have no capacity to take over the assets amid capital flight. Prices adjust instead of quantities. This adjustment works through rising risk premia that convince foreign agents to stay. In rich countries, $\Delta_{GE}(w)$ approaches $\Delta_{PE}(w)$. Local investors actively retrench. Even if $\gamma(t)$ fell to zero, countries with large accumulated wealth ($w \longrightarrow \infty$) would simply take over their entire foreign liabilities. The right panel of Figure 3 shows the quantity of shares transacted: $h^*(w)\Delta_{PE}(\cdot)$ as the dashed line and $h^*(w)\Delta_{GE}(\cdot)$ as the solid line. The units of the vertical axis are the total supply of assets. The difference between the two lines is the unsold stock of assets that measures how much adjustment has to happen through prices.



(a) Percentage changes in asset prices on impact $100 \cdot \tilde{p}(w, 0)/p(w)$ and the two components.

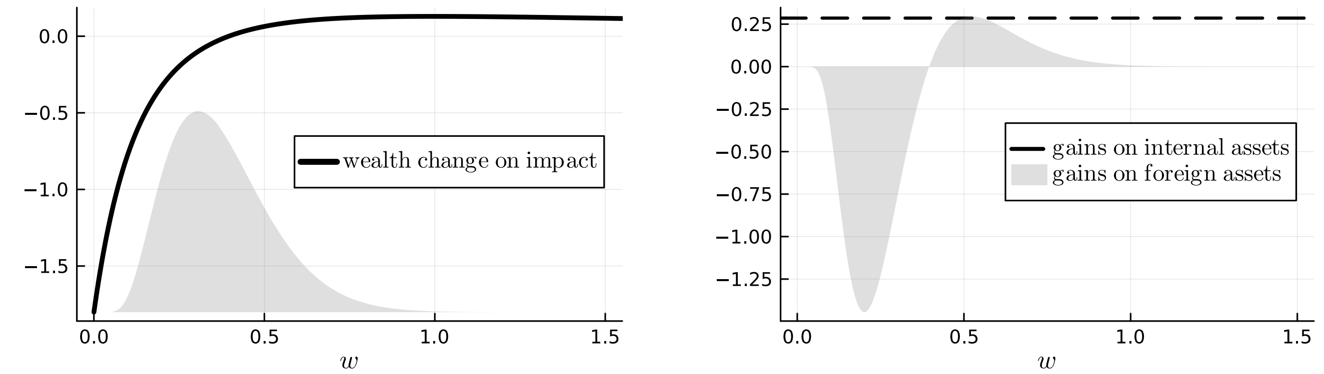
(b) Foreign sales $h^*(w)\Delta_{PE}(w)$ and $h^*(w)\Delta_{GE}(w)$ as a share of the total outstanding stock of trees.

Figure 3: Asset price changes and foreign asset sales on impact for a 20% negative shock to $\gamma(t)$.

Wealth redistribution and exorbitant duty. Investors in regular countries hold two assets on their balance sheets: trees and short-term bonds. There is no revaluation of bonds, so all changes in wealth come from the capital gains or losses on the trees. Since $\tilde{p}(w, 0)$ is negative for small w and positive for large, wealth is redistributed from poor countries to rich ones. The left panel of Figure 4 shows wealth changes on impact as a share of the country's wealth before the shock.

The right panel of Figure 4 shows capital gains and losses the intermediary makes on its portfolio when the shock hits. They are weighted with the wealth density of the regular countries: the shaded area adds up to total external gains and losses. The intermediary runs losses on the left tail of the wealth distribution in times of capital flight, which is a well-known empirical regularity. This is what the literature refers to as "exorbitant duty". By absorbing part of the losses on risky assets in regular countries, the special country essentially makes a wealth transfer to them. This insurance mechanism is present in models of Maggiori (2017), Sauzet (2023), Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022), and others.

A less-known outcome is that the special country makes capital gains on the right tail of the wealth distribution. Assets in the right-tail countries, although intrinsically risky, endogenously behave as safe and appreciate in capital flight events. The rest of the world partly insures the US. Another implication is that, through the US, rich countries also partly insure poor ones. The intermediary's net worth is a state variable for all asset prices, and capital gains that accrue to the US stabilize everyone's asset prices by supporting the intermediary's balance sheet in bad times.



(a) Percentage changes in regular countries' wealth on impact $100 \cdot \tilde{p}(w,0)h(w)/w$.

(b) Percentage change in intermediary's wealth on impact $100 \cdot \tilde{p}(w,0)h^*(w)/w^*$ and $100 \cdot \tilde{p}^*(0)q^*/w^*$.

Figure 4: Wealth changes on impact for a 20% negative shock to $\gamma(t)$. On the left: relative changes in regular countries' wealth. On the right: impact gains of the intermediary on its risky assets (weighted with the wealth density of regular countries) and on the special country's tree (dashed).

Finally, the dashed line shows the capital gains on domestic assets, the special country's tree. These are unambiguously positive after a negative shock to $\gamma(t)$. These capital gains can increase the special country's wealth in bad times even if it runs net losses on its net foreign asset position. Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022) highlight accounting for internal holdings as a way to reconcile the exorbitant duty of the US with the fact that its domestic currency appreciates in bad times. As pointed out by Maggiori (2017), if the US becomes relatively poorer as it transfers wealth to the rest of the world, the dollar should depreciate, absent other frictions and currency demand shocks. Capital gains on internal asset markets can help the US increase its wealth share despite the wealth transfer to other countries. Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022) argue that this is empirically true.

In my model, the sign of the net change in the special country's wealth after the shock to $\gamma(t)$ is ambiguous and depends on the calibration. However, it is possible to say that losses on external assets dominate when q^* is small. Without large internal holdings, the special country surely becomes relatively poorer, taking into account both its capital gains on rich regular countries and its losses on the poor ones. I formulate this as a corollary to Proposition 6.

Corollary 1. *If q^* is small enough, the impact change in the special country's wealth after a negative shock to $\gamma(t)$ is negative.*

While the presence of domestic US assets does not guarantee that the US wealth share increases in bad times in the model, their absence guarantees the opposite, and exorbitant duty obtains. The reason is that, in the steady state, the intermediary invests more in risky assets in the left tail of the wealth distribution, targeting their high idiosyncratic risk premia. These countries with high idiosyncratic risk are negatively affected by global risk, making returns on the US external portfolio procyclical. In my calibration, the US makes net gains on impact, consistent with the evidence presented by Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022).

6.2 Real shocks and identification

I now add global real shocks. These shocks affect expected output $\nu(t)$ in all countries. The exercise is the same as in the case of financial shocks: consider an unanticipated change $\tilde{\nu}(t) = \delta e^{-\mu_\nu t}$ that reverts to zero after hitting the economy that rests at the steady state coming up to $t = 0$. To make real shocks comparable to financial shocks in magnitude, I assume that their size is small: $\delta = O(\sigma^2)$. This puts real and financial shocks on an equal footing, since financial shocks are intrinsically a second-moment phenomenon.[1]

Proposition 7. *The first-order deviations of the interest rate and asset prices are, up to $O(\sigma^4)$,*

$$\tilde{r}(t) = -\mu_\nu \frac{\tilde{\nu}(t)}{\nu}$$
$$\tilde{p}^*(t) = \frac{\tilde{\nu}(t)}{\rho}$$
$$\tilde{p}(w,t) = \frac{\tilde{\nu}(t)}{\rho}$$

The global interest rate deviates from its steady-state value to account for expected consumption growth, hence the mean-reversion parameter μ_ν in the expression. A positive shock $\tilde{\nu}(t)$ is expected to revert back to zero, so it causes the interest rate to fall. All other asset prices are equally affected. Their changes reflect the changing net present value of dividends. The mean-reversion rate of the shock μ_ν drops out because dividends are discounted with the risk-free rate that also changes in proportion to μ_ν.

The fact that the impact of real shocks on asset prices is flat across the wealth distribution, while the impact of financial shocks has a clear wealth gradient, suggests that financial and real shocks can be identified through simple rotations of the panel of asset prices. Define the following asset price aggregates $p(t)$ and $x(t)$:

$$p(t) = \int p(w,t) dG(w,t)$$
$$x(t) = \int p(w,t)\delta(w,t) dG(w,t)$$

Here $\delta(w,t)$ is a weighting function: $\delta(w,t) = \mathbb{1}\{w \geq \text{med}(t)[w]\} - \mathbb{1}\{w < \text{med}[w](t)\}$. This

[1] An alternative approach that is becoming common in the literature is taking a double limit of small shocks and large risk aversion to bring risk premium shocks into the first order of importance. This would correspond to taking $\gamma(t)$ to infinity in a way that keeps $\gamma(t)\sigma^2$ finite. See Borovicka and Hansen (2013), Kekre and Lenel (2024), Oskolkov and Perez (2026) for examples.

weighting defines a high-minus-low component of the panel: asset prices in rich asset-rich economies compared to those in asset-poor ones. Taken together, $p(t)$ and $x(t)$ are a level and a slope factor that jointly capture both the common variation in asset prices and its wealth gradient, and hence should isolate the real and financial shocks. I formalize this result in the following proposition.

Proposition 8. *In the first order, the level and slope factors are, up to $O(\sigma^4)$,*

$$\tilde{p}(t) = \frac{1}{\rho} \cdot \tilde{\nu}(t) + \frac{w^* q^*}{(\rho + \mu_\gamma)(1 + q^*)} \int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w) \cdot \sigma^2 \tilde{\gamma}(t)$$
$$\tilde{x}(t) = \frac{w^*}{(\rho + \mu_\gamma)(1 + q^*)} \left(\int_{\overline{w}}^{\infty} \boldsymbol{\pi}(w)^2 d\mathcal{G}(w) - \int_0^{\overline{w}} \boldsymbol{\pi}(w)^2 d\mathcal{G}(w) \right) \cdot \sigma^2 \tilde{\gamma}(t)$$

The level factor $\tilde{p}(t)$ is strongly affected by the aggregate dividend. The impact of the financial shock $\tilde{\gamma}(t)$ is limited by the special country's asset supply q^*. If $q^* = 0$, financial shocks do not impact the average asset price at all. The reason is that this preference shock is purely redistributive: with a unit elasticity of intertemporal substitution, aggregate wealth is a fixed multiplier of the total consumption, which is not affected by $\tilde{\gamma}(t)$. In my calibration, $q^* > 0$, but quantitatively, real shocks affect the average price more strongly, while the high-minus-low slope factor is only affected by the financial shock.

By establishing an invertible mapping from the aggregate disturbances $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ to the level and slope factors, Proposition 8 suggest identification of shocks from the cross-section. To recover $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ from $\tilde{p}(t)$ and $\tilde{x}(t)$, I only need steady-state objects and the mean-reversion parameter μ_γ of the financial shock. I leverage this identification result to recover the real and financial shocks from asset prices below.

7 Empirical Analysis

In this section, I describe my calibration and the recovery of the financial and real shocks from the asset price data. Specifically, I use the identification result in Proposition 8 to back out aggregate shocks from the level and slope factors in the cross-section of MSCI indices. I then show that the recovered shocks are related to indirect proxies: financial shocks are correlated with the VIX and EBP time series, while real shocks are correlated to the US and OECD output series. Finally, I show that the model reproduces well the relationship between the asset-liability ratio of a country and the sensitivity of its asset prices to the global financial cycle, which is an untargeted moment.

7.1 Calibration

I calibrate the model using its steady state and estimate the processes for aggregate shocks to the intermediary's risk-taking capacity $\gamma(t)$ and global output $\nu(t)$ using linearized transition dynamics. The steady state of the model is determined by seven parameters: $\{\rho, \nu, \sigma, \lambda, \hat{\lambda}, \gamma, q^*\}$. The number of degrees of freedom is six: the interest rate r and the drift and variance of regular countries' wealth $\mu_w(w)$ and $\sigma_w(w)^2$ are homogeneous of degree one in $\{\rho, \nu, \sigma^2, \lambda, \lambda^*\}$, while asset prices $p(w)$ and p^*, the special country's wealth w^*, and the wealth density $g(w)$ are homogeneous of degree zero. With these degrees of freedom, I discipline the parameters by bringing the model to reproduce ten empirical moments.

Table 3: steady-state calibration.

	model	target	source
aggregates:			
global risk-free rate	3.8%	3.8%	Bertaut, Curcuru, Faia, and Gourinchas (2024)
average risk premium	5.1%	5.1%	Bertaut, Curcuru, Faia, and Gourinchas (2024)
emerging market premium	2.9%	2.9%	MSCI
US wealth share	37.8%	32.3%	Credit Suisse (2022)
US output share	11.8%	22.8%	World Bank
ratio A/L**:**			
mean	0.69	0.68	IFS (IMF)
standard deviation	0.50	0.49	IFS (IMF)
q25	0.33	0.31	IFS (IMF)
q50	0.56	0.56	IFS (IMF)
q75	0.89	0.89	IFS (IMF)

First, I use recent evidence from security-level holdings provided by Bertaut, Curcuru, Faia, and Gourinchas (2024). I take the average return on the US bond liabilities as a target for the risk-free rate in my model. I then take the difference between the average return on the US equity claims and its bond liabilities as a target for the average excess return earned by the intermediary. To discipline the differences in excess returns between advanced economies and emerging markets, I take MSCI equity indices for developed countries and emerging markets and compute the average annualized difference in returns. I take the US wealth and output share from Credit Suisse (2022) and the World Bank. The model underestimates the US output share, overestimating the special country's ability to generate wealth out of a given stock of productive assets. One reason for this might be that in the data, global intermediaries are not domiciled exclusively in the US, with some of the largest banks housed in Europe. The model, however, fully confines global financial intermediation to the special country, directing all intermediation profits to its residents.

To discipline the wealth distribution, I take the following statistic. For every country, I compute the ratio of external assets $A(w) = (1 - \theta(w))w$ to external liabilities $L(w) = p(w)h^*(w)$. I then take the distribution of this variable and bring its mean, standard deviation, and three quartiles close to the data. As an analog in the data, I take stocks of assets and liabilities corresponding to "private flows" in the terminology of Forbes and Warnock (2012) and Forbes and Warnock (2021): portfolio debt, portfolio equity, and banking flows. I exclude FDI and reserves. The data come from the International Financial Statistics database provided by the IMF.

The ratio $A(w)/L(w)$ is important because it shows the retrenchment capacity of the local agents. It determines how effectively they can replace foreign investors in a capital flight event. Table 3 shows that, while not exact, the match between the model and the data on the five moments of A/L is close. Table 4 shows the parameter values.

Table 4: model parameters.

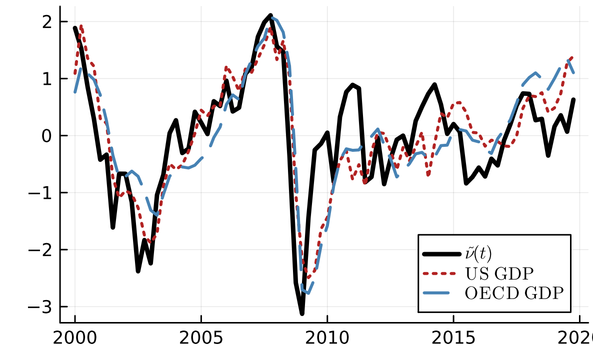
parameter	value	meaning
regular countries		
ρ	0.0719	discount rate
ν	0.0384	output rate
σ	0.1152	output volatility
λ	0.0418	wealth emigration rate
special country		
q^*	0.1333	asset stock
γ	1.8818	risk-taking capacity
λ^*	0.0857	wealth emigration rate

Turning to aggregate shocks, I postulate Ornstein-Uhlenbeck processes for the global risk-taking capacity $\gamma(t)$ and global output $\nu(t)$. Given persistence $\{\mu_\gamma, \mu_\nu\}$ and loadings $\{\sigma_\gamma, \sigma_\nu\}$,

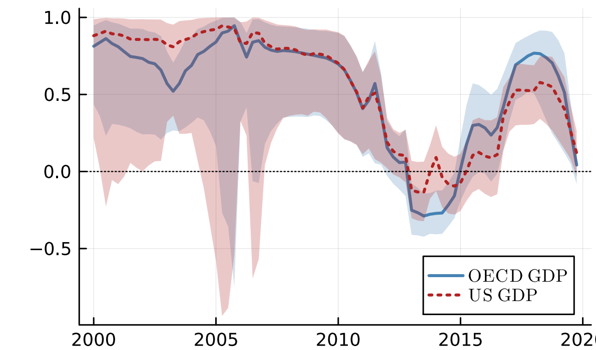
$$d\gamma(t) = \mu_\gamma(\gamma - \gamma(t))dt + \sigma_\gamma \cdot dW(t)$$
$$d\nu(t) = \mu_\nu(\nu - \nu(t))dt + \sigma_\nu \cdot dW(t)$$

Here $dW(t)$ is a two-dimensional standard Brownian motion and σ_γ and σ_ν are two-dimensional vectors. For deviations $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$, this specification implies $d\tilde{\gamma}(t) = \mu_\gamma\tilde{\gamma}(t) + \sigma_\gamma \cdot dW(t)$ and $d\tilde{\nu}(t) = \mu_\nu\tilde{\nu}(t) + \sigma_\nu \cdot dW(t)$. To recover the shocks $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$ from the level and slope factors $\tilde{p}(t)$ and $\tilde{x}(t)$, I only need steady-state objects specified in Proposition 8 and the mean-reversion parameter of the financial shock μ_γ. However, μ_γ only scales $\tilde{x}(t)$, so I can first recover $\tilde{\gamma}(t)$ from $\tilde{x}(t)$ up to a multiplier, and then estimate its persistence to determine the scale.

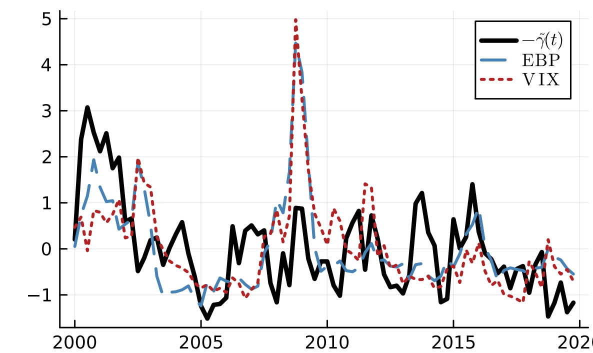
Figure 5a compares the recovered disturbance $\tilde{\nu}(t)$ to the HP-filtered series of the US GDP and the combined GDP of all OECD countries. Figure 5b shows rolling window correlation estimates with HAC-robust standard errors from Newey and West (1986) computed over 20-quarter windows. The rolling-window correlation stays positive for most of the sample. The recovered real shocks



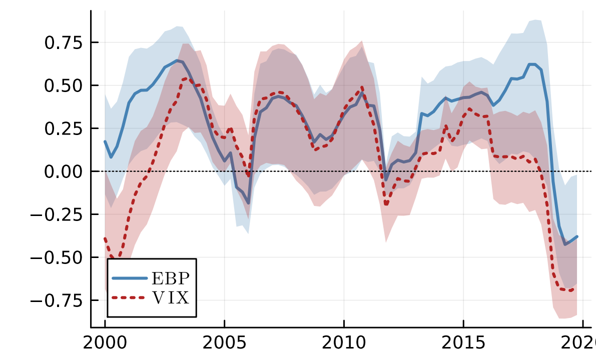
(a) recovered $\tilde{\nu}(t)$ and the GDP of US and OECD.



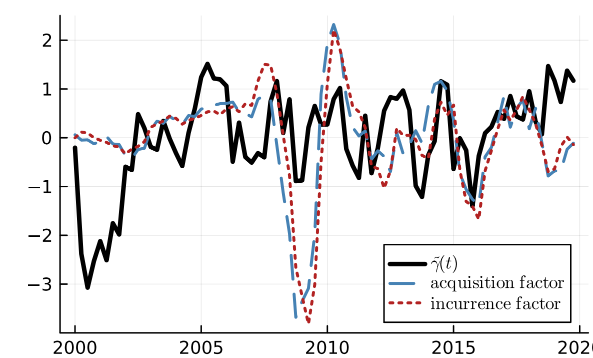
(b) rolling-window correlation of $\tilde{\nu}(t)$ and GDP.



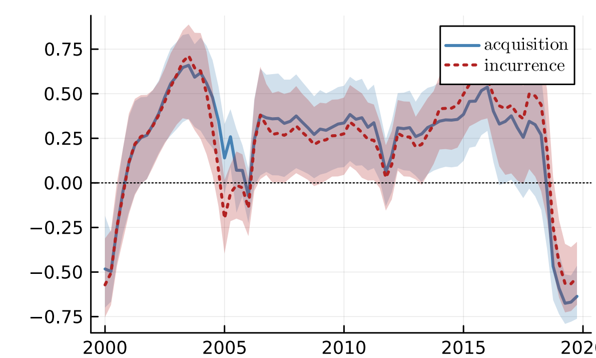
(c) recovered $\tilde{\gamma}(t)$ with the measures of financial stress VIX and EBP.



(d) rolling-window correlation of $\tilde{\gamma}(t)$ with the measures of financial stress VIX and EBP.



(e) recovered $\tilde{\gamma}(t)$ and the principal components of flows f_t^{acq} and f_t^{inc}.



(f) rolling-window correlation of $\tilde{\gamma}(t)$ with the principal components f_t^{acq} and f_t^{inc}.

compares almost equally well with the US and OECD series for GDP.

I compare the recovered financial shock $\tilde{\gamma}(t)$ to the four time series used in Section 2: the principal components of outward and inward flows, VIX, and EBP. These series time the global financial cycle. Figure 5c compares the recovered path of $\tilde{\gamma}(t)$ to the EBP and VIX. Figure 5d shows rolling window correlation estimates. The rolling-window correlation is mostly positive and is especially pronounced for EBP. Figure 5e and Figure 5f do the same for the financial flow aggregates. The rolling-window plot on Figure 5f shows that the correlation stays positive for most of the sample, except at the extremes.

7.2 Global financial cycle in cross-section

I now validate the model by comparing the structure of the cross-section of asset prices that it produces with the data. To do that, I combine the expressions for the impact of aggregate shocks on individual countries in Proposition 6 and Proposition 7 with the identification result in Proposition 8 that maps aggregate shocks to the level and slope factors $\tilde{p}(t)$ and $\tilde{x}(t)$. These results together imply that the cross-section of asset prices has a tractable level-slope decomposition.

PROPOSITION 9. *In the first order with respect to aggregate shocks and up to $O(\sigma^4)$,*

$$\tilde{p}(w,t) = \tilde{p}(t) + \beta(w)\tilde{x}(t) \tag{7}$$

Here $\beta(w)$ is

$$\beta(w) = (1+q^*) \cdot \frac{\boldsymbol{\pi}(w)^2 - \displaystyle\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x)}{\displaystyle\int_{\overline{w}}^{\infty} \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \int_0^{\overline{w}} \boldsymbol{\pi}(x)^2 d\mathcal{G}(x)}$$

The average risky asset price $\tilde{p}(t)$ is a true level factor, since it loads equally on all individual prices. The high-minus-low slope factor that computes asset price deviations in high-wealth countries against those in low-wealth ones is a slope factor, since its loading $\beta(w)$ is a monotone function of wealth w that integrates to zero. The slope factor is positive when the global economy is in financial distress: intermediaries' risk-taking capacity is low and capital flows recede. This beta is negative for low-wealth countries, whose external assets are small relative to liabilities: domestic agents cannot replace foreign investors in times of aggregate capital flight, so these countries underperform in global downturns. High-wealth countries, who have large foreign assets relative to liabilities, have a positive beta on the global risk-off due to their high retrenchment potential.

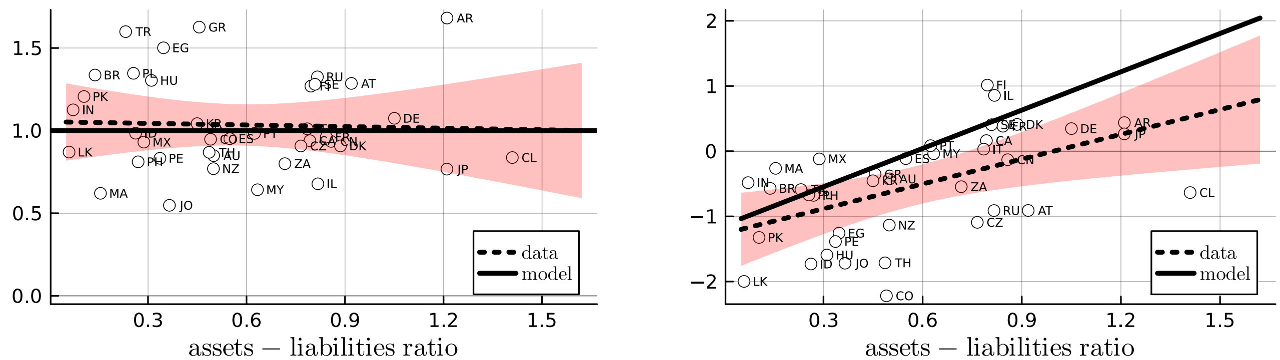
These global financial cycle betas are a natural untargeted moment. One last step I need to take to validate the model is recovering $\boldsymbol{\pi}(w)$, which $\beta(w)$ is a function of, from the available data: I do not directly observe w and have to use the asset-liability ratio instead. The following proposition establishes the mapping from this ratio, denoted by $\boldsymbol{\zeta}(w)$, to $\boldsymbol{\pi}(w)$.

PROPOSITION 10. *The steady-state risk prices are, up to $O(\sigma^4)$,*

$$\boldsymbol{\pi}(w) = \begin{cases} \dfrac{\sqrt{(\rho\gamma w^* - \nu)^2 + 4\rho\gamma\nu w^* \boldsymbol{\zeta}(w)} - \rho\gamma w^* - \nu}{2\gamma\nu w^*(\boldsymbol{\zeta}(w) - 1)}, & \text{if } \boldsymbol{\zeta}(w) \neq 1 \\ \dfrac{\rho}{\rho\gamma w^* + \nu}, & \text{if } \boldsymbol{\zeta}(w) = 1 \end{cases}$$

With this result, I compute model-implied betas $\beta(w)$ from the regression in equation (7) and compare them to the betas $\{\beta_i\}$ in the data:

$$\tilde{p}_{it} = \beta_i^{\text{level}} \tilde{p}_t + \beta_i^{\text{slope}} \tilde{x}_t + \epsilon_{it} \tag{8}$$



(a) level factor loadings.

(b) slope factor loadings.

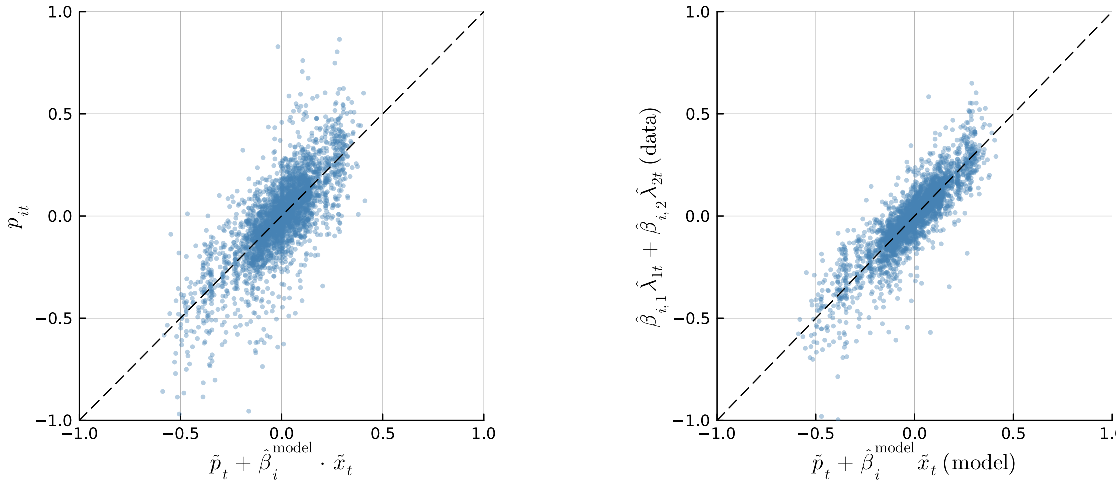
I also compare the coefficients $\{\alpha_i\}$ on the level factor to the model-implied benchmark of $\alpha_i = 1$. Figure 6a and Figure 6b show the fit of estimated regressions of alphas and betas on the asset-liability ratios in the data and in the model.

To assess if the two-factor structure in Proposition 9 is a good description of the cross-section of asset prices, I compare explanatory power of the factor regression equation (8) to the first two principal components:

$$\tilde{p}_{it} = \beta_{i,1}\lambda_{1,t} + \beta_{i,2}\lambda_{2,t} + \epsilon_{it} \tag{9}$$

The average R^2 of this regression is 73%. The average R^2 of the model-implied level-slope regression is 70%, which is remarkably close to the the theoretical upper bound achieved by the principal components. Moreover, the level factor almost coincides with the first principal component, while the correlation between the slope factor and the second principal component is 0.53. This indicates that the slope-level structure with model-implied sensitivities to the high-minus-low slope factor captures essential co-movement in asset prices well.

As a final experiment, I take the realized values of the level and slope factors $\tilde{p}(t)$ and $\tilde{x}(t)$ and predict asset prices in all country-quarter pairs in my sample using the model-implied sensitivities. Figure 7a shows the realized asset prices in my sample against those predicted by the level and slope components with model-implied sensitivities. The predictive R^2 of this exercise is 57%, reflecting relatively large idiosyncratic shocks. Figure 7b shows a more direct comparison: aggregate components of asset prices in the data, as captured by the two principal components, against those predicted by the model. Here the predictive power is 76%, demonstrating the good fit of the model-implied sensitivities and the close correspondence between the slope factor and the second principal component.



(a) asset prices in the data and their aggregate components in the model.

(b) aggregate components of prices in the data and predicted by the model.

8 Conclusion

I construct a model of the global financial cycle that is driven by shocks to risk-taking capacity of the global financial intermediaries. The shock makes the intermediaries reconsider their models for idiosyncratic shocks, which leads to repricing of country risk and generates aggregate capital flight. The key cross-sectional variable that determines how the global financial cycle affects individual countries is their external asset-liability ratio. Countries with low external assets see their risk premia rise sharply in global downturns. In countries with large ratios, domestic investors actively retrench in crises. These markets show high elasticity of domestic demand: large movements in quantities and stable asset prices. As a result, advanced economies face larger outflows of foreign investors in equilibrium, but their risky assets outperform those in emerging markets to due a higher retrenchment capacity local agents.

I derive asset price responses to aggregate shocks analytically and find that risk-off shocks have a wealth gradient in responses: calculating the relative performance of asset-rich countries compared to asset-poor ones isolates the financial component of the global cycle. I use this identification result to recover financial and real aggregate shocks from the panel of equity prices. The resulting real shock series strongly co-move with GDP series of the US and OECD, and the resulting financial shock series co-move with VIX, EBP, and principal components in financial flows. I then show that the level-slope factor structure of asset prices implied by the model has has empirical relevance, explaining 57% of the variation in global equities and 76% of the variation that can be statistically attributed to global shocks.

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Online Appendix for A Heterogeneous-Country Model of the Global Financial Cycle

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A Details of calibration and estimation.

Calibration. To calibrate the model, I construct the empirical distribution of assets-to-liability ratios A/L. I take private assets and liabilities: portfolio debt and equity and "other" claims. My sample starts in Q1 of 2003 and extends to 2024. I exclude the US to account for the fact it is a special country in the model. To adjust for the fact that not all countries are present in the sample throughout the period, I assign weights $\{\omega_i\}$ to all observations of a country i, where ω_i is the inverse of the share of the quarters the country i appears in the panel. I solve for the steady state of the model in full, not using the second-order approximation. Figure A.1 compares the steady-state objects from the full model to those obtained in the second-order approximation.

Technical remark. The technical content of Proposition 6 is that the main components of the interest rate and asset price deviations from the steady state only depend on the current level of $\tilde{\gamma}(t)$. Typically, they depend on the whole history of shocks, which leads to expressions like

$$\tilde{p}(w,t) = \int_0^\infty J(w,t,s)\tilde{\gamma}(t-s)\eta(ds)$$

Here $J(\cdot)$ is a sequence-space Jacobian that linearly maps the whole sequence $\{\tilde{\gamma}(s)\}_{s=-\infty}^{s=\infty}$ to the sequence of first-order deviations $\{\tilde{p}(\cdot,t)\}_{t\geq 0}$. Integration has to be done with a measure $\eta(\cdot)$ to accommodate the fact that $\tilde{\gamma}(t)$ may impact $\tilde{p}(w,t)$ discretely, in addition to continuous impact of $\{\tilde{\gamma}(s)\}_{s<t}$. The reason $J(w,t,s)$ typically depends on $s<t$ is that aggregate shocks move aggregate states, such as the wealth distribution, and these effects accumulate over $[0,t]$, adding to the direct effect of $\tilde{\gamma}(t)$.[2] Aggregate states still move in response to $\tilde{\gamma}(t)$ in my model, but Proposition 6 shows that, up to the fourth order of σ, these accumulating effects do not show up in individual asset prices and the interest rate. The Jacobian $J(w,t,s)$ is simply $J(w)$, and for one-dimensional variables like the global risk-free rate $r(t)$ or the special country's tree price $p^*(t)$, the corresponding Jacobian is a scalar. All that is needed to compute the Jacobians are the steady-state wealth distribution and price of risk.

[2] Auclert, Bardóczy, Rognlie, and Straub (2021) provide a detailed explanation.

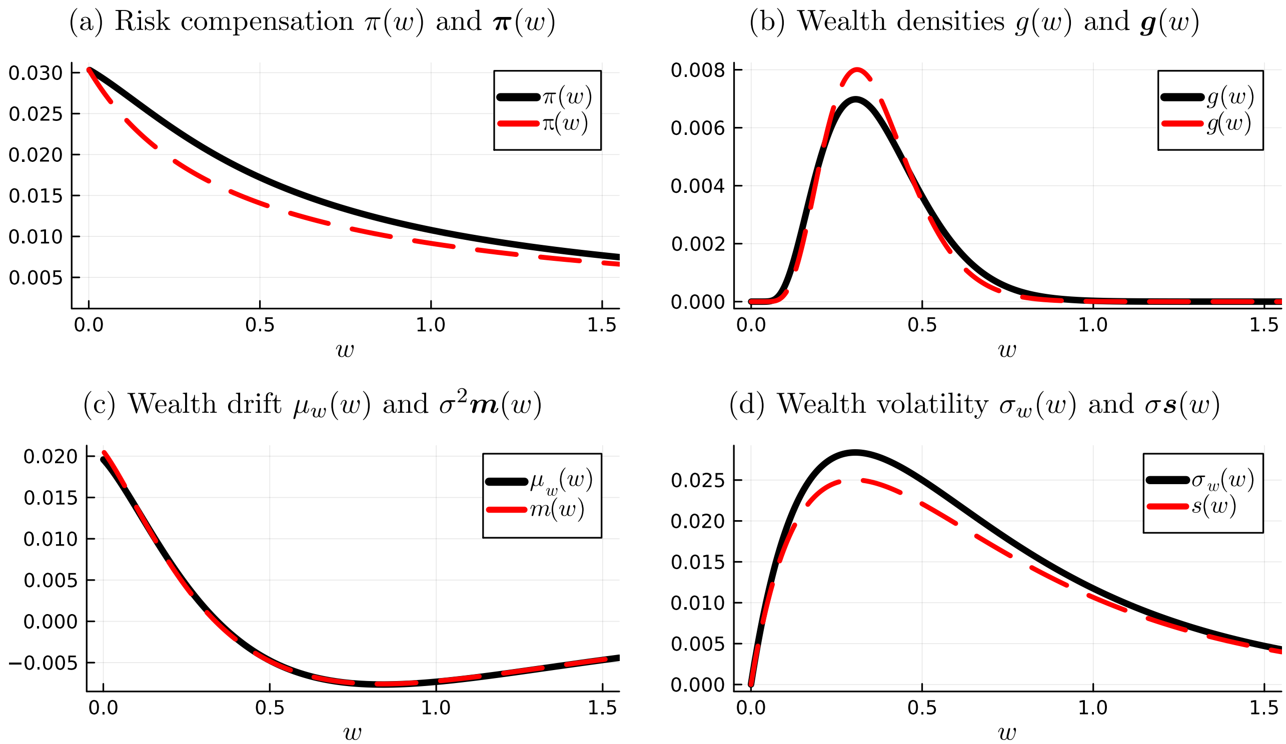


Figure A.1: Exact solutions and second-order approximations from Proposition 5.

B Proofs.

Proof of Lemma 1. Start with local agents. Denote by w their individual wealth and let x be all other state variables. Excess returns are

$$dR = \mu_R(x,t)dt + \sigma_R(x,t)dZ$$

Since aggregate dynamics are deterministic, x only loads on the same diffusion dZ. Let $\mu_X(x,t)$ and $\sigma_X(x,t)$ be the drift and loading vectors of x:

$$dx = \mu_X(x,t)dt + \sigma_X(x,t)dZ$$

The HJB equation for the local agent's value $V(w,x,t)$ is

$$\begin{aligned}
\rho V(w,x,t) - \partial_t V(w,x,t) = \max_{c,\theta}\ & \rho\log(c) + (r(t)w - c + \theta\mu_R(x,t)w)\partial_w V(w,x,t) \\
&+ \frac{\theta^2\sigma_R(x,t)^2w^2}{2}\partial^2_{ww}V(w,x,t) + \mu_X(x,t)\cdot\partial_x V(w,x,t) \\
&+ \frac{1}{2}\mathrm{tr}[\sigma_X(x,t)'\partial^2_{xx}V(w,x,t)\sigma_X(x,t)] + \sigma_X(x,t)\cdot\partial^2_{wx}V(w,x,t)\theta\sigma_R(x,t)w
\end{aligned}$$

Guess $V(w,x,t) = \log(w) + \eta(x,t)$. Under this and $\sigma_R(x,t) > 0$, $\partial^2_{wx}V(w,x,t) = 0$, $c = \rho w$, and

$$\theta = \frac{\mu_R(x,t)}{\sigma_R(x,t)^2}$$

Plugging this into the HJB and canceling terms,

$$\rho\eta(x,t) - \partial_t\eta(x,t) = r(t) - \rho + \frac{\mu_R(x,t)^2}{2\sigma_R(x,t)^2} + \mu_X(x,t)\cdot\partial_x\eta(x,t) + \frac{1}{2}\mathrm{tr}[\sigma_X(x,t)'\partial^2_{xx}\eta(x,t)\sigma_X(x,t)]$$

This verifies the conjecture that $V(w,x,t)$ is separable over w and (x,t).

Continue with the intermediary's trading desks assigned to i. First, given the drift correction h_{it} they choose, the entropy penalty is

$$dm_{it} = -h_{it}dZ_{it} - \frac{1}{2}h_{it}^2dt$$

The desk evaluates the expectation of this penalty under $\mathbb{Q}_t$, which is a probability measure under which $d\hat{Z}_{it} = dZ_{it} - h_{it}dt$ is a true standard Brownian increment. Hence

$$\mathbb{E}^{\mathbb{Q}_t}[dm_{it}] = \mathbb{E}^{\mathbb{Q}_t}\left[-h_{it}dZ_{it} - \frac{1}{2}h_{it}^2dt\right] = \mathbb{E}^{\mathbb{Q}_t}\left[-h_{it}d\hat{Z}_{it} + \frac{1}{2}h_{it}^2dt\right] = \frac{1}{2}h_{it}^2dt$$

The return process from the desk's perspective is

$$dR_{it} = (\mu^R_{it} - h_{it}\sigma^R_{it})dt + \sigma^R_{it}d\hat{Z}_{it}$$

The desk's budget constraint is

$$dw^*_{it} = (r_tw^*_{it} - c^*_{it})dt + (\theta^*_{it}(\mu^R_{it} - h_{it}\sigma^R_{it}) - \mu^\pi_{it})w^*_{it}dt + (\theta^*_{it}\sigma^R_{it} - \sigma^\pi_{it})w^*_{it}d\hat{Z}_{it}$$

Here μ^π_{it} and σ^π_{it} are the drift and volatility of the profit rebate:

$$\frac{w^*_{it}}{\underline{w}^*_{it}}d\pi_{it} = \mu^\pi_{it}w^*_{it}dt + \sigma^\pi_{it}w^*_{it}d\hat{Z}_{it}$$

The fact that $d\hat{Z}_{it}$ is the only source of randomness in profit rebates is due to the law of large numbers: all other shocks that affect the intermediary's total profits integrate out. The state variables of an individual desk are hence the same as those of the local investor, since all aggregates are deterministic and are subsumed in t. Given this, the desk's value $V^*(w^*, x, t)$ solves

$$\begin{aligned}\rho V^*(w^*, x, t) - \partial_t V^*(w^*, x, t) = \max_{c^*,\theta^*,\eta^*} \ \min_h \ &\rho \log(c^*) + \frac{\gamma h^2}{2} \\ &+ (r(x,t)w^* - c^* + \theta^* w^*(\mu_R(x,t) - h\sigma_R(x,t)) - w^*\mu_\pi(x,t))\partial_{w^*}V^*(w^*, x, t) \\ &+ \frac{(\theta^* w^* \sigma_R(x,t) - w^*\sigma_\pi(x,t))^2}{2}\partial^2_{w^*w^*}V^*(w^*, x, t) + \mu_X(x,t)\cdot\partial_x V^*(w^*, x, t) \\ &+ \frac{1}{2}\mathrm{tr}[\sigma_X(x,t)'\partial^2_{xx}V^*(w^*, x, t)\sigma_X(x,t)] \\ &+ \sigma_X(x,t)\cdot\partial^2_{w^*x}V^*(w^*, x, t)(\theta^* w^*\sigma_R(x,t) - w^*\sigma_\pi(x,t))\end{aligned}$$

Here I plug in $dR^*(x,t) = 0$ since the special country's tree does not carry any risk and hence will not provide excess returns in equilibrium. Guess $V^*(w^*, x, t) = \log(w^*) + \eta^*(x,t)$. Under this, $\partial^2_{w^*x}V^*(w^*, x, t) = 0$, $c^* = \rho w$, and

$$h = \frac{1}{\gamma}\theta^*\sigma_R(x,t)$$

$$\theta^*\sigma_R(x,t)^2 - \sigma_\pi(x,t)\sigma_R(x,t) = \mu_R(x,t) - h\sigma_R(x,t)$$

This implies

$$\theta^* = \frac{\gamma}{\gamma+1}\cdot\frac{\mu_R(x,t) + \sigma_\pi(x,t)\sigma_R(x,t)}{\sigma_R(x,t)^2}$$

Now recall that profit rebates keep wealth constant across desks assigned to all countries. This means $\sigma_\pi(x,t) = \theta^*\sigma_R(x,t)$, which leads to

$$\theta^* = \gamma\frac{\mu_R(x,t)}{\sigma_R(x,t)^2}$$

The final step is to verify the conjecture that $V^*(w^*, x, t) = \log(w^*) + \eta^*(x,t)$. Plugging the optimal solution into the HJB and canceling terms,

$$\begin{aligned}\rho\eta^*(x,t) - \partial_t\eta^*(x,t) = r(x,t) - \rho + \frac{\gamma}{2}\cdot\frac{\mu_R(x,t)}{\sigma_R(x,t)} - \mu_\pi(x,t) + \mu_X(x,t)\cdot\partial_x\eta^*(x,t) \\ + \frac{1}{2}\mathrm{tr}[\sigma_X(x,t)'\partial^2_{xx}\eta^*(x,t)\sigma_X(x,t)]\end{aligned}$$

This verifies the conjecture that $V^*(w^*, x, t)$ is separable over w^* and (x,t) completes the proof. □

Proof of Proposition 1. Portfolio shares in Lemma 1, the fact that $h_{it} = \theta_{it}w_{it}/p_{it}$ and $h^*_{it} =$

$\theta^*_{it} w^*_t / p_{it}$, and market clearing for each country's tree ($h_{it} + h^*_{it} = 1$) lead to the following expression:

$$p_{it} = (w_{it} + \gamma_t w^*_t) \cdot \frac{\mu^R_{it}}{(\sigma^R_{it})^2}$$

From this expression, it follows that $\theta_{it} = p_{it}/(w_{it} + \gamma_t w^*_t)$ and $\theta^*_{it} = \gamma_t p_{it}/(w_{it} + \gamma_t w^*_t)$m which leads to $h_{it} = w_{it}/(w_{it} + \gamma_t w^*_t)$ and $h^*_{it} = \gamma_t w^*_t/(w_{it} + \gamma_t w^*_t)$. Finally, equation (4) follows from applying the relation between $(\mu^R_{it}, \sigma^R_{it})$ and $(\mu^p_{it}, \sigma^p_{it})$. □

Proof of Proposition 2. Applying Itô's lemma to $p(w, t)$,

$$\begin{aligned} \mu_p(w,t) &= \partial_t p(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww} p(w,t) \\ \sigma_p(w,t) &= \sigma_w(w,t)\partial_w p(w,t) \end{aligned}$$

Now plug this into equation (4):

$$\begin{aligned} r(t)p(w,t) - \partial_t p(w,t) = \nu(t) &- \frac{(\sigma_w(w,t)\partial_w p(w,t) + \sigma)^2}{w + \gamma(t)w^*(t)} \\ &+ \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww} p(w,t) \end{aligned}$$

Plug the optimal consumption and portfolio choice into the local agent's budget constraint:

$$\begin{aligned} \mu_w(w,t) &= (r(t) - \rho - \lambda)w + \lambda^* w^*(t) + \frac{\mu_R(w,t)^2}{\sigma_R(w,t)^2} w \\ \sigma_w(w,t) &= \frac{\mu_R(w,t)}{\sigma_R(w,t)} w \end{aligned}$$

Eliminating $\mu_R(w,t)$ and $\sigma_R(w,t)$ from the expression for $\mu_w(w,t)$,

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \lambda^* w^*(t) + \frac{\sigma_w(w,t)^2}{w}$$

Next, using the fact that $\sigma_w(w,t) = \theta(w)\sigma_R(w,t)w$, the definition $\sigma_R(w,t) = (\sigma + \sigma_p(w,t))/p(w,t)$, and $\theta(w) = p(w)/(w + \gamma w^*(t))$, which follows from market clearing,

$$\sigma_w(w,t) = \frac{(\sigma + \sigma_p(w,t))w}{w + \gamma(t)w^*(t)} = \frac{(\sigma + \sigma_w(w,t)\partial_w p(w,t))w}{w + \gamma(t)w^*(t)} = \frac{\sigma w}{w + \gamma(t)w^*(t) - w\partial_w p(w,t)}$$

With this, the pricing equation can be transformed to

$$r(t)p(w,t) - \partial_t p(w,t) = \nu(t) - \frac{\sigma^2(w + \gamma(t)w^*(t))}{[w + \gamma(t)w^*(t) - w\partial_w p(w,t)]^2} + \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww}p(w,t)$$

Define the risk compensation

$$\pi(w,t) = \frac{w + \gamma(t)w^*(t)}{[w + \gamma(t)w^*(t) - w\partial_w p(w,t)]^2}$$

This completes the description of $p(w,t)$, $\pi(w,t)$, $\mu_w(w,t)$, and $\sigma_w(w,t)$. To derive the expression for the interest rate, multiply equation (6) by the density $g(w,t)$ and integrate:

$$r(t)\int p(w,t)g(w,t)dw - \int \partial_t p(w,t)g(w,t)dw = \nu(t) - \sigma^2 \int \pi(w,t)g(w,t)dw + \int \mu_w(w,t)g(w,t)\partial_w p(w,t)dw + \int \frac{\sigma_w(w,t)^2 g(w,t)}{2}\partial^2_{ww}p(w,t)dw$$

Integrate the last two terms by parts to get

$$\int r(t)p(w,t)g(w,t)dw - \int \partial_t p(w,t)g(w,t)dw = \nu(t) - \int \sigma^2\pi(w,t)g(w,t)dw - \int \partial_w(\mu_w(w,t)g(w,t))p(w,t)dw + \int \frac{\partial^2_{ww}(\sigma_w(w,t)^2 g(w,t))}{2}p(w,t)dw$$

Using equation (5), replace the last two terms

$$r(t)\int p(w,t)g(w,t)dw - \int \partial_t p(w,t)g(w,t)dw = \nu(t) - \sigma^2\int \pi(w,t)g(w,t)dw + \int \partial_t g(w,t)p(w,t)dw \tag{A.1}$$

This implies

$$r(t)\int p(w,t)g(w,t)dw - \int \partial_t(p(w,t)g(w,t))dw = \nu(t) - \sigma^2\int \pi(w,t)g(w,t)dw \tag{A.2}$$

Now use the fact that expected returns on the special country's tree are zero:

$$r(t)p^*(t) - \partial_t p^*(t) = \nu(t)$$

Multiplying this equation by q^* and adding to equation (A.2),

$$r(t)\left(\int p(w,t)g(w,t)dw + q^*p^*(t)\right) - \partial_t\left(\int p(w,t)g(w,t)dw + q^*p^*(t)\right) = (1+q^*)\nu(t) - \sigma^2\int \pi(w,t)g(w,t)dw$$

Consumption market clearing and $c(w,t) = \rho w$ and $c^*(t) = \rho w^*(t)$ imply

$$\int p(w,t)g(w,t)dw + q^*p^*(t) = \int wg(w,t)dw + w^*(t) = \frac{(1+q^*)\nu(t)}{\rho}$$

Hence,

$$\frac{(1+q^*)r(t)\nu(t)}{\rho} - \frac{(1+q^*)\nu'(t)}{\rho} = (1+q^*)\nu(t) - \sigma^2\int \pi(w,t)g(w,t)dw$$

Reorganizing this,

$$r(t) = \rho + \frac{\nu'(t)}{\nu(t)} - \frac{\rho\sigma^2}{(1+q^*)\nu(t)}\int \pi(w,t)g(w,t)dw$$

This completes the proof. □

Proof of Proposition 3. Consider the volatility of wealth first. Since $p'(w) \longrightarrow 0$ as $w \longrightarrow \infty$,

$$\sigma_w(w) = \frac{\sigma}{1 + w^*/w - p'(w)} \longrightarrow \sigma$$

From this, it immediately follows that

$$\frac{\mu_w(w)}{w} = r - \rho - \lambda + \frac{\lambda^* w^*}{w} + \frac{\sigma_w(w)^2}{w^2} \longrightarrow r - \rho - \lambda$$

The risk price $\pi(w)$ converges to zero as well:

$$\pi(w) = \frac{w + \gamma w^*}{[w + \gamma w^* - wp'(w)]^2} = \frac{1}{w}\cdot\frac{1 + \gamma w^*/w}{[1 + \gamma w^*/w - p'(w)]^2} \longrightarrow 0$$

Now consider the pricing equation

$$rp(w) = \nu - \sigma^2\pi(w) + \mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w)$$

The claim is that $p(w) \longrightarrow \nu/r$ as $w \longrightarrow \infty$. Suppose, toward a contradiction, that there exists $\varepsilon > 0$ such that for all $W > 0$ there is a $w > W$ such that $|p(w) - \nu/r| > \varepsilon$. Then, since $p''(w) \longrightarrow 0$ and $\sigma_w(w) \longrightarrow 0$ as $w \longrightarrow \infty$, it must be that there exists $\varepsilon > 0$ such that for all $W > 0$ there is a $w > W$ such that $|\mu_w(w)p'(w)| > \varepsilon$. This last inequality can be rewritten as

$$\left|\frac{\mu_w(w)}{w} \cdot p'(w)w\right| > \varepsilon \Longrightarrow \left|\frac{\mu_w(w)}{w}\right| \cdot |p'(w)w| > \varepsilon$$

But since $\mu_w(w)/w \longrightarrow r - \rho - \lambda$, there exists a $\overline{W} > 0$ such that for all $w > \overline{W}$, it holds that $|\mu_w(w)/w| < 2|r - \rho - \lambda|$. Hence, there exists a $\overline{W} > 0$ such that for all $w > \overline{W}$,

$$|p'(w)|w > |p'(w)w| > \frac{\varepsilon}{2|r - \rho - \lambda|}$$

This implies that $p'(w)$ does not change its sign on $(\overline{W}, \infty)$. Hence, on all of the $(\overline{W}, \infty)$,

$$\text{either } p'(w) > \frac{\varepsilon}{2w|r - \rho - \lambda|} \text{ or } p'(w) < -\frac{\varepsilon}{2w|r - \rho - \lambda|}$$

Integrating $p'(w)$ from $\overline{W}$ to $(1 + \tilde{})\overline{W}$,

$$\text{either } p((1 + \tilde{})\overline{W}) > p(\overline{W}) + \frac{\log(1 + \tilde{})\varepsilon}{2|r - \rho - \lambda|} \text{ or } p((1 + \tilde{})\overline{W}) < p(\overline{W}) - \frac{\log(1 + \tilde{})\varepsilon}{2|r - \rho - \lambda|}$$

This contradicts $p(\cdot)$ having a finite limit at infinity. Hence, $p(w) \longrightarrow \nu/r$ as $w \longrightarrow \infty$. □

Proof of Proposition 4. In the steady state, $dw^*(t) = 0$. Using the intermediary's budget constraint and setting the perpetual youth terms to zero, $\lambda = \lambda^* = 0$,

$$\begin{aligned}
c^* &= rw^* + \int w^*\theta^*(w)\mu_R(w)dG(w) \\
&= r\left(\int p(w)h^*(w)dG(w) - b^* + q^*p^*\right) + \int w^*\theta^*(w)\theta(w)\sigma_R(w)^2 dG(w) \\
&= q^*\nu + r\left(\int p(w)h^*(w)dG(w) - b^*\right) + \int \frac{\gamma w^*(\sigma_w(w)p'(w) + \sigma)^2}{(w + \gamma w^*)^2} dG(w) \\
&= q^*\nu + r\left(\int p(w)h^*(w)dG(w) - b^*\right) + \int \frac{\gamma w^*\sigma^2}{[w + \gamma w^* - wp'(w)]^2} dG(w) \\
&= q^*\nu + r\left(\int p(w)h^*(w)dG(w) - b^*\right) + \int \sigma^2 h^*(w)\pi(w)dG(w)
\end{aligned}$$

The first equality here uses the definition of wealth w^* and fact that $\theta(w) = \mu_R(w)/\sigma_R(w)^2$, the second equality uses $p^* = \nu/r$ and plugs in $\sigma_R(w) = (\sigma_p(w) + \sigma)/p(w) = (\sigma_w(w)p'(w) + \sigma)/p(w)$, the third one plugs in $\sigma_w(w)$, and the last one uses the expressions for $\pi(w)$ and $h^*(w)$. This proves the first part.

On the other hand,

$$c^* = rw^* + \int w^*\theta^*(w)\mu_R(w)dG(w) = r\left(\int p(w)h^*(w)dG(w) - b^* + q^*p^*\right)$$
$$+ \int w^*\theta^*(w)\frac{\mu_p(w) + \nu - rp(w)}{p(w)}dG(w)$$
$$= q^*\nu + r\left(\int p(w)h^*(w)dG(w) - b^*\right) + \int \gamma w^*\frac{\mu_p(w) + \nu - rp(w)}{w + \gamma w^*}dG(w)$$
$$= q^*\nu + r\left(\int p(w)h^*(w)dG(w) - b^*\right) + \int h^*(w)(\nu - rp(w))dG(w) + \int h^*(w)\mu_p(w)dG(w)$$

Here the first equality uses the definition of wealth w^* and expected excess returns $\mu_R(w)$, the second equality uses $p^* = \nu/r$ and $\theta^*(w) = \gamma\theta(w) = \gamma p(w)/(w + \gamma w^*)$, and the third one uses the expression for $h^*(w)$. This proves the second part. □

Proof of Proposition 5. I first restate the proposition.
In the limit $\sigma \longrightarrow 0$, the interest rate and asset prices are

$$r = \rho - \frac{\rho\sigma^2}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) + O(\sigma^4) \tag{A.3}$$
$$p^* = \frac{\nu}{\rho} + \frac{\sigma^2}{(1+q^*)\rho}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) + O(\sigma^4) \tag{A.4}$$
$$p(w) = \frac{\nu}{\rho} + \frac{\sigma^2}{\rho}\cdot\left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) - \boldsymbol{\pi}(w)\right] + O(\sigma^4) \tag{A.5}$$

The second-order term in $p(w)$ is monotone in w, with $p(0) < \nu/\rho$ and $p(w) > \nu/\rho$ when $w \longrightarrow \infty$. The functions $\boldsymbol{\pi}(\cdot)$, $\boldsymbol{g}(\cdot)$, $\boldsymbol{m}(\cdot)$, and $\boldsymbol{s}(\cdot)$ approximate the risk compensation, the steady-state wealth density, and the drift and volatility of wealth:

$$\pi(w) = \boldsymbol{\pi}(w) + O(\sigma^2) \tag{A.6}$$
$$g(w) = \boldsymbol{g}(w) + O(\sigma^2) \tag{A.7}$$
$$\mu_w(w) = \sigma^2\boldsymbol{m}(w) + O(\sigma^4) \tag{A.8}$$
$$\sigma_w(w) = \sigma\boldsymbol{s}(w) + O(\sigma^3)$$

Start with the pricing equation:

$$rp(w) = \nu - \sigma^2\pi(w) + \mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w) \tag{A.9}$$

Take the interest rate:

$$r = \rho - \frac{\rho\sigma^2}{(1+q^*)\nu}\int \pi(x)dG(x) = \rho + O(\sigma^2) \tag{A.10}$$

This implies

$$\begin{aligned}\mu_w(w) &= (r - \rho - \boldsymbol{\lambda}\sigma^2)w + \boldsymbol{\lambda}^*\sigma^2 w^* + \frac{\sigma_w(w)^2}{w} \\ &= \boldsymbol{\lambda}^*\sigma^2 w^* - \left(\frac{\rho}{(1+q^*)\nu}\int \pi(x)dG(x) + \boldsymbol{\lambda}\right)\sigma^2 w + \frac{\sigma^2 w}{[w + \gamma w^* - wp'(w)]^2} = O(\sigma^2)\end{aligned} \tag{A.11}$$

Since $\mu_w(w) = O(\sigma^2)$, $\sigma_w(w)^2 = O(\sigma^2)$, and $r = \rho + O(\sigma^2)$, the pricing equation can be cast as

$$\rho p(w) = \nu + O(\sigma^2) \Longrightarrow p(w) = \frac{\nu}{\rho} + O(\sigma^2) \tag{A.12}$$

This immediately implies $p'(w) = O(\sigma^2)$ and $p''(w) = O(\sigma^2)$, leading to

$$\pi(w) = \frac{1}{w + \gamma w^*} + O(\sigma^2) \equiv \boldsymbol{\pi}(w) + O(\sigma^2) \tag{A.13}$$

Hence,

$$r = \rho - \frac{\rho\sigma^2}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)dG(x) + O(\sigma^4) \tag{A.14}$$

This establishes the approximation for r. Using this and $\mu_w(w)p'(w) + \sigma_w(w)^2 p''(w)/2 = O(\sigma^4)$,

$$\left(\rho - \frac{\rho\sigma^2}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)dG(x)\right)p(w) = \nu - \sigma^2\boldsymbol{\pi}(w) + O(\sigma^4) \tag{A.15}$$

Reorganizing,

$$p(w) = \frac{\nu}{\rho} + \frac{\sigma^2}{\rho}\left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)dG(x) - \boldsymbol{\pi}(w)\right] + O(\sigma^4) \tag{A.16}$$

Similarly, $rp^* = \nu$ implies

$$p^* = \frac{\nu}{\rho} + \frac{\sigma^2}{(1+q^*)\rho}\int \boldsymbol{\pi}(x)dG(x) + O(\sigma^4) \tag{A.17}$$

This establishes the approximation for asset prices. Finally, wealth dynamics are

$$\sigma_w(w) = \frac{\sigma w}{w+\gamma w^*} + O(\sigma^3) \equiv \sigma \boldsymbol{s}(w) + O(\sigma^3) \tag{A.18}$$

$$\begin{aligned}\mu_w(w) &= \sigma^2 \left[\boldsymbol{\lambda^*} w^* + \left(\boldsymbol{\pi}(w)^2 - \frac{\rho}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)dG(x) - \boldsymbol{\lambda}\right) w\right] + O(\sigma^4) \\ &\equiv \sigma^2 \boldsymbol{m}(w) + O(\sigma^4)\end{aligned} \tag{A.19}$$

Take the Kolmogorov forward equation,

$$(\mu_w(w)g(w))' = \frac{1}{2}(\sigma_w(w)^2 g(w))'' \tag{A.20}$$

Define $\boldsymbol{g}(\cdot)$ by

$$(\boldsymbol{m}(w)\boldsymbol{g}(w))' = \frac{1}{2}(\boldsymbol{s}(w)^2\boldsymbol{g}(w))'' \tag{A.21}$$

Expanding the drift and volatility,

$$\begin{aligned}(\boldsymbol{m}(w)g(w))' - \frac{1}{2}(\boldsymbol{s}(w)^2 g(w))'' &= (\boldsymbol{m}(w)[g(w)-\boldsymbol{g}(w)])' - \frac{1}{2}(\boldsymbol{s}(w)^2[g(w)-\boldsymbol{g}(w)])'' \\ &= O(\sigma^2)\end{aligned} \tag{A.22}$$

Hence, $g(w) = \boldsymbol{g}(w) + O(\sigma^2)$. This completes the proof. □

Proof of Corollary 1. The impact revaluation of the intermediary's wealth is, up to $o(\sigma^3)$,

$$\begin{aligned}\tilde{w}^*(0) &= \int \tilde{p}(w,0)h^*(w)d\mathcal{G}(w) + \tilde{p}^*(0)q^* \\ &= -\frac{\tilde{\gamma}(0)\sigma^2}{\rho+\mu_\gamma}\left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\left(\int h^*(w)d\mathcal{G}(w) + q^*\right) - \int h^*(w)\boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\right] \\ &= -\frac{\tilde{\gamma}(0)\sigma^2}{\rho+\mu_\gamma}\left[\int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\int h^*(w)d\mathcal{G}(w) - \int \boldsymbol{\pi}(w)^2 h^*(w)d\mathcal{G}(w) + \frac{q^*}{1+q^*}\Omega(w^*)\right] \\ &= \frac{\tilde{\gamma}(0)\sigma^2}{\rho+\mu_\gamma}\left[\mathbb{C}\left[h^*(w),\boldsymbol{\pi}(w)^2\right] - \frac{q^*}{1+q^*}\Omega(w^*)\right]\end{aligned}$$

Here $\Omega(w^*)$ is given by

$$\Omega(w^*) = \left[1 - \int h^*(w)d\mathcal{G}(w)\right]\int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w) \leq \int \frac{1}{(w+\gamma w^*)^2}d\mathcal{G}(w) \leq \frac{\rho^2}{((1+q^*)\nu - (1-\gamma)\rho w^*)^2}$$

Here the first inequality uses $0 \leq h^*(\cdot) \leq 1$ and $\boldsymbol{\pi}(w) = (w+\gamma w^*)^{-2}$, and the second uses Jensen's inequality: given w^*, $\mathbb{E}[w] = (1+q^*)\nu/\rho - w^*$, and $(w+\gamma w^*)^{-2}$ is a convex positive function of

x, so $\mathbb{E}[(w+\gamma w^*)^{-2}] \leq (\mathbb{E}[w]+\gamma w^*)^{-2} = ((1+q^*)\nu/\rho - (1-\gamma)w^*)^2$. Since $\Omega(w^*)$ is a bounded positive function, for small enough q^*,

$$\mathbb{C}\left[h^*(w), \boldsymbol{\pi}(w)^2\right] - \frac{q^*}{1+q^*}\Omega(w^*) > 0$$

This follows from the fact that $h^*(w) = \gamma w^* \boldsymbol{\pi}(w)$ is a positive decreasing function, which is hence positively correlated with another positive decreasing function $\boldsymbol{\pi}(w)^2$. This fact means that for small enough q^*, $\tilde{w}^*(0)$ has the same sign as $\tilde{\gamma}(0)$. This completes the proof. □

Proof of Proposition 6. I first restate the proposition.

The $\tilde{\gamma}(t)$-first-order deviations of the interest rate and asset prices are

$$\tilde{r}(t) = \tilde{\gamma}(t)\sigma^2 \cdot \frac{\rho w^*}{(1+q^*)\nu} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4) \tag{A.23}$$

$$\tilde{p}^*(t) = -\tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{\rho+\mu_\gamma} \cdot \frac{1}{1+q^*} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4) \tag{A.24}$$

$$\tilde{p}(w,t) = -\tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{\rho+\mu_\gamma}\left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4) \tag{A.25}$$

The function $\tilde{p}(w,t)$ is monotone in w, and it has the opposite signs at $w=0$ and $w \longrightarrow \infty$ for all $t \geq 0$. The change in the risk compensation is

$$\tilde{\pi}(w,t) = -\tilde{\gamma}(t) w^* \boldsymbol{\pi}(w)^2 + O(\sigma^2) \tag{A.26}$$

Consider a constant $\nu(t) \equiv \nu$. Take the pricing equation (6) and linearize it with respect to $\tilde{\gamma}(t) \equiv \gamma(t) - \gamma$. Take the first-order deviations $\tilde{r}(t)$, $\tilde{w}^*(t)$, $\tilde{p}(w,t)$, $\tilde{\pi}(w,t)$, $\tilde{\mu}_w(w,t)$ and $\tilde{\sigma}_w(w,t)$:

$$\begin{aligned}\tilde{r}(t)p(w) + r\tilde{p}(w,t) - \tilde{p}_t(w,t) &= -\sigma^2\tilde{\pi}(w,t) + \tilde{\mu}_w(w,t)p'(w) + \mu_w(w)\partial_w\tilde{p}(w,t) \\ &\quad + \tilde{\sigma}_w(w,t)\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial^2_{ww}\tilde{p}(w,t)\end{aligned} \tag{A.27}$$

Doing the same with the equation for $p^*(t)$,

$$\tilde{r}(t)p^* + r\tilde{p}^*(t) - \tilde{p^*}'(t) = 0 \tag{A.28}$$

The expression for $r(t)$ expands as

$$\tilde{r}(t) = \frac{\sigma^2\rho}{(1+q^*)\nu}\int \tilde{\pi}(x,t)dG(x) - \frac{\sigma^2\rho}{(1+q^*)\nu}\int \pi(x)\tilde{g}(x,t)dx \tag{A.29}$$

Here $\tilde{g}(x,t)$ is the first-order deviation of $g(w,t)$ from $g(w)$ satisfying

$$\begin{aligned}\tilde{g}_t(w,t) &= -\partial_w(\mu_w(w)\tilde{g}(w,t) + \tilde{\mu}_w(w,t)g(w)) \\ &\quad + \partial^2_{ww}\left[\frac{\sigma_w(w)^2\tilde{g}(w,t)}{2} + 2\tilde{\sigma}_w(w,t)\sigma_w(w)g(w)\right]\end{aligned} \tag{A.30}$$

The first-order deviations of the drift and volatility are

$$\tilde{\mu}_w(w,t) = \tilde{r}(t)w + \lambda^*\tilde{w}(t) + \frac{2\tilde{\sigma}_w(w,t)\sigma_w(w)}{w} \tag{A.31}$$

$$\tilde{\sigma}_w(w,t) = \frac{\sigma w[\tilde{p}_w(w,t)w - \tilde{\gamma}(t)w^* - \gamma\tilde{w}^*(t)]}{[w + \gamma w^* - wp'(w)]^2} \tag{A.32}$$

The firs-order deviation of the risk compensation is

$$\tilde{\pi}(w,t) = \frac{\tilde{\gamma}(t)w^* + \gamma\tilde{w}^*(t)}{[w + \gamma w^* - wp'(w)]^2} - \frac{2\pi(w)(\tilde{\gamma}(t)w^* + \gamma\tilde{w}(t) - w\tilde{p}_w(w,t))}{w + \gamma w^* - wp'(w)} \tag{A.33}$$

The first-order deviation of the special country's wealth is

$$\tilde{w}(t) = -\int x\tilde{g}(x,t)dx \tag{A.34}$$

Now consider the second-order approximation around $\sigma = 0$. Let $\lambda^* = \boldsymbol{\lambda}^*\sigma^2$ and $\lambda = \boldsymbol{\lambda}\sigma^2$. First, since $\tilde{r}(t) = O(\sigma^2)$, $\tilde{\mu}_w(w,t) = O(\sigma^2)$ as well. Together with $\tilde{\sigma}_w(w,t) = O(\sigma)$, $\mu_w(w) = O(\sigma^2)$, and $\sigma_w(w) = O(\sigma)$, this implies

$$r\tilde{p}(w,t) - \partial_t\tilde{p}(w,t) = O(\sigma^2) \tag{A.35}$$

This means $\tilde{p}(w,t) = O(\sigma^2)$, leading to $\tilde{p}_w(w,t) = O(\sigma^2)$ and $\tilde{p}_{ww}(w,t) = O(\sigma^2)$. Next, $\tilde{\mu}_w(w) = O(\sigma^2)$, $\tilde{\sigma}_w(w,t) = O(\sigma)$, $\mu_w(w) = O(\sigma^2)$, and $\sigma_w(w) = O(\sigma)$ imply $\tilde{g}_t(w,t) = O(\sigma^2)$ and hence $\tilde{g}(w,t) = O(\sigma^2)$. This means $\tilde{w}(t) = O(\sigma^2)$, leading to

$$\tilde{\pi}(w,t) = -\tilde{\gamma}(t)w^*\boldsymbol{\pi}(w)^2 + O(\sigma^2) \tag{A.36}$$

Plugging this into the expression for $\tilde{r}(t)$,

$$\begin{aligned}\tilde{r}(t) &= \tilde{\gamma}(t)\sigma^2 \cdot \frac{\rho w^*}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)^2 dG(x) + O(\sigma^4) \\ &= \tilde{\gamma}(t)\sigma^2 \cdot \frac{\rho w^*}{(1+q^*)\nu}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4)\end{aligned} \tag{A.37}$$

Using this in the differential equation for $\tilde{p}^*(t)$,

$$\rho\tilde{p^*}(t) - \tilde{p^*}'(t) = \tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4) \tag{A.38}$$

Since $\tilde{\gamma}(t) = \delta e^{-\mu_\gamma t}$ and the terminal condition $\tilde{p}^*(t) \longrightarrow 0$ as $t \longrightarrow \infty$, this easily integrates:

$$\tilde{p^*}(t) = -\tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{\rho+\mu_\gamma} \cdot \frac{1}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4) \tag{A.39}$$

The last remaining piece is the pricing equation for $\tilde{p}(w,t)$. Using the fact that $\tilde{\mu}_w(w,t)p'(w) = O(\sigma^4)$, $\mu_w(w)\tilde{p}_w(w,t) = O(\sigma^4)$, $\tilde{\sigma}_w(w,t)\sigma_w(w)p''(w) = O(\sigma^4)$, and $\sigma_w(w)^2\tilde{p}_{ww}(w,t) = O(\sigma^4)$,

$$\rho\tilde{p}(w,t) - \partial_t\tilde{p}(w,t) = \tilde{\gamma}(t)\sigma^2 \cdot w^* \cdot \left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4) \tag{A.40}$$

With $\tilde{\gamma}(t) = \tilde{e}^{-\mu_\gamma t}$ and the terminal condition $\tilde{p}(w,t) \longrightarrow 0$ as $t \longrightarrow \infty$, this integrates to

$$\tilde{p}(w,t) = -\tilde{\gamma}(t)\sigma^2 \cdot \frac{w^*}{\rho+\mu_\gamma} \cdot \left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4) \tag{A.41}$$

This completes the proof. □

Proof of Proposition 7. I first restate the proposition.
The $\tilde{\nu}(t)$-first-order deviations of the interest rate and asset prices are

$$\tilde{r}(t) = -\frac{\mu_\nu}{\nu}\tilde{\nu}(t) + O(\sigma^4) \tag{A.42}$$

$$\tilde{p^*}(t) = \frac{1}{\rho}\tilde{\nu}(t) + O(\sigma^4) \tag{A.43}$$

$$\tilde{p}(w,t) = \frac{1}{\rho}\tilde{\nu}(t) + O(\sigma^4) \tag{A.44}$$

Take the first-order deviations $\tilde{r}(t)$, $\tilde{w}^*(t)$, $\tilde{p}(w,t)$, $\tilde{\pi}(w,t)$, $\tilde{\mu}_w(w,t)$ and $\tilde{\sigma}_w(w,t)$:

$$\begin{aligned}\tilde{r}(t)p(w) + r\tilde{p}(w,t) - \tilde{p}_t(w,t) &= \tilde{\nu}(t) - \sigma^2\tilde{\pi}(w,t) + \tilde{\mu}_w(w,t)p'(w) + \mu_w(w)\partial_w\tilde{p}(w,t) \\ &\quad + \tilde{\sigma}_w(w,t)\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial^2_{ww}\tilde{p}(w,t)\end{aligned} \tag{A.45}$$

Expanding the equation for $p^*(t)$,

$$\tilde{r}(t)p^* + r\tilde{p^*}(t) - \tilde{p^*}'(t) = \tilde{\nu}(t) \tag{A.46}$$

The expression for $r(t)$ expands as

$$\tilde{r}(t) = \frac{\tilde{\nu}'(t)}{\nu} + \frac{\rho\sigma^2\tilde{\nu}(t)}{(1+q^*)\nu^2}\int \pi(x)dG(x) - \frac{\rho\sigma^2}{(1+q^*)\nu}\left[\int \tilde{\pi}(x,t)dG(x) + \int \pi(x)\tilde{g}(x,t)dx\right] \tag{A.47}$$

Since $\tilde{\nu}'(t) = O(\sigma^2)$, the interest rate deviation is of the same order: $\tilde{r}(t) = O(\sigma^2)$. The first-order deviation of the wealth density $\tilde{g}(x,t)$ solves

$$\tilde{g}_t(w,t) = -\partial_w(\mu_w(w)\tilde{g}(w,t) + \tilde{\mu}_w(w,t)g(w)) + \partial^2_{ww}\left[\frac{\sigma_w(w)^2\tilde{g}(w,t)}{2} + 2\tilde{\sigma}_w(w,t)\sigma_w(w)g(w)\right] \tag{A.48}$$

The first-order deviations of the drift and volatility are

$$\tilde{\mu}_w(w,t) = \tilde{r}(t)w + \boldsymbol{\lambda}^*\sigma^2\tilde{w}(t) + \frac{2\tilde{\sigma}_w(w,t)\sigma_w(w)}{w} \tag{A.49}$$

$$\tilde{\sigma}_w(w,t) = \frac{\sigma w[\tilde{p}_w(w,t)w - \gamma\tilde{w}^*(t)]}{[w+\gamma w^* - wp'(w)]^2} = \sigma w\boldsymbol{\pi}(w)[\tilde{p}_w(w,t)w - \gamma\tilde{w}^*(t)] + O(\sigma^3) \tag{A.50}$$

Since $\tilde{\mu}_w(w,t) = O(\sigma^2)$, all terms on the right in the pricing equation (A.45) are $O(\sigma^2)$. Hence

$$r\tilde{p}(w,t) - \tilde{p}_t(w,t) = O(\sigma^2) \tag{A.51}$$

This implies $\tilde{p}(w,t) = O(\sigma^2)$, $\tilde{p}_w(w,t) = O(\sigma^2)$, and $\tilde{p}_{ww}(w,t) = O(\sigma^2)$. Hence, all terms on the right in equation (A.45) are of order $O(\sigma^4)$, except for $\tilde{\nu}(t) - \sigma^2\tilde{\pi}(w,t)$, where

$$\tilde{\pi}(w,t) = \frac{\gamma\tilde{w}^*(t)}{[w+\gamma w^* - wp'(w)]^2} - \frac{2\pi(w)(\gamma\tilde{w}(t) - w\tilde{p}_w(w,t))}{w+\gamma w^* - wp'(w)} = O(\sigma^2) \tag{A.52}$$

The equality follows from the fact that the first-order deviation of the special country's wealth is

$$\tilde{w}(t) = \frac{(1+q^*)\tilde{\nu}(t)}{\rho} - \int x\tilde{g}(x,t)dx \tag{A.53}$$

But since $\tilde{\mu}_w(w,t) = O(\sigma^2)$, $\mu_w(w) = O(\sigma^2)$, $\tilde{\sigma}_w(w,t) = O(\sigma)$, and $\sigma_w(w) = O(\sigma)$, the expanded Kolmogorov forward equation (A.48) implies $\tilde{g}_t(w,t) = O(\sigma^2)$. This in turn implies $\tilde{g}(w,t) = O(\sigma^2)$. This leads to $\tilde{w}^*(t) = O(\sigma^2)$, which implies $\tilde{\pi}(w,t) = O(\sigma^2)$ because $\tilde{p}_w(w,t) = O(\sigma^2)$ too.

Coming back to the expression for the interest rate in equation (A.47),

$$\tilde{r}(t) = \frac{\tilde{\nu}'(t)}{\nu} + O(\sigma^4) \tag{A.54}$$

The interest rate only reacts to consumption growth expectations in the second order. Since all terms on the right in equation (A.45) are of order $O(\sigma^4)$ except for $\tilde{\nu}(t)$,

$$\rho\tilde{p}(w,t) - \tilde{p}_t(w,t) = \tilde{\nu}(t) - \frac{\nu}{\rho}\cdot\frac{\tilde{\nu}'(t)}{\nu} + O(\sigma^4) = \tilde{\nu}(t)\cdot\frac{\mu_\nu+\rho}{\rho} + O(\sigma^4) \tag{A.55}$$

This uses $\tilde{\nu}(t) = \delta_\nu e^{-\mu_\nu t}$. Integrating,

$$\tilde{p}(t) = \frac{\tilde{\nu}(t)}{\rho} \tag{A.56}$$

In the second order, prices only react to the expected discounted dividend changes. □

Proof of Proposition 8. I first restate the proposition.
The deviation of the high-minus-low factor is

$$\tilde{x}(t) = \frac{\tilde{\gamma}(t)\sigma^2 w^*}{(\rho+\mu_\gamma)(1+q^*)}\left(\int_{\overline{w}}^{\infty}\boldsymbol{\pi}(w)^2 d\mathcal{G}(w) - \int_0^{\overline{w}}\boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\right) + O(\sigma^4) \tag{A.57}$$

Here $\overline{w}$ is the median of w at the distribution $\mathcal{G}(\cdot)$. The deviation of the average price is

$$\tilde{p}(t) = \frac{1}{\rho}\cdot\tilde{\nu}(t) + \frac{w^* q^*}{(\rho+\mu_\gamma)(1+q^*)}\int\boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\cdot\sigma^2\tilde{\gamma}(t) \tag{A.58}$$

Take the definition of $\tilde{x}(t)$:

$$\begin{aligned}
\tilde{x}(t) &= \int p(w,t)\delta(w,t)dG(w,t) - \int p(w)\delta(w)dG(w) \\
&= \int \tilde{p}(w,t)\delta(w)dG(w) + \int p(w)(\delta(w,t)g(w,t) - \delta(w)g(w))dw \\
&= \int \tilde{p}(w,t)\boldsymbol{\delta}(w)d\mathcal{G}(w) + \sigma^2\int P_1(w)[\delta(w,t)\tilde{g}(w,t) + g(w)(\delta(w,t)-\delta(w))]dw + O(\sigma^4) \\
&= \int \tilde{p}(w,t)\boldsymbol{\delta}(w)d\mathcal{G}(w) + O(\sigma^4)
\end{aligned} \tag{A.59}$$

Here $P_1(w)$ is the second term in $p(w) = \nu/\rho + P_1(w)\sigma^2 + O(\sigma^4)$. The last equality here uses the fact that $\tilde{g}(w,t) = O(\sigma^2)$ and that the medians of $G(w,t)$ and $G(w)$ are also $O(\sigma^2)$ apart. The penultimate equality uses the fact that the difference between $G(w)$ and $\mathcal{G}(w)$ is $O(\sigma^2)$, that their medians are also $O(\sigma^2)$ apart, and that $\tilde{p}(w,t) = O(\sigma^2)$. It also uses the fact that $\delta(w,t)g(w,t) - \delta(w)g(w)$ integrates to zero.

Now, using Proposition 6,

$$\tilde{x}(t) = \tilde{\gamma}(t)\sigma^2\frac{w^*}{(\rho+\mu_\gamma)(1+q^*)}\left(\int_{\overline{w}}^{\infty}\boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \int_0^{\overline{w}}\boldsymbol{\pi}(x)^2 d\mathcal{G}(x)\right) + O(\sigma^4) \tag{A.60}$$

Finally, using Proposition 6 and Proposition 7,

$$\tilde{p}(w,t) = \frac{\tilde{\nu}(t)}{\rho} - \frac{\tilde{\gamma}(t)\sigma^2 w^*}{\rho + \mu_\gamma} \cdot \left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4) \tag{A.61}$$

Integrating this with $d\mathcal{G}(w)$,

$$\tilde{p}(t) = \frac{\tilde{\nu}(t)}{\rho} + \frac{\tilde{\gamma}(t)\sigma^2 w^*}{\rho + \mu_\gamma} \cdot \frac{q^*}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4) \tag{A.62}$$

This completes the proof. □

Proof of Proposition 9. Using Proposition 6 and Proposition 7,

$$\tilde{p}(w,t) = \frac{\tilde{\nu}(t)}{\rho} - \frac{\tilde{\gamma}(t)\sigma^2 w^*}{\rho + \mu_\gamma} \cdot \left[\frac{1}{1+q^*}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4) \tag{A.63}$$

Using Proposition 8,

$$\tilde{p}(w,t) = \tilde{p}(t) + \frac{\tilde{\gamma}(t)\sigma^2 w^*}{\rho + \mu_\gamma}\left[\boldsymbol{\pi}(w)^2 - \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x)\right] + O(\sigma^4) \tag{A.64}$$

Using the expression for $\tilde{x}(t)$,

$$\tilde{p}(w,t) = \tilde{p}(t) + \tilde{x}(t)(1+q^*) \cdot \frac{\boldsymbol{\pi}(w)^2 - \displaystyle\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x)}{\displaystyle\int_{\overline{w}}^{\infty} \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \int_0^{\overline{w}} \boldsymbol{\pi}(x)^2 d\mathcal{G}(x)} + O(\sigma^4) \tag{A.65}$$

Defining $\beta(w)$ by $\tilde{p}(w,t) = \tilde{p}(t) + \beta(w)\tilde{x}(t) + O(\sigma^4)$ completes the proof. □

Proof of Proposition 10. The assets-to-liabilities ratio in the steady state is

$$\begin{aligned}\zeta(w) = \frac{(1-\theta(w))w}{h^*(w)p(w)} &= \frac{(w+\gamma w^* - p(w))w}{\gamma w^* p(w)} = \frac{w(\rho(w+\gamma w^*) - \nu)}{\gamma \nu w^*} + O(\sigma^2) \\ &= \frac{(1-\gamma w^* \boldsymbol{\pi}(w))(\rho - \nu\boldsymbol{\pi}(w))}{\boldsymbol{\pi}(w)^2 \gamma \nu w^*} + O(\sigma^2) \equiv \boldsymbol{\zeta}(w) + O(\sigma^2)\end{aligned} \tag{A.66}$$

This leads to the following quadratic equation for $\boldsymbol{\pi}(w)$ in terms of $\boldsymbol{\zeta}(w)$:

$$(\boldsymbol{\zeta}(w) - 1)\gamma\nu w^* \boldsymbol{\pi}(w)^2 + (\rho\gamma w^* + \nu)\boldsymbol{\pi}(w) - \rho = 0 \tag{A.67}$$

If $\boldsymbol{\zeta}(w) = 1$, then $\boldsymbol{\pi}(w) = \rho/(\rho\gamma w^* + \nu)$. If $\boldsymbol{\zeta}(w) \neq 1$, then this quadratic equation has two solutions:

$$\pi_{1,2} = -\frac{\rho\gamma w^* + \nu \pm \sqrt{(\rho\gamma w^* + \nu)^2 + 4\rho\gamma\nu w^*(\boldsymbol{\zeta}(w) - 1)}}{2\gamma\nu w^*(\boldsymbol{\zeta}(w) - 1)} \tag{A.68}$$

The root that converges to $\rho/(\rho\gamma w^* + \nu)$ as $\boldsymbol{\zeta}(w) \longrightarrow 1$ is

$$\boldsymbol{\pi}(w) = \frac{\sqrt{(\rho\gamma w^* + \nu)^2 + 4\rho\gamma\nu w^*(\boldsymbol{\zeta}(w) - 1)} - (\rho\gamma w^* + \nu)}{2\gamma\nu w^*(\boldsymbol{\zeta}(w) - 1)} \tag{A.69}$$

This completes the proof. $\square$

C Additional details for empirics.

In this appendix, I report additional specifications of the motivating regressions in Section 2. Table 5 reports results on weighted least squares regressions with weights inversely proportional to the standard error of the dependent variable, which is an estimate from the first stage. Table 6 reports ordinary least squares regressions with the sample trimmed to exclude outliers (countries with asset-liability ratios above two). Table 7 reports weighted least squares results for the trimmed sample. Table 8, Table 9, and Table 10 report the same for asset flows as the outcome variable.

Table 5: Cyclicality of asset prices and asset-liability ratios. Prices multiplied by 100 to show percentage changes. Weighted least-squares regressions with first-stage estimates weighted by the inverse standard error. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	9.16	10.50	-10.72	-11.62
	(9.67)	(11.94)	(-10.06)	(-8.89)
Γ	-3.20	-3.75	1.99	4.03
	(-2.64)	(-3.24)	(1.35)	(2.19)
R^2	0.21	0.27	0.05	0.11
N	28	30	39	39
T	80	80	80	80

Table 6: Cyclicality of asset prices and asset-liability ratios. Prices multiplied by 100 to show percentage changes. Sample restricted to countries with the asset-liability ratio below 2. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	9.51	10.83	-11.51	-11.52
	(8.83)	(10.62)	(-10.81)	(-9.24)
Γ	-3.07	-3.32	2.41	3.69
	(-2.20)	(-2.43)	(1.61)	(2.11)
R^2	0.16	0.17	0.07	0.11
N	28	30	39	39
T	80	80	80	80

Table 7: Cyclicality of asset prices and asset-liability ratios. Prices multiplied by 100 to show percentage changes. Weighted least-squares regressions with first-stage estimates weighted by the inverse standard error. Sample restricted to countries with the asset-liability ratio below 2. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	9.16	10.50	-10.72	-11.62
	(9.67)	(11.94)	(-10.06)	(-8.89)
Γ	-3.20	-3.75	1.99	4.03
	(-2.64)	(-3.24)	(1.35)	(2.19)
R^2	0.21	0.27	0.05	0.11
N	28	30	39	39
T	80	80	80	80

Table 8: Cyclicality of asset flows and asset-liability ratios. Prices multiplied by 100 to show percentage changes. Weighted least-squares regressions with first-stage estimates weighted by the inverse standard error. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	-0.38	-0.37	0.26	0.32
	(-1.74)	(-1.70)	(1.52)	(1.38)
Γ	3.56	3.46	-2.27	-3.00
	(8.82)	(8.42)	(-7.17)	(-7.23)
R^2	0.47	0.45	0.37	0.38
N	89	89	89	89
T	80	80	80	80

Table 9: Cyclicality of asset flows and asset-liability ratios. Prices multiplied by 100 to show percentage changes. Sample restricted to countries with the asset-liability ratio below 2. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	0.19	-0.08	-1.00	-0.10
	(0.30)	(-0.14)	(-1.32)	(-0.13)
Γ	2.17	2.57	-0.05	-2.23
	(2.27)	(2.91)	(-0.04)	(-1.90)
R^2	0.06	0.09	0.00	0.04
N	84	84	84	84
T	80	80	80	80

Table 10: Cyclicality of asset flows and asset-liability ratios. Prices multiplied by 100 to show percentage changes. Weighted least-squares regressions with first-stage estimates weighted by the inverse standard error. Sample restricted to countries with the asset-liability ratio below 2. T-statistics reported in brackets.

	f^{acq}	f^{inc}	VIX	EBP
α	-0.37	-0.34	0.25	0.32
	(-1.65)	(-1.50)	(1.43)	(1.32)
Γ	3.54	3.37	-2.24	-2.99
	(8.36)	(7.85)	(-6.94)	(-6.94)
R^2	0.46	0.43	0.37	0.37
N	84	84	84	84
T	80	80	80	80

I next report the samples of countries that went into the baseline regressions.

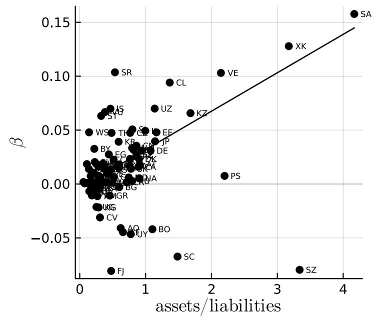
- Balanced panel for the acquisition factor construction (N=53): AM, AR, AU, AW, BD, BG, BO, BR, BY, CA, CL, CO, CZ, DE, DK, EE, ES, FI, FR, GT, HR, HU, ID, IL, IS, IT, JP, KG, KH, KR, KZ, LK, LS, LT, LV, MD, MK, MX, PA, PE, PG, PH, PT, RO, RU, SE, SI, SK, TH, TR, UA, VU, ZA.

- Balanced panel for the incurrence factor construction (N=56): AM, AR, AU, AW, BD, BG, BO, BR, BY, CA, CL, CO, CZ, DE, DK, EC, EE, ES, FI, FR, GE, GT, HR, HU, ID, IL, IN, IS, IT, JP, KG, KH, KR, KZ, LK, LS, LT, LV, MD, MK, MX, PA, PE, PG, PH, PK, PT, RO, RU, SE, SI, SK, TH, TR, UA, ZA.

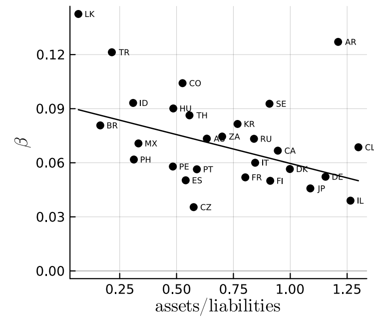
- Price regressions (N=39): AR, AT, AU, BR, CA, CL, CN, CO, CZ, DE, DK, EG, ES, FI, FR, GR, HU, ID, IL, IN, IT, JO, JP, KR, LK, MA, MX, MY, NZ, PE, PH, PK, PL, PT, RU, SE, TH, TR, ZA.

- Flows regressions (N=90): AF, AL, AM, AO, AT, AU, BA, BD, BG, BO, BR, BY, CA, CL, CN, CO, CR, CV, CZ, DE, DK, DO, EE, EG, ES, FI, FJ, FR, GE, GR, GT, HN, HR, HU, ID, IL, IN, IS, IT, JM, JO, JP, KG, KR, KZ, LK, LT, LV, MA, MD, MG, MK, MN, MX, MZ, NA, NI, NZ, PA, PE, PH, PK, PL, PS, PT, PY, RO, RS, RU, SA, SC, SE, SI, SK, SR, ST, SV, SZ, TH, TJ, TR, UA, UG, UY, UZ, VE, VU, WS, XK, ZA.

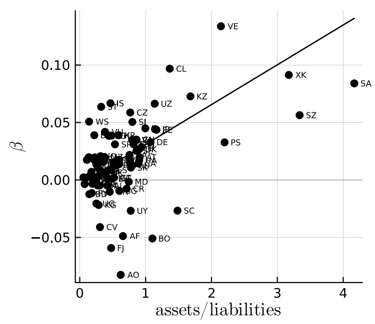
Finally, I present the scatterplots behind the eight baseline regressions.



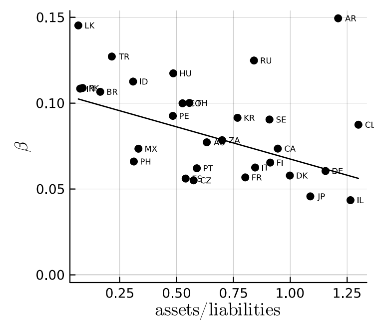
(a) flow cyclicality $\beta^{a,\text{acq}}$ and asset-liability ratios



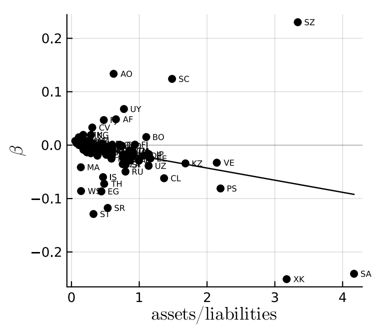
(b) price cyclicality $\beta^{p,\text{acq}}$ and asset-liability ratios



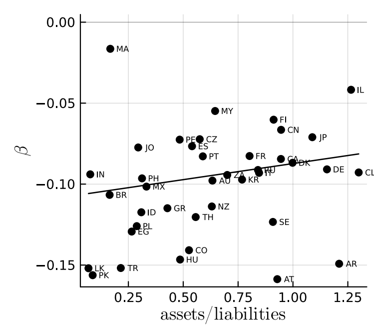
(c) flow cyclicality $\beta^{a,\text{inc}}$ and asset-liability ratios



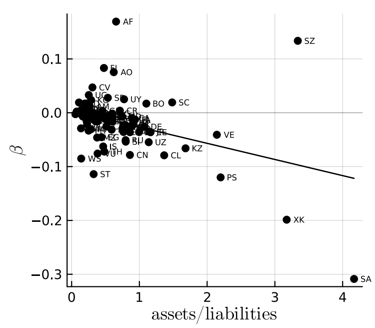
(d) price cyclicality $\beta^{p,\text{inc}}$ and asset-liability ratios



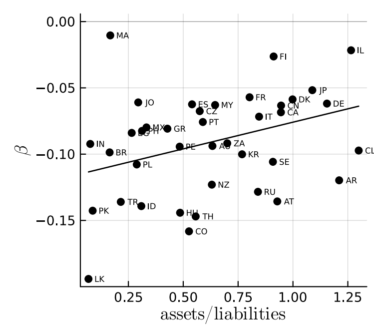
(e) flow cyclicality $\beta^{a,\text{VIX}}$ and asset-liability ratios



(f) price cyclicality $\beta^{p,\text{VIX}}$ and asset-liability ratios



(g) flow cyclicality $\beta^{a,\text{EBP}}$ and asset-liability ratios



(h) price cyclicality $\beta^{p,\text{EBP}}$ and asset-liability ratios