Heterogeneous Impact of the Global Financial Cycle*

Aleksei Oskolkov

Yale University

aleksei.oskolkov@yale.edu

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Abstract

I develop a heterogeneous-country model of the world economy with global financial shocks. Imperfect international risk-sharing creates a wealth distribution across countries. Shocks to the risk-taking capacity of global intermediaries trigger aggregate capital flight. The global rise in risk premia is concentrated in poor countries, while rich countries experience "retrenchment": domestic agents repatriate external assets to replace foreign investors and stabilize asset prices. Asset markets in rich countries endogenously gain the status of safe havens. This helps explain why, in the data, advanced economies experience larger outflows of foreign investment in global busts, while assets depreciate more in emerging markets.

Key Words: capital flows, risk premium, global financial cycle, heterogeneity, retrenchment

JEL Classification Numbers: F30, F40, G15

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1 Introduction

There is an aggregate cycle in international capital flows and asset prices. Miranda-Agrippino and Rey (2022) show that one global factor explains more that 20% of the variation in gross capital flows across the world. This factor is strongly correlated with measures of global risk-taking capacity and the dominant component in asset prices. In booms, when asset prices are high, investors tend to accumulate foreign holdings. In global downturns, they sell foreign assets and shift their portfolios toward domestic markets, which is what the international finance literature calls "retrenchment".

Countries are not equally exposed to the cycle. Emerging markets are especially strongly affected by changes in the global risk appetite, their asset prices are more volatile, and risk premia on their assets are higher. At the same time, capital flows are more strongly correlated with global aggregates in advanced economies. In downturns, advanced economies see larger outflows of foreign investment. Their local investors, on the other hand, sell more of their external assets than investors from emerging markets, responding to negative shocks with more active retrenchment.

I study global financial shocks in a dynamic heterogeneous-country model that reconciles these two facts in equilibrium. In the model, countries issue risky assets exposed to country-specific risk. Markets are segmented: the world relies on a global intermediary who trades with local agents in all countries and facilitates capital flows. Shocks to the intermediary's risk-taking capacity cause aggregate capital flight events as it sells risky assets around the world. The reaction of local investors to capital flight is heterogeneous. In advanced economies, local investors have substantial external assets and readily use them to buy their domestic assets that the global intermediary tries to sell. These local investors can take on additional risk without a major increase in risk premia. Foreign demand shocks lead to large asset sales with little movement in expected returns. In emerging economies, the pool of domestic investors is thin. They cannot buy as much of the asset stock from the global intermediary, and their asset markets adjust to foreign demand shocks through prices rather than quantities. Risk premia rise sharply in these countries, as foreign investors have nobody to sell to, and higher expected returns have to convince them to stay.

I show empirically that advanced economies do indeed see larger outflows of foreign investment in times of aggregate capital flight, both in dollars and as a share of their countries' external liabilities. Local investors in these countries sell more of their foreign assets, again, both in absolute terms and relative to external liabilities. At the same time, equities and government bonds in advanced economies outperform those in emerging markets in capital flight events. The model interprets this as evidence for a higher elasticity of local asset demand in developed economies. Adjustment to shifts in foreign demand mainly operates through the quantity margin, insulating asset prices from global shocks. I show that the model generates these heterogeneous elasticities of domestic demand without intrinsic differences between countries: all local investors have the same preferences, and all risky assets have the same properties of cash flows. The only difference between countries is the history of domestic shocks, which makes their local agents rich or poor, and hence better or worse equipped to replace foreign investors in times of global capital flight. I use the model to study general equilibrium implications of the heterogeneous responses to capital flight, derive the responses of the entire cross-section of asset prices and the global risk-free rate, and characterize international wealth redistribution caused by global financial shocks.

The model economy consists of a continuum of countries and a global intermediary. Each country is endowed with a Lucas tree, with output shocks independent across countries. Local agents hold these domestic trees and buy foreign assets for diversification. Markets are segmented: agents cannot invest in other countries directly. Instead, they hold risk-free bonds issued by the global intermediary, who invests in all trees. The intermediary is based in a special country, a stand-in for the US, and is fully responsible for international capital flows. The only purpose of the US in the model is to house the intermediary, and I exclude it from my analysis of cross-country heterogeneity because of its central position in the global financial system.

The intermediary's risk-taking capacity is limited by a value-at-risk constraint, which prevents it from taking advantage of a fully diversified portfolio with a continuum of uncorrelated assets. The intermediary demands a premium for holding country-specific risk. As a result, in equilibrium, international risk-sharing is incomplete. Agents retain exposure to domestic shocks. The history of dividends determines their current wealth. There is a non-degenerate wealth distribution, and countries continually move around the distribution following good and bad output spells.

A country's wealth at a given point in time determines the market premium for its idiosyncratic risk and portfolio composition of local investors. In the left tail of the distribution, wealth is scarce. Country risk premia are high, and local investors hold risky portfolios in which domestic trees dominate the intermediary's bonds, the riskless foreign asset. The intermediary partly compensates for the scarcity of domestic wealth, but its demand is limited by the value-at-risk constraint. In the right tail of the wealth distribution, local investors have large external holdings. They are saturated with safe assets and do not demand high risk premia at home. Their portfolios have low volatility, and these rich countries enjoy stable wealth and consumption dynamics.

Uninsured risk creates precautionary motives and depresses the global interest rate. The magnitude of the interest rate depression is related to the average country-specific price of risk, for which the model provides an analytical expression. I show that, for small shocks, prices of trees in all countries can be decomposed into three parts: the present discounted value of dividends, the country-specific discount for output risk, and the surcharge coming from the low global rates. Local wealth determines the size of the risk discount. In poor countries, this term dominates, reflecting a thin investor base. In rich countries, the risk discount is small, the surcharge term dominates, and their risky assets are overpriced compared to the fundamental cash flows.

The tightness of the intermediary's value-at-risk constraint serves as a source of the global shock. An exogenous tightening makes holding country risk on its balance sheet more costly, and the intermediary tries to sell risky assets around the world. The left tail of the wealth distribution is affected more: in the run-up to the shock, the intermediary's position is larger in riskier countries with higher expected returns, and this is where it tries to sell more. Emerging markets are the primary target of a sudden stop. However, the main difference between them and advanced economies is in the reaction of local investors. In rich countries, local investors retrench, selling external assets to buy domestic ones, and easily replace foreign demand. In poor countries, they cannot absorb as much as the intermediary is willing to sell, and prices adjust instead. The rise in risk premia is concentrated in the left tail, and the outflow of foreign investors in the right.

Importantly, in general equilibrium, risky assets in rich countries act as safe assets: they appreciate in bad times. The reason is that the global risk-free rate falls in times of capital flight because a tighter value-at-risk constraint increases the price of risk in all markets, strengthening precautionary motives. This inflates asset prices, counteracting the rise in risk premia. With small shocks, I show analytically that this interest rate effect dominates in rich countries, and the prices of their risky assets rise amid capital flight. This outcome is endogenous. More elastic domestic demand in advanced economies, more active retrenchment in equilibrium, and rising asset prices in downturns are only due to larger accumulated external assets.

Global financial shocks redistribute wealth. Advanced economies enjoy capital gains on their assets while emerging markets make losses, and wealth dispersion increases. The intermediary is exposed to risky assets in emerging markets as well. Absorbing part of the losses on these, the intermediary provides insurance to poorer countries. This outcome looks like a wealth transfer from the special country, where the intermediary is housed, to the rest of the world. These wealth transfers are well-documented: Maggiori (2017) notes their large size and generates them in a model where the US has more financial depth and takes on the role of the global insurer. Gourinchas and Rey (2022) refer to this pattern as "exorbitant duty" of the financial hegemon.

Like in Maggiori (2017), Kekre and Lenel (2021), Sauzet (2023), Devereux, Engel, and Wu (2023), and other models, exorbitant duty in my model accompanies "exorbitant privilege": intermediation profits in normal times allow the special country to run perpetual trade and current account deficits despite its negative net foreign asset position. Unlike prior work, I show that the intermediary also makes capital gains on its holdings in advanced economies in bad times. Wealth transfers flow from advanced economies through the special country to emerging markets. Rich countries indirectly insure poor ones by limiting the intermediary's losses, which is important because emerging markets are exposed to the intermediary's net worth especially strongly.

Taking the model to the data, I add global output shocks to financial shocks and estimate it with the time series of global equity prices and aggregate capital flows. Without targeting moments of advanced economies and emerging markets separately, I show that the model can qualitatively reproduce their differences: equity prices are more volatile in the latter, while outward financial flows, computed relative to countries' external liabilities, are more volatile in the former.

I extract the principal component from the cross-country panel of capital flows and use it as the empirical measure of the global financial cycle. As an alternative, I use two measures of the global risk-taking capacity: VIX and the excess bond premium of Gilchrist and Zakrajšek (2012). These variables are strongly, although imperfectly, correlated with aggregate flows, and empirical results are qualitatively similar if I use them as a measure of the global financial cycle instead. A one standard deviation fall in aggregate capital flows is associated with advanced economies losing 2*pp* more of their external liabilities. This measure does not include valuation changes, only counting transactions. Foreign investors in advanced economies sell more of their holdings in bad times. At the same time, the relative performance of equities in advanced economies is countercyclical: advanced economies outperform emerging markets when capital flows recede. This is consistent with the mechanism in the model: retrenchment is more active in rich countries, and their risky assets weather aggregate capital flight events better. In the data, global sales of 10% of foreign assets are associated with a 1.6*pp* excess return on equities in advanced economies compared to emerging markets. In the model, the same aggregate foreign asset sales generate 0.8*pp* of excess returns. Although the model cannot capture all empirical variation due to differences in institutions, asset characteristics, and financial development, it reproduces a significant portion of the variation with wealth heterogeneity alone, suggesting that asset demand elasticities are an important determinant of the global response to financial shocks.

1.1 Related literature

The most closely related paper is Caballero and Simsek (2020). They show how retrenchment stabilizes domestic asset markets in a model where capital flight is driven by idiosyncratic shocks. Liquidity needs trigger fire sales by foreign investors. Local investors use their foreign holdings to pick up the unwanted asset and support its price. This mechanism is also present in Jeanne and Sandri (2023). I build a dynamic version of this model in the style of Brunnermeier and Sannikov (2014) with global intermediaries, aggregate shocks, and endogenous differences wealth, focusing on the distributional consequences of aggregate capital flight.

My model shares the focus on cross-sections of advanced and emerging economies with closely related papers by Morelli, Ottonello, and Perez (2022) and Bai, Kehoe, and Perri (2019). Morelli, Ottonello, and Perez (2022) model a global intermediary invests in emerging markets. They account for the feedback from the cross-section of returns to the intermediary's net worth and find that shocks to the intermediary's balance sheet are an important driver of borrowing costs around the world. Bai, Kehoe, and Perri (2019) use a similar model to measure the relative importance of global and local shocks in explaining the cross-section of sovereign spreads. My main contribution relative to these papers is adding gross capital flows that are jointly determined with asset prices as the primary mechanism behind heterogeneous responses to global shocks.

Davis and Van Wincoop (2022) construct a multicountry model to generate gross flows after a shock to global risk aversion and show the importance of within-country heterogeneity. Davis and Van Wincoop (2023) additionally show that symmetric shocks to identical countries can generate positive co-movement between gross flows in and out as long as more risk-tolerant investors invest more abroad. I focus instead on the distributional consequences of global shocks in intermediated markets with endogenous heterogeneity between ex-ante identical countries.

Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022) build a multicountry model with time-varying appetites for risk coming from deep habits. They show how an adverse output shock in a large country leads to an appreciation of its currency and an increase in its wealth share. I arrive at regressive redistribution in downturns through capital flows. In relative terms, rich countries become richer because they easily replace foreign demand for their assets. In absolute terms, they become richer because the shock to risk-taking capacity decreases the interest rate, while risk premia are held down by retrenchment, and asset prices rise.

Farboodi and Kondor (2022) study heterogeneous boom-buts dynamics with imperfect information about asset quality. In their model, shocks determine what investors learn about firms. In bad times, they flee from emerging markets to advanced economies, inducing a recession in the former and stabilizing output in the latter. Fu (2023) models joint determination of capital flows and exchange rates, showing that currency betas are lower in countries where domestic investors have a higher propensity to retrench than foreign ones. The resulting link between retrenchment and cyclicality of returns is similar to that in my model. Zhou (2023) shows how exposure of assets to investor-specific shocks depends on the willingness of other market participants to absorb additional supply, focusing on the composition of the foreign investor base. A similar mechanism involving domestic investors is at the heart of my model.

There is a large literature on heterogeneous responses to global shocks that focuses on the US and the rest of the world, endowing the US with intrinsic advantages such as less tight leverage constraints, non-pecuniary benefits attached to its bonds, and higher risk tolerance. This literature includes, among others, Bruno and Shin (2015), Maggiori (2017), Farhi and Maggiori (2018), Jiang, Krishnamurthy, and Lustig (2020), Kekre and Lenel (2021), Sauzet (2023), Devereux, Engel, and Wu (2023), and Jiang (2024). In Jiang, Krishnamurthy, and Lustig (2020) and Kekre and Lenel (2021), the dollar carries a convenience yield. Kekre and Lenel (2021) study flight to safety caused by a shock to this convenience yield in a model with nominal frictions and investment.

A large empirical literature explores global drivers of international capital flows and asset prices. Miranda-Agrippino and Rey (2022) provide a comprehensive review. The dominant global factor in a large panel of risky asset prices has been extracted by Miranda-Agrippino, Nenova, and Rey (2020) and more recently updated by Miranda-Agrippino and Rey (2020). Similarly strong co-movement has been documented for capital flows. Forbes and Warnock (2012) and Forbes and Warnock (2021) show co-movement between gross flows. Barrot and Serven (2018) identify common components in gross flows and show that these common components are strongly related to aggregate variables such as VIX, US dollar exchange rate, and interest rates.

Part of this literature deals with heterogeneity between advanced economies and emerging markets. Barrot and Serven (2018) and Cerutti, Claessens, and Puy (2019) show that flows in advanced economies are more responsive to common factors. This fact is at the heart of the model, which is built to generate more elastic asset markets in rich countries.

The literature studying distributions of returns and flows includes Chari, Stedman, and Lundblad (2020), Gelos, Gornicka, Koepke, Sahay, and Sgherri (2022), and Eguren Martin, O'Neill, Sokol, and von dem Berge (2021). Kalemli-Özcan (2019) and Bräuning and Ivashina (2020) show that US monetary policy spillovers have a more pronounced effect on emerging markets. Chari, Stedman, and Lundblad (2020) show the outsized effect of risk-off episodes on the worst realizations, the left tail. This is the response to a shock to risk-taking capacity in my model: the left tail of returns shifts significantly further, while the average stays very close to normal times.

The paper is organized as follows. Section 2 lays out the model and defines equilibrium. Section 3 characterizes equilibrium. Section 4 provides analytical results on the steady state and the shock to risk-taking capacity. Section 5 describes the calibration and estimation procedure. Section 6 lays out the empirical results.

2 Model

Time is continuous and runs forever. There is no aggregate uncertainty. The world is a unit measure of countries indexed by $i \in [0,1]$ and a large special country populated by intermediaries. Each country has a Lucas tree in fixed unit supply. Output is homogeneous across countries. Cumulative output of i's tree up to time t is denoted by y_{it}, and flow output is $dy_{it} = \nu_t dt + \sigma dZ_{it}$. Expected output ν_t is common to all countries and evolves deterministically. The volatility σ is constant, and the random increments dZ_{it} are standard Brownian, independent across countries.

Agents from these countries only invest in their domestic trees and bonds issued by global intermediaries. Bonds are riskless and short-term, paying interest $r_t dt$. This is the risk-free rate of the economy. The price p_{it} of i's tree is an endogenous stochastic process. The instantaneous excess return on trees is the dividend yield and capital gains over and above the risk-free rate:

$$dR_{it} = \frac{dy_{it} + dp_{it}}{p_{it}} - r_t dt$$

Each country i houses a continuum of identical agents. Denote the wealth of the representative agent in i by w_{it} and the aggregate wealth of her country by $\underline{w}_{it}$. These processes will coincide in equilibrium, but agents take $\underline{w}_{it}$ as given.

For stationarity, the model includes a version of perpetual youth with additional wealth-sharing across countries. Agents in countries $i \in [0,1]$ die with a Poisson intensity λ. In this event, the dying agent's wealth is sent to the special country, and she is replaced with a newborn. All survivors transfer a share of their wealth to the newborn to make her net worth the same as her predecessor's. Similarly, agents in the special country die with a Poisson intensity $\hat{\lambda}$, and their wealth is sent uniformly to regular countries $i \in [0,1]$, where it is shared between local residents in proportion to their net worth. In country i, the evolution of individual agent's wealth w_{it} is

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} - \frac{w_{it}}{\underline{w}_{it}} \cdot \lambda \underline{w}_{it} dt + \frac{w_{it}}{\underline{w}_{it}} \cdot \hat{\lambda} \underline{\hat{w}}_t dt \tag{1}$$

Here c_{it} is consumption. The second term represents returns on the tree, where θ_{it} is its portfolio share. The third and fourth terms reflect the perpetual youth adjustment. The third one is the transfers to newborns: the total flow of the dying agents' wealth is $\lambda \underline{w}_{it}$, and everyone in i compensates the newborns according to her own wealth share $w_{it}/\underline{w}_{it}$. The fourth term is the inflow of wealth from the dying agents in the special country: $\hat{w}_t$ is its aggregate wealth, $\hat{\lambda}$ is the death rate, and all arriving wealth is again shared in proportion to the net worth of local residents.

The sequence problem of the agent in the country i is

$$\max_{\{c_{is}, \theta_{is}\}_{s \geq t}} \mathbb{E}_t \left[\rho \int_t^{\infty} e^{\rho(t-s)} \log(c_{is}) ds \right]$$

subject to equation (1) and $w_{it} \geq 0$. Since everyone in i is the same, in equilibrium $w_{it} = \underline{w}_{it}$, and

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} + (\hat{\lambda} \hat{w}_t - \lambda w_{it})dt$$

I next describe the special country. It is special for two reasons. First, its asset supply is different. Second, its local agents are global intermediaries. I explain these two properties below, using notation with hats to separate this country from regular ones.

In contrast to regular countries, the special country is large, with a finite measure $\hat{q}$ of trees that are pooled together in a fund. The random components of their yields wash out, so the total output over dt in the special country is $\hat{q}\nu_t dt$. These trees can only be traded as one, in a bundle with equal weights. I refer to this fund as the special country's tree for convenience. Its price is $\hat{p}_t$, and the excess return is $d\hat{R}_t = (\nu dt + d\hat{p}_t)/\hat{p}_t - r_t dt$.

The wealth $\hat{w}_t$ of the local representative agent, who is also the global intermediary, is its holdings of trees less bonds outstanding. It evolves as

$$d\hat{w}_t = (r_t \hat{w}_t - \hat{c}_t)dt + \int [\hat{\theta}_{it} \hat{w}_t dR_{it}] di + \hat{\theta}_t \hat{w}_t d\hat{R}_t + \frac{\hat{w}_t}{\underline{\hat{w}}_t} \cdot \lambda w_t dt - \frac{\hat{w}_t}{\underline{\hat{w}}_t} \cdot \hat{\lambda} \underline{\hat{w}}_t dt \tag{2}$$

Here $\hat{c}_t$ is consumption. The second and third terms are excess returns on trees in all countries with portfolio weights $\{\hat{\theta}_{it}\}$ on regular ones and $\hat{\theta}_t$ on the special country. The last two terms mirror the perpetual youth terms in equation (1): there is an inflow $\lambda w_t dt$, where w_t is the aggregate wealth of all regular countries, and an outflow $\hat{\lambda}\underline{\hat{w}}_t dt$, where $\underline{\hat{w}}_t$ is the special country's aggregate wealth.

The intermediary's risk-taking capacity is limited. It faces a value-at-risk constraint:

$$\int \mathbb{V}[\hat{\theta}_{it} dR_{it}]di \leq \gamma_t \int \mathbb{E}[\hat{\theta}_{it} dR_{it}]di \tag{3}$$

This constraint aggregates idiosyncratic uncertainty over the intermediary's returns in all countries and bounds it by a multiple of expected profits on these assets. The parameter γ_t is key: it determines the intermediary's ability to hold country-specific risk. With $\gamma_t = \infty$, the intermediary could in principle take advantage of access to a continuum of uncorrelated assets and fully insure other agents, absorbing all idiosyncratic risk. A finite γ_t leads to non-trivial portfolios, and negative shocks to γ_t trigger capital flight events. The problem of the intermediary is

$$\max_{\{\hat{c}_s, \hat{\boldsymbol{\theta}}_{\boldsymbol{s}}\}_{s\geq t}} \mathbb{E}_t \left[\rho \int_t^\infty e^{\hat{\rho}(s-t)} \log(\hat{c}_s) ds\right]$$

subject to $\hat{w}_t \geq 0$, equation (2), and equation (3). Since $\hat{w}_t = \underline{\hat{w}}_t$ in equilibrium, wealth evolves as

$$d\hat{w}_t = (r_t \hat{w}_t - \hat{c}_t)dt + \int [\hat{\theta}_{it}\hat{w}_t dR_{it}]di + \hat{\theta}_t \hat{w}_t d\hat{R}_t + (\lambda w_t - \hat{\lambda}\hat{w}_t)dt$$

This completes the description of the environment. To define equilibrium, one needs notation for tree and bond holdings instead of positions $\{\theta_{it}, \hat{\theta}_{it}, \hat{\theta}_t\}$. Denote bond holdings of country i by b_{it} and their holdings of domestic trees by h_{it}. By construction, i's wealth is $w_{it} = b_{it} + p_{it}h_{it}$, and the risky share θ_{it} determines the split: $\theta_{it}w_{it} = p_{it}h_{it}$ and $(1 - \theta_{it})w_{it} = b_{it}$.

To track the intermediary's holdings of trees and bond issuance, let $\hat{b}_t$ be the total bonds issued, let $\{\hat{h}_{it}\}$ be its holdings of trees in regular countries, and let $\hat{h}_t$ be its holdings of the special country's tree. By construction,

$$\hat{w}_t = \int p_{it}\hat{h}_{it}di + \hat{p}_t\hat{h}_t - \hat{b}_t$$
$$\hat{b}_t = \left(\int \hat{\theta}_{it}di + \hat{\theta}_t - 1\right)\hat{w}_t$$

Portfolio weights $\{\hat{\theta}_{it}\}$ satisfy $\hat{\theta}_{it}\hat{w}_t = p_{it}\hat{h}_{it}$ for all i, and $\hat{\theta}_t$ satisfies $\hat{\theta}_t\hat{w}_t = \hat{p}_t\hat{h}_t$. Figure 1 uses this notation for a crude example of balance sheets. I can now define equilibrium.

Definition 1. *Given the processes $\{\nu_t, \gamma_t, \{Z_{it}\}\}_{t\geq 0}$ and the associated filtrations, an equilibrium is a collection of adapted price processes $\{r_t, \{p_{it}\}, \hat{p}_t\}_{t\geq 0}$, wealth processes $\{\{w_{it}\}, \{\underline{w}_{it}\}, \hat{w}_t, \underline{\hat{w}}_t\}_{t\geq 0}$, consumption processes $\{\{c_{it}\}, \hat{c}_t\}_{t\geq 0}$, and processes for asset holdings $\{\{h_{it}\}, \{\hat{h}_{it}\}, \{b_{it}\}, \hat{b}_t, \hat{h}_t\}_{t\geq 0}$ such that all agents optimize and*

- *aggregate wealth process agrees with individual wealth:* $w_{it} = \underline{w}_{it}$ *for all* i *and* $\hat{w}_t = \underline{\hat{w}}_t$*,*
- *bond market clears:* $\int b_{it} di = \hat{b}_t$*,*
- *markets for regular country trees clear:* $h_{it} + \hat{h}_{it} = 1$ *for all* $i \in [0, 1]$*,*
- *market for the special country tree clears:* $\hat{h}_t = \hat{q}$*,*
- *market for consumption goods clears:* $\int c_{it} di + \hat{c}_t = (1 + \hat{q})\nu$*.*

Markets for trees clear country by country, and the market for the intermediary's bonds clears globally. The market for consumption goods clears automatically as soon as other markets do.

assets	**liabilities**
b_{it}	w_{it}
$p_{it} h_{it}$	

agent in country i

assets	**liabilities**
$\int p_{it} \hat{h}_{it} di$	$\hat{b}_t$
$\hat{p}_t \hat{h}_t$	$\hat{w}_t$

global intermediary

Figure 1: Schematic balance sheets of agents from a country i and the global intermediary.

3 Equilibrium

This section characterizes equilibrium. The main object of interest is asset prices p_{it}. Solving for them requires characterizing equilibrium excess returns dR_{it} and using market clearing conditions. There is only idiosyncratic uncertainty in this economy. Only agents from country i and the intermediary have access to country i's tree, so prices and excess returns dR_{it} only load on dZ_{it}:

$$dp_{it} = \mu^p_{it} dt + \sigma^p_{it} dZ_{it}$$
$$dR_{it} = \mu^R_{it} dt + \sigma^R_{it} dZ_{it}$$

These equations define the drift and volatility of prices and excess returns $(\mu^p_{it}, \mu^R_{it}, \sigma^p_{it}, \sigma^R_{it})$. These are equilibrium objects related to each other by

$$\mu^R_{it} = \frac{\nu_t + \mu^p_{it}}{p_{it}} - r_t$$
$$\sigma^R_{it} = \frac{\sigma + \sigma^p_{it}}{p_{it}}$$

As usual, the volatility of returns has an exogenous and an endogenous component.

The first result that helps characterize equilibrium concerns consumption and portfolio choice.

Lemma 1. *Suppose $\sigma_{it}^R \neq 0$ for all countries. Agents consume a constant fraction of their wealth, $c_{it} = \rho w_{it}$ and $\hat{c} = \rho \hat{w}_t$, and choose mean-variance portfolios:*

$$\theta_{it} = \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \text{ and } \hat{\theta}_{it} = \gamma_t \frac{\mu_{it}^R}{(\sigma_{it}^R)^2}$$

The condition that $\sigma_{it}^R \neq 0$ rules out pathological equilibria with dividend shocks exactly offsetting capital gains on all sample paths. I describe these equilibria in the proof. The results for the regular countries are not surprising given the log utility. The non-trivial part of this lemma is the intermediary's consumption and portfolio choice. It turns out that adding the value-at-risk constraint to log utility does not break the constant-share consumption rule, and the only change in portfolio choice is the new effective risk-tolerance coefficient γ_t.

Two things about the value-at-risk constraint are especially convenient. First, it allows for time-varying risk aversion while keeping tractability. Other ways to achieve time-varying risk aversion include, for example, recursive preferences of Duffie and Epstein (1992) or habits of Campbell and Cochrane (1999). Recursive preferences create hedging motives in portfolio choice, making portfolios less tractable and introducing additional state variables that solve partial differential equations. Habits create additional state variables as well, increasing the number of endogenous objects to solve for. The value-at-risk constraint instead introduces an exogenous state variable γ_t and keeps portfolios tractable and easy to aggregate. Oskolkov (2024) describes the general treatment of value-at-risk constraints and provides a foundation through a version of robust preferences.

Second, the value-at-risk constraint generates non-trivial portfolios in a world without aggregate uncertainty and with perfectly diversifiable shocks. The intermediary has access to a continuum of assets. Unlike, for example, in preferred habitat models like Vayanos and Vila (2021), they provide uncorrelated returns. Despite that, the intermediary still faces a risk-return tradeoff at the level of individual assets. This is due to the functional form of the constraint that includes the sum of idiosyncratic variances in all countries instead of the total statistical variance of the entire portfolio, which is zero by the law of large numbers. As a result, the model features country-specific idiosyncratic risk premia in a potentially fully diversifiable world.

A useful feature of portfolio shares θ_{it} and $\hat{\theta}_{it}$ in Lemma 1 is that they are proportional to each other, simplifying aggregation in each country's market. The following proposition describes asset holdings and the main pricing equation for each i.

Proposition 1. *In each country i, the local agent's tree holdings are*

$$h_{it} = \frac{w_{it}}{w_{it} + \gamma_t \hat{w}_t} \text{ and } \hat{h}_{it} = \frac{\gamma_t \hat{w}_t}{w_{it} + \gamma_t \hat{w}_t}$$

The asset pricing equation for i's tree is

$$\underbrace{\mu^p_{it} + \nu_t - r_t p_{it}}_{\textit{risk premium}} = \underbrace{(\sigma^p_{it} + \sigma)^2}_{\textit{quantity of risk}} \cdot \frac{1}{\gamma_t \hat{w}_t + w_{it}} \tag{4}$$

There are two participants in each country's market, and they split the tree according to their wealth shares and effective risk tolerance, which equals one for local agents due to log utility and γ_t for the intermediary. Domestic ownership of the tree is monotone in local wealth, converging to one in extremely rich countries ($w_{it} \longrightarrow \infty$) and zero in extremely poor ones ($w_{it} \longrightarrow 0$).

Equation (4) is the main asset pricing equation in the model. It shows that the price of risk in every country is the weighted sum of the wealth in its market, where risk tolerance coefficients are the weights. The price of risk can rise due to local factors, such as negative output shocks that make the local agent poorer. It can also rise due to global factors: a negative shock to the intermediary's wealth or risk-taking capacity γ_t. These global factors are common to all countries, inducing synchronous movements in risk premia and ultimately asset prices.

A useful benchmark is the $\gamma_t = \infty$. In this case, the intermediary can take advantage of full diversification, with country-specific risk washing out. Equation (4) shows that expected excess returns μ^R_{it} have to be zero in equilibirum. Otherwise, the intermediary would demand assets in unbounded quantities. Since $\mu^R_{it} = 0$, local agents are unwilling to hold their trees because they cannot take advantage of the continuum of foreign assets, and local shocks for them continue to be aggregate. As a result, the intermediary holds the entire global supply of risky assets: $\hat{h}_{it} = 1$ for all i. Local agents are not exposed to risk and all have the same wealth in the long run.

With a finite γ_t, the intermediary demands a positive risk premium, which induces local agents to participate as well. A non-degenerate wealth distribution emerges in equilibrium due to exposure to shocks. This distribution is a global state variable that determines the interest rate and asset prices. Characterizing them requires taking equation (4) one step further, which is what I do next.

3.1 Asset prices

I will characterize asset prices and other quantities as functions of two variables: local wealth and time. This is possible because aggregate dynamics are deterministic, and the evolution of aggregate states, including the wealth distribution $G(w,t)$, can be indexed by time t only. Domestic wealth w replaces the index i since the local dynamics of two countries with the same wealth are identical.

Define the drift and volatility of local wealth w and tree prices $p(w,t)$ by

$$\begin{aligned} dw &= \mu_w(w,t)dt + \sigma_w(w,t)dZ \\ dp(w,t) &= \mu_p(w,t)dt + \sigma_p(w,t)dZ \end{aligned}$$

The density $g(w,t)$ associated with the distribution $G(w,t)$ solves a Kolmogorov forward equation

$$\partial g_t(w,t) = -\partial_w(\mu_w(w,t)g(w,t)) + \frac{1}{2}\partial^2_{ww}(\sigma_w(w,t)^2 g(w,t)) \tag{5}$$

subject to an initial condition on $g(w,0)$. The corresponding Kolmogorov backward equation is usually reserved for the agents' value functions. In my model, log utility makes solving for them redundant, and the backward equation instead applies to asset prices. The following proposition characterizes $p(w,t)$ and $r(t)$ for given exogenous processes $\{\gamma(t), \nu(t)\}$.

PROPOSITION 2. *Asset prices $p(w,t)$ and the interest rate $r(t)$ solve the following equations:*

$$r(t)p(w,t) - \partial_t p(w,t) = \nu(t) \underbrace{-\ \sigma^2\pi(w,t)}_{\textit{risk adjustment}} + \underbrace{\mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww}p(w,t)}_{\textit{growth term}} \tag{6}$$

$$r(t) = \rho + \frac{\nu'(t)}{\nu(t)} - \frac{\rho\sigma^2}{(1+\hat{q})\nu(t)}\int \pi(w,t)dG(w,t)$$

Here the drift and variance of wealth are

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}\hat{w}(t) + \frac{\sigma_w(w,t)^2}{w}$$
$$\sigma_w(w,t) = \frac{\sigma w}{w\ +\ \gamma(t)\hat{w}(t) \underbrace{-\ w\partial_w p(w,t)}_{\textit{amplification}}}$$

The risk compensation $\pi(w,t)$ is

$$\pi(w,t) = \frac{w + \gamma(t)\hat{w}(t)}{[w + \gamma(t)\hat{w}(t) - w\partial_w p(w,t)]^2}$$

The backward equation for prices follows from applying Itô's lemma to equation (4). There are two additional terms relative to fair-value pricing $r(t)p(w,t) - \partial_t p(w,t) = \nu(t)$. Risk adjustment reflects the market compensation for country-specific shocks. The growth term reflects the fact that prices are a function of wealth, so its drift introduces a drift in $p(w,t)$. Since prices are a non-linear function, the volatility of wealth introduces additional drift. The interest rate is related to the subjective discount rate and the growth rate of aggregate consumption, which is equal to the total output. The last term in the expression for the interest rate reflects uninsured risk as given by the average risk adjustment across all countries.

Wealth volatility $\sigma_w(w,t)$ originates in the dividend risk and is proportional to σ and wealth w. The denominator has two terms. The first is the total market wealth $w + \gamma(t)\hat{w}(t)$, which reflects risk sharing between the two participants. The second term is $-w\partial_w p(w,t)$, which accounts for endogenous risk amplification: dividend shocks hit wealth, which translates to prices and feeds back into wealth through them. The drift in wealth has a consumption-savings part $(r(t)-\rho)w$, the perpetual youth part $\hat{\lambda}\hat{w}(t) - \lambda w$, and the last term is the risk premium that investors receive for exposing their wealth to the trees. The risk compensation $\pi(w,t)$ inherits the term $-w\partial_w p(w,t)$ from the wealth volatility because local wealth w is a key variable driving asset returns. Without endogenous volatility, and hence this endogenous part in the denominator, $\pi(w,t)$ would simply be equal to $(\gamma(t)\hat{w}(t)+w)^{-1}$, the partial equilibrium price of risk as seen in equation (4).

Finally, the special country's wealth $\hat{w}(t)$ is an endogenous aggregate state. This is the last piece of equilibrium required to solve for asset prices. Following from consumption market clearing and the fact that aggregate consumption is a share ρ of aggregate output,

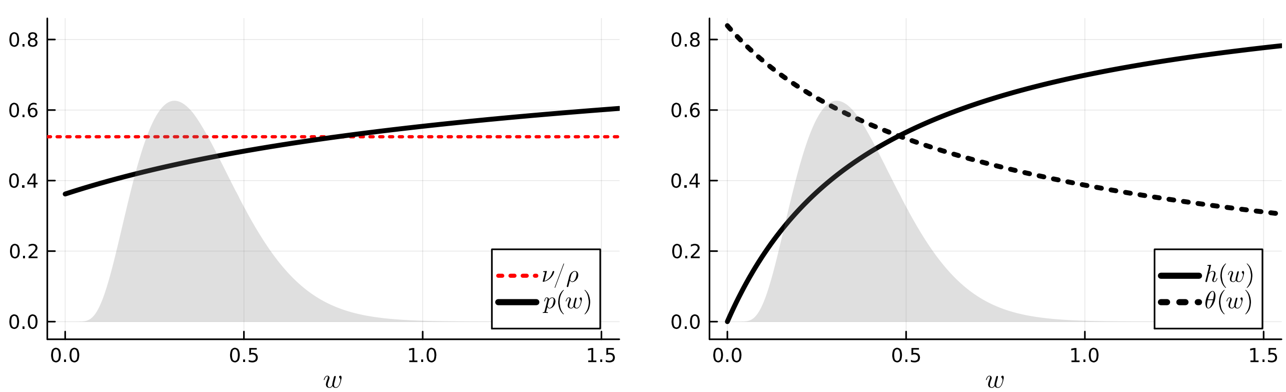
$$\hat{w}(t) = \frac{(1+\hat{q})\nu(t)}{\rho} - \int w dG(w,t)$$

The price of the special country's tree satisfies the fair value pricing equation $r(t)\hat{p}(t) - \hat{p}'(t) = \nu(t)$ because there is no associated risk.

I will study the paths of aggregate states $\{\gamma(t), \nu(t)\}$ that lead the economy to converge to a global steady state. This steady state provides terminal conditions for equation (6). It is also an illustrative focal point that exposes some properties of the economy that I describe below.

3.2 Steady state

In a steady state, all equations from Proposition 2 continue to hold for the time-independent version of all functions such as $p(w)$ and $g(w)$, and aggregates such as r and $\hat{w}$. Idiosyncratic shocks continue to hit regular countries and move them around the wealth distribution. A good sequence of dividends can take a country to the right tail, and a bad sequence can drag it back to the middle or to the left tail.



(a) Asset prices and the fair value

(b) Holdings and risky shares of local agents

Figure 2: Panel (a): asset prices $p(w)$ and the fair value limit. Panel (b): tree holdings of local agents $h(w)$ and the risky portfolio shares $\theta(w)$.

Figure 2 shows asset prices $p(w)$, domestic tree holdings $h(w)$, and portfolio shares $\theta(w)$ allocated to the trees by local agents. The steady-state wealth density $g(w)$ is in the background. Prices and the wealth density solve the following coupled system of ordinary differential equations:

$$rp(w) = \nu - \sigma^2\pi(w) + \mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(ww) \tag{7}$$

$$0 = (\mu_w(w)g(w))' - \frac{1}{2}(\sigma_w(w)g(w))'' \tag{8}$$

The interest rate is

$$r = \rho - \frac{\rho\sigma^2}{(1+\hat{q})\nu}\int \pi(x)dG(x)$$

The coefficients in the coupled system of equation (7) and equation (8) are

$$\begin{aligned}
\pi(w) &= \frac{w+\gamma\hat{w}}{[w+\gamma\hat{w}-wp'(w)]^2} \\
\sigma_w(w) &= \frac{\sigma w}{w+\gamma\hat{w}-wp'(w)} \\
\mu_w(w) &= (r-\rho-\lambda)w + \hat{\lambda}\hat{w} + \frac{\sigma_w(w)^2}{w}
\end{aligned}$$

Finally, the special country's wealth is

$$\hat{w} = \frac{(1+\hat{q})\nu}{\rho} - \int xdG(x)$$

Despite the simplicity of the setup, solving the coupled system of equation (7) and equation (8) is not straightforward. Endogenous risk makes the differential equation (7) non-linear. In practice, it can be solved iteratively: take a conjecture for $p(w)$, $g(w)$, compute the risk compensation $\pi(w)$, and use it to calculate the interest rate r and the drift and volatility of wealth $\mu_w(w)$ and $\sigma_w(w)$. Then, plug them in equation (7) and solve for the updated $p(w)$ as if it were a linear ODE. Finally, update the conjecture for $g(w)$ by solving the linear Kolmogorov forward equation (8). The pair $p(w)$ and $g(w)$ are a fixed point of this operation.

The right panel of Figure 2 shows local tree holdings and risky portfolio shares. The closed-form solution for $h(w) = w/(w+\gamma\hat{w})$ comes from Proposition 1, and the risky share equals $\theta(w) = p(w)h(w)/w = p(w)/(w+\gamma\hat{w})$. Section 5 describes the parameters used for the figure.

The special country enjoys its central position in the global financial system. The intermediary takes positions in trees all around the wealth distribution. With access to a continuum of uncorrelated returns, it earns excess returns with certainty. Its liabilities, on the contrary, are low-yielding risk-free bonds. This leads to "exorbitant privilege": profits allow the special country to sustain perpetual trade deficits even if it runs a net debt to the rest of the world. As shown, for example, by Gourinchas and Rey (2022), this corresponds to the empirical description of the US, whose external liabilities are dominated by safe assets, and external assets are mostly risky.

The expression for the special country's trade deficit follows from its budget constraint and the equilibrium tree holdings, setting the perpetual youth terms to zero for simplicity.

Corollary 1. *Suppose $\lambda = \hat{\lambda} = 0$. The special country's trade deficit is*

$$\begin{aligned}\hat{c} - \hat{q}\nu &= r \cdot \underbrace{\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right)}_{\textit{net foreign assets}} + \underbrace{\int \sigma^2 \pi(w)\hat{h}(w)dG(w)}_{\textit{risk compensation}} \\ &= r \cdot \left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \underbrace{\int (\nu - rp(w))\hat{h}(w)dG(w)}_{\textit{risk discount}} + \underbrace{\int \mu_p(w)\hat{h}(w)dG(w)}_{\textit{trading profits}}\end{aligned}$$

The trade deficit $\hat{c} - \hat{q}\nu$ can be funded through interest payments on the net foreign asset position and the risk compensation. In the data, the US has negative net foreign assets as it runs trade deficits, for which the risk compensation term must be positive. This term can be further decomposed into two sources. First, the risk discount reflects the fact that trees are priced at less than their fair value. The intermediary takes advantage of that, borrowing at r and buying claims to a stream of dividends ν at a price $p(w) < \nu/r$. Second, trading profits arise because countries move around the steady-state wealth distribution due to idiosyncratic shocks. This churn generates steady-state capital flows as the intermediary trades trees with local agents. Its trading strategy takes advantage of the drift in prices. The average drift in prices is zero in the steady state, but the intermediary takes positions $\hat{h}(w)$ that skew towards growing countries. As shocks reshuffle the wealth distribution, these countries become richer, their assets appreciate, and the intermediary sells them to buy cheaper assets from countries that arrive to the left tail.

The trading profits term is also equal to the financial account surplus of the special country, while the trade deficit and factor payments (interest paid on debt less dividends received) combine into the current account deficit. The fact that trading profits are positive implies that the special country can run perpetual current account deficits as well as trade deficits.

In addition to the special country, it is possible to characterize the dynamics in the right tail of the wealth distribution. To do that, I choose equilibria with a finite limit of $p(w)$ as $w \longrightarrow \infty$ and $\hat{p} = \nu/r$, ruling out bubbles in all assets. For these equilibria, Proposition 2 implies the following.

Corollary 2. *Suppose $p(w)$ has a finite limit as $w \longrightarrow \infty$. Then,*

$$\lim_{w \longrightarrow \infty} \sigma_w(w) = \sigma$$
$$\lim_{w \longrightarrow \infty} \frac{\mu_w(w)}{w} = r - \rho - \lambda < 0$$
$$\lim_{w \longrightarrow \infty} p(w) = \hat{p} = \nu/r > \nu/\rho$$

As a country becomes rich, the volatility of its wealth converges to a constant. Riskiness does not scale with wealth, and rich countries enjoy safe payoffs and consumption in relative terms: $\sigma_w(w)/w \longrightarrow 0$. The trend of their net worth goes negative, as they simply consume their accumulated wealth and drift back to the middle of the wealth distribution. Their asset prices converge to the safe asset benchmark. As a country grows richer, it increasingly looks like the special country, except it cannot get stuck in the right tail forever.

Importantly, as the global economy still contains uninsured idiosyncratic risk, the interest rate is depressed relative to the subjective discount rate: $r < \rho$. Safe assets are overvalued relative to fundamental cash flows: $\hat{p} = \nu/r > \nu/\rho$. It is intuitive that this should apply to the special country that has no inherent risk. Corollary 2 shows that rich regular countries become so safe that they also inherit this safety premium.

Characterizing asset prices in general is challenging because of the non-linearity of the key coupled system. However, it is possible to obtain analytical insights under a small-shock approximation. The next section describes asset prices under this approximation both in the steady state and following shocks to the global risk-taking capacity that trigger capital flight events.

4 Analytical Results

I consider the limit of small idiosyncratic shocks: $\sigma \longrightarrow 0$. This approximation generates closed-form expressions for risk premia and clarifies the magnitudes of the different moving parts in equilibrium. Unsurprisingly, risk premia are comparable to the variance of idiosyncratic shocks σ^2. Less intuitively, it turns out that endogenous risk and wealth amplification effects only contribute terms of order $O(\sigma^4)$ to risk premia and asset prices. This has important implications for aggregate shocks and heterogeneity. Shifts in the wealth distribution created by aggregate shocks do affect risk premia, but these effects turn out to be qualitatively smaller than the direct impact of changing preferences for risk or dividends. I show that accounting for this difference in magnitudes substantially simplifies characterizing shocks to the intermediary's risk-taking capacity $\gamma(t)$. Key impulse responses are available in closed form.

Start with the steady state. Define a price of risk function $\boldsymbol{\pi}(w)$:

$$\boldsymbol{\pi}(w) \equiv \frac{1}{w + \gamma \hat{w}}$$

Let $\lambda = \boldsymbol{\lambda}\sigma^2$ and $\hat{\lambda} = \hat{\boldsymbol{\lambda}}\sigma^2$ and define also a pair of functions $\boldsymbol{m}(\cdot)$ and $\boldsymbol{s}(\cdot)$ as

$$\begin{aligned} \boldsymbol{m}(w) &= w\left(\boldsymbol{\pi}(w)^2 - \frac{\rho}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) - \boldsymbol{\lambda}\right) + \hat{\boldsymbol{\lambda}}\hat{w} \\ \boldsymbol{s}(w) &= w\boldsymbol{\pi}(w) \end{aligned}$$

Here $\mathcal{G}(\cdot)$ denotes the distribution with an associated density $\boldsymbol{g}(\cdot)$ that solves

$$(\boldsymbol{m}(w)\boldsymbol{g}(w))' = \frac{1}{2}(\boldsymbol{s}(x)^2\boldsymbol{g}(w))''$$

Finally, the special country's wealth $\hat{w}$ in the expression for $\boldsymbol{\pi}(\cdot)$ is

$$\hat{w} = \frac{(1+\hat{q})\nu}{\rho} - \int x d\mathcal{G}(x)$$

With these ingredients, the following proposition characterizes the limiting steady state.

PROPOSITION 3. *In the limit $\sigma \longrightarrow 0$, the interest rate and asset prices are*

$$\begin{aligned} r &= \rho - \frac{\rho\sigma^2}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) + O(\sigma^4) \\ \hat{p} &= \frac{\nu}{\rho} + \frac{\sigma^2}{(1+\hat{q})\rho}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) + O(\sigma^4) \\ p(w) &= \frac{\nu}{\rho} + \frac{\sigma^2}{\rho}\cdot\left[\frac{1}{1+\hat{q}}\int \boldsymbol{\pi}(x)d\mathcal{G}(x) - \boldsymbol{\pi}(w)\right] + O(\sigma^4) \end{aligned}$$

The second-order term in $p(w)$ is monotone in w, with $p(0) < \nu/\rho$ and $p(w) > \nu/\rho$ when $w \longrightarrow \infty$. The functions $\boldsymbol{\pi}(\cdot)$, $\boldsymbol{g}(\cdot)$, $\boldsymbol{m}(\cdot)$, and $\boldsymbol{s}(\cdot)$ approximate the risk compensation, the steady-state wealth density, and the drift and volatility of wealth:

$$\begin{aligned} \pi(w) &= \boldsymbol{\pi}(w) + O(\sigma^2) \\ g(w) &= \boldsymbol{g}(w) + O(\sigma^2) \\ \mu_w(w) &= \sigma^2\boldsymbol{m}(w) + O(\sigma^4) \\ \sigma_w(w) &= \sigma\boldsymbol{s}(w) + O(\sigma^3) \end{aligned}$$

The first important property of the small-σ approximation is that the risk compensation $\boldsymbol{\pi}(\cdot)$ is missing the amplification term $-wp'(w)$ in the denominator. This amplification channel reflects the equilibrium feedback loop from dividend shocks to wealth to prices. It turns out that this channel is of the next order of importance compared to the total wealth-weighted risk tolerance $w+\gamma\hat{w}$ in the market. The "partial equilibrium" price of risk $(w+\gamma\hat{w})^{-1}$ is a good approximation for the general equilibrium risk compensation.

This fact simplifies computations. The strength of precautionary motives that depress the interest rate relative to the subjective discount rate is the average price of risk. Risky asset prices have an especially revealing functional form too. On the one hand, country-specific price of risk $\boldsymbol{\pi}(w)$ depresses $p(w)$ relative to the fundamental cash flows ν/ρ. On the other hand, because of precautionary motives that lower the interest rate, all asset prices are inflated by the average price

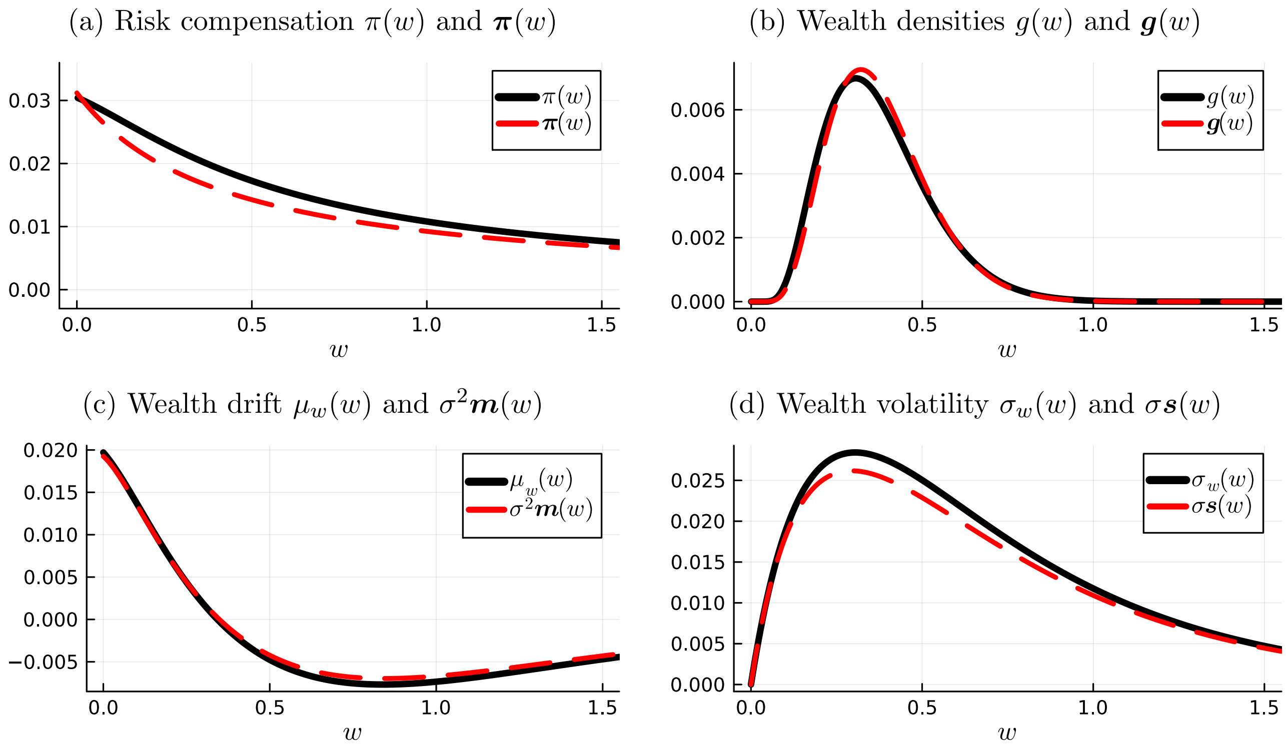


Figure 3: Exact solutions and second-order approximations from Proposition 3.

of risk. Since $\boldsymbol{\pi}(w) \longrightarrow 0$ as $w \longrightarrow \infty$, the second effect dominates for rich countries. Their risky assets trade at a premium relative to the fundamentals. At the other end, when $w \longrightarrow 0$, the first effect dominates: $p(0) < \nu/\rho$. This follows from the monotonicity of $\boldsymbol{\pi}(\cdot)$. Finally, the special country's tree does not have an associated price of risk, which means it is inflated relative to ν/ρ by exactly as much as the interest rate is depressed relative to ρ.

All approximation errors are proportional to σ^2 instead of σ. This fact is due to symmetry: replacing σ with $-\sigma$ should not change anything in the economy because the underlying shocks are Brownian motions. Figure 3 demonstrates the quality of the approximation by plotting the exact solutions for $\{\pi(\cdot), g(\cdot), \mu_w(\cdot), \sigma_w(\cdot)\}$ and the corresponding approximations for my preferred calibration described in Section 5.

4.1 Shocks to risk-taking capacity

I now describe a negative shock that hits the intermediary's risk-taking capacity $\gamma(t)$. This shock operates through the value-at-risk constraint and makes it more costly for the intermediary to hold country-specific risk on its balance sheet. The shock does not discriminate between countries, inducing the intermediary to sell the same proportion of its position in all countries. The impact on countries, however, is unequal for two reasons. First, since the intermediary holds a larger share of the market in poor countries, the resulting sudden stop targets the left tail of the wealth distribution more. Second, the response of local agents depends on their wealth. In rich countries, local agents have accumulated large stocks of foreign assets, and they can buy large quantities of domestic assets without a sharp rise in expected excess returns. In poor countries, local agents cannot absorb much more of their domestic risk. Expected excess returns have to rise instead, convincing the intermediary to sell less in equilibrium. Adjustment in these markets happens through prices rather than quantities, and risk premia rise sharply in the left tail. Markets behave more elastically in countries with large stocks of foreign assets.

On top of these responses across individual countries, the global risk-free rate changes in general equilibrium. The price of risk rises because aggregate risk tolerance falls in all markets, so uninsured idiosyncratic risk depresses the risk-free rate more than before. With the global interest rate falling, asset prices are inflated relative to the steady state. This effect counteracts the rise in risk premia, and it dominates in rich countries, where domestic agents' retrenchment keeps the risk premia from rising too much. As a result, risky asset prices in rich countries rise after a negative shock to global risk-taking capacity. These assets behave like colloquial "safe" assets because they have a countercyclical component in returns. Importantly, their safe asset status is endogenous: the physical properties of dividends and asset supply are fixed across countries. There are no fundamental differences in the cross-section, just the stock of wealth accumulated before the shock.

To show this analytically, I again take advantage of the second-order approximation around $\sigma = 0$ and additionally linearize the model with respect to aggregate shocks. The benefit of this approach is that endogenous risk is not among the second-order terms. This pushes the impact of initial wealth revaluation into the fourth order of σ: the fact that countries' net worth is immediately hit by the change in $\gamma(t)$ does not feed into their asset prices. This leaves asset prices affected by two things only: the change in the price of risk on the country's local market and the change in the global risk-free rate.

Formally, consider a single unanticipated shock $\Delta\gamma(t) = \delta e^{-\mu_\gamma t}$. I will use $\Delta\gamma(t) < 0$, a negative shock, as the running example. The economy approaches $t = 0$ resting at the steady state. The following proposition characterizes the deviations $\Delta p(w,t)$, $\Delta\hat{p}(t)$, and $\Delta r(t)$ from the steady-state values $p(w)$, $\hat{p}$, and r.

Proposition 4. *The $\Delta\gamma(t)$-first-order deviations of the interest rate and asset prices are*

$$\Delta r(t) = \Delta\gamma(t)\sigma^2 \cdot \frac{\rho\hat{w}}{(1+\hat{q})\nu} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4)$$
$$\Delta\hat{p}(t) = -\Delta\gamma(t)\sigma^2 \cdot \frac{\hat{w}}{\rho+\mu_\gamma} \cdot \frac{1}{1+\hat{q}} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4)$$
$$\Delta p(w,t) = -\Delta\gamma(t)\sigma^2 \cdot \frac{\hat{w}}{\rho+\mu_\gamma} \left[\frac{1}{1+\hat{q}} \int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4)$$

The function $\Delta p(w,t)$ is monotone in w, and it has the opposite signs at $w=0$ and $w \longrightarrow \infty$ for all $t \geq 0$. The change in the risk compensation is

$$\Delta\pi(w,t) = -\Delta\gamma(t)\hat{w}\boldsymbol{\pi}(w)^2 + O(\sigma^2)$$

The technical significance of this result is that the main components of the interest rate and asset price deviations from the steady state only depend on the current level of $\Delta\gamma(t)$. Typically, they depend on the whole history of the aggregate shock, which leads to expressions like

$$\Delta p(w,t) = \int_0^\infty J(w,t,s)\Delta\gamma(t-s)\eta(ds)$$

Here $J(\cdot)$ is a sequence-space Jacobian that linearly maps the whole sequence $\{\Delta\gamma(s)\}_{s=-\infty}^{s=\infty}$ to the sequence of first-order deviations $\{\Delta p(\cdot,t)\}_{t\geq 0}$. Integration has to be done with a measure $\eta(\cdot)$ to accommodate the fact that $\Delta\gamma(t)$ may impact $\Delta p(w,t)$ discretely, in addition to continuous impact of $\{\Delta\gamma(s)\}_{s<t}$. The reason $J(w,t,s)$ typically depends on $s<t$ is that aggregate shocks move aggregate states, such as the wealth distribution, and these effects accumulate over $[0,t]$, adding to the direct effect of $\Delta\gamma(t)$.[1]

Aggregate states still move in response to $\Delta\gamma(t)$ in my model, but Proposition 4 shows that, up to the fourth order of σ, these accumulating effects do not show up in individual asset prices and the interest rate. The Jacobian $J(w,t,s)$ is simply $J(w)$, and for one-dimensional variables like the global risk-free rate $r(t)$ or the special country's tree price $\hat{p}(t)$, the corresponding Jacobian is a scalar. All that is needed to compute the Jacobians are the steady-state wealth distribution and price of risk. Besides providing analytical insights, this approximation makes estimating the model with GMM particularly easy, as model-generated moments have closed-form expressions and only depend on a few steady-state objects.

The analytical content of Proposition 4 clarifies what happens to the price of risk and asset prices across the wealth distribution. The change in the risk compensation is zeroth-order in σ. The intermediary's risk tolerance affects it directly. This impacts the global interest rate: uninsured idiosyncratic risk generates more precautionary motives if $\Delta\gamma(t)<0$, and the interest rate falls. The price of the special country's tree rises, reacting to a lower discount rate.

Prices of risky assets in regular countries are subject to two effects: they fall due to a rising local price of risk, but the global increase in the price of risk depresses the risk-free rate, and this pushes all asset prices up. The left panel on Figure 4 plots the change in $p(w)$ on impact as a function of the country's wealth before the shock, along with the two components.

Because of monotonicity in $\boldsymbol{\pi}(\cdot)$, the first effect surely dominates around $w=0$, and risky asset prices fall on impact in poor countries. The second effect comes out on top in rich countries, where

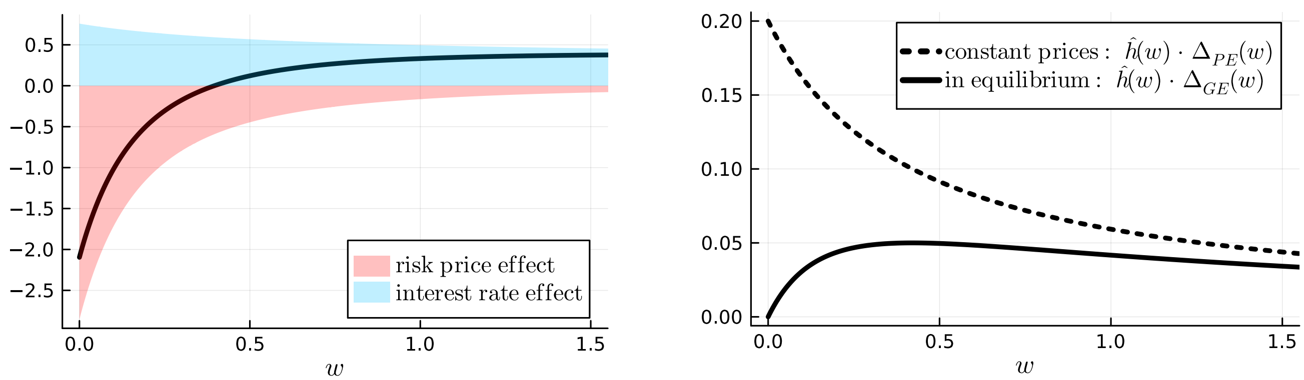
[1] Auclert, Bardóczy, Rognlie, and Straub (2021) provide a detailed explanation.

$\boldsymbol{\pi}(w)^2 \longrightarrow 0$. Steady-state risk premia are small in those countries due to large domestic investors saturated with safe assets: the market does not require a high premium to hold an additional unit of risk. The same large domestic investors pour in when there is capital flight, replacing foreign investors without asking for much higher expected excess returns.

To see this more clearly, consider the following first-order quantity: the fraction of its position the intermediary tries to sell in a given country at constant prices. This is simply how much $\gamma(t)$ falls: $\Delta_{PE}(w) = -\Delta\gamma(t)/\gamma$. On the other hand, the general equilibrium change in foreign holdings of the country's tree as a share of the foreign-held stock is

$$\Delta_{GE}(w) = \frac{\Delta \hat{h}(w,t)}{\hat{h}(w)} = -\frac{\Delta\gamma(t)}{\gamma} \cdot \frac{w}{w + \gamma\hat{w}}$$

This is how much domestic investors actually buy from the intermediary in equilibrium. This function converges to zero in poor countries, since local investors have no capacity to take over their risky assets amid capital flight. Prices have to adjust in those markets instead of quantities. This adjustment works through rising risk premia that convince foreign agents to stay. In rich countries, $\Delta_{GE}(\cdot)$ is maximized. Local investors actively retrench. Even if $\Delta\gamma(t)$ were to fall almost to zero, countries with large accumulated wealth ($w \longrightarrow \infty$) would simply take over their entire foreign liabilities.

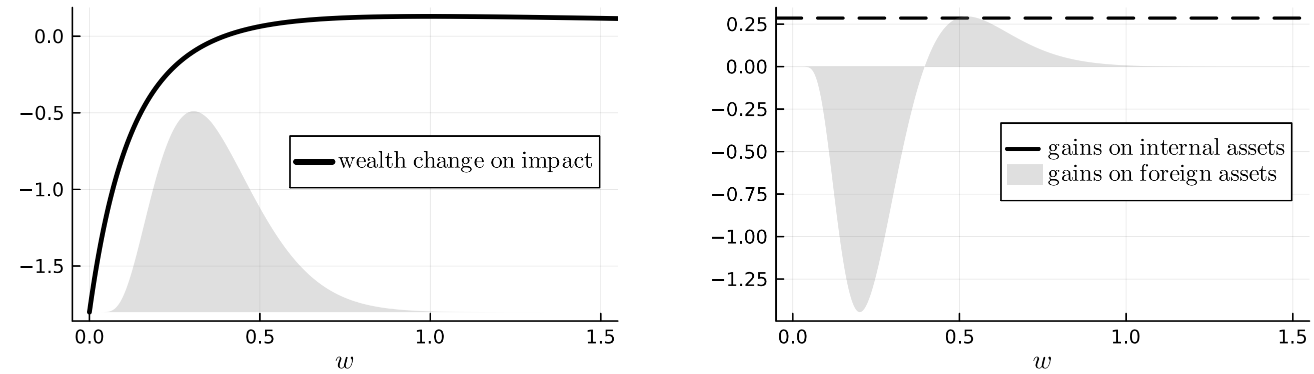


(a) Percentage changes in asset prices on impact $100 \cdot \Delta p(w,0)/p(w)$ and the two components.

(b) Foreign sales $\hat{h}(w)\Delta_{PE}(w)$ and $\hat{h}(w)\Delta_{GE}(w)$ as a share of the total outstanding stock of trees.

Figure 4: Asset price changes and foreign asset sales on impact for a 20% negative shock to $\gamma(t)$.

The right panel of Figure 4 shows asset sales by the intermediary as a function of the country's wealth before the shock. The units are the share of the total stock of outstanding trees. The counterfactual constant-price sales $\hat{h}(w)\Delta_{PE}(w)$ simply mimic the size of the intermediary's position in the country before the shock. Since this is maximized in poor countries, the sudden stop hits the left tail of the wealth distribution especially hard. On top of this, there are no large domestic investors in poor countries to sell to. The actual sales $\hat{h}(w)\Delta_{GE}(w)$ converge to zero at $w \longrightarrow 0$.



(a) Percentage changes in regular countries' wealth on impact $100 \cdot \Delta p(w, 0)h(w)/w$.

(b) Percentage change in intermediary's wealth on impact $100 \cdot \Delta p(w, 0)\hat{h}(w)/\hat{w}$ and $100 \cdot \Delta\hat{p}(0)\hat{q}/\hat{w}$.

Figure 5: Wealth changes on impact for a 20% negative shock to $\gamma(t)$. On the left: relative changes in regular countries' wealth. On the right: impact gains the intermediary makes on its foreign assets (weighted with the wealth density of regular countries) and the impact gains it makes on the special country's tree (dashed line).

Intuitively, the difference between the two lines is the unsold stock of assets that measures how much adjustment has to happen through prices.

Finally, Proposition 4 has direct implications for wealth redistribution in capital flight events. Investors in regular countries hold two assets on their balance sheets: trees and short-term bonds. There is no revaluation of the latter, so all changes in wealth come from the capital gains or losses on the trees. Since $\Delta p(w, 0)$ is negative for small w and positive for large, wealth is redistributed from poor countries to rich ones. The left panel of Figure 5 shows wealth changes on impact resulting from the risk-taking capacity shock as a share of the country's wealth before the shock.

The right panel of Figure 5 shows capital gains and losses the intermediary makes on its portfolio when the shock hits. They are weighted with the wealth density of the regular countries, so the shaded area adds up to total external gains and losses. The intermediary runs losses on the left tail of the wealth distribution in times of capital flight, which is a well-known empirical regularity. This is what the literature refers to as "exorbitant duty". By absorbing part of the losses on risky assets in regular countries, the special country essentially makes a wealth transfer to them. This insurance mechanism is present in models of Maggiori (2017), Sauzet (2023), Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022), and others.

A less-known outcome is that the special country makes capital gains on the right tail of the wealth distribution. Assets in the right-tail countries, although intrinsically risky, endogenously behave as safe and appreciate in capital flight events. This way, the rest of the world partly insures the US. Another implication is that, through the US, rich countries also partly insure poor ones. The intermediary's net worth is an important state variable for all asset prices, and capital gains that accrue to the US stabilize everyone's asset prices by supporting the intermediary's balance sheet in bad times.

Finally, the dashed line shows the capital gains on domestic assets, the special country's tree. These are unambiguously positive after a negative shock to $\gamma(t)$. These capital gains can increase the special country's wealth in bad times even if it runs net losses on its net foreign asset position. Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022) highlight accounting for internal holdings as a way to reconcile the exorbitant duty of the US with the fact that its domestic currency appreciates in bad times. As pointed out by Maggiori (2017), with home bias in consumption, a country whose wealth share is falling should see its real exchange rate depreciate. If the US becomes relatively poorer as it transfers wealth to the rest of the world, its real exchange rate should depreciate, absent other frictions and currency demand shocks. Capital gains on internal asset markets can help the US increase its wealth share despite the wealth transfer to other countries. Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022) argue that this is empirically true.

In my model, the sign of the net change in the special country's wealth after the shock to $\gamma(t)$ is ambiguous and depends on the calibration. However, it is possible to say that losses on external assets dominate when $\hat{q}$ is small. Without large internal holdings, the special country surely becomes relatively poorer, taking into account both its capital gains on rich regular countries and its losses on the poor ones. I formulate this as a corollary to Proposition 4.

Corollary 3. *If $\hat{q}$ is small enough, the impact change in the special country's wealth after a negative shock to $\gamma(t)$ is negative: $\Delta\hat{w}(0) < 0$.*

While the presence of domestic US assets does not guarantee that the US wealth share increases in bad times in the model, their absence guarantees the opposite, and exorbitant duty obtains. The reason is that, in the steady state, the intermediary invests more in risky assets in the left tail of the wealth distribution, targeting their high idiosyncratic risk premia. These countries with high idiosyncratic risk end up being negatively affected by global risk too, making returns on the US external portfolio procyclical. In my calibration, the US makes net gains on impact, consistent with the evidence presented by Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2022).

5 Calibration and Estimation

I calibrate the model using its steady state and estimate the processes for aggregate shocks to the intermediary's risk-taking capacity $\gamma(t)$ and global output $\nu(t)$ using linearized transition dynamics. The steady state of the model is determined by seven parameters: $\{\rho, \nu, \sigma, \lambda, \hat{\lambda}, \gamma, \hat{q}\}$. The number of degrees of freedom is six: the interest rate r and the drift and variance of regular countries' wealth $\mu_w(w)$ and $\sigma_w(w)^2$ are homogeneous of degree one in $\{\rho, \nu\,\sigma^2, \lambda, \hat{\lambda}\}$, while asset prices $p(w)$ and $\hat{p}$, the special country's wealth $\hat{w}$, and the wealth density $g(w)$ are homogeneous of degree zero. With these degrees of freedom, I discipline the parameters by bringing the model to reproduce ten empirical moments.

Table 1: steady-state calibration.

	model	target	source
aggregates:			
global risk-free rate	3.8%	3.8%	Bertaut, Curcuru, Faia, and Gourinchas (2024)
average risk premium	5.1%	5.1%	Bertaut, Curcuru, Faia, and Gourinchas (2024)
emerging market premium	2.9%	2.9%	MSCI
US wealth share	37.7%	32.3%	Credit Suisse (2022)
US output share	12.7%	22.8%	World Bank
ratio A/L:			
mean	0.72	0.71	IFS (IMF)
standard deviation	0.51	0.47	IFS (IMF)
q25	0.35	0.34	IFS (IMF)
q50	0.59	0.64	IFS (IMF)
q75	0.94	0.92	IFS (IMF)

First, I use recent evidence from security-level holdings provided by Bertaut, Curcuru, Faia, and Gourinchas (2024). I take the average return on the US bond liabilities as a target for the risk-free rate in my model. I then take the difference between the average return on the US equity claims and its bond liabilities as a target for the average excess return earned by the intermediary. To discipline the differences in excess returns between advanced economies and emerging markets, I take MSCI equity indices for developed countries and emerging markets and compute the average annualized difference in returns. I take the US wealth and output share from Credit Suisse (2022) and the World Bank. The model underestimates the US output share, overestimating the special country's ability to generate wealth out of a given stock of productive assets. One reason for this might be that in the data, global intermediaries are not domiciled exclusively in the US, with some of the largest banks housed in Europe. The model, however, fully confines global financial intermediation to the special country, directing all intermediation profits to its residents.

To discipline the wealth distribution, I take the following statistic. For every country, I compute the ratio of external assets $A(w) = (1 - \theta(w))w$ to external liabilities $L(w) = p(w)\hat{h}(w)$. I then take the distribution of this variable and bring its mean, standard deviation, and three quartiles close to the data. As an analog in the data, I take stocks of assets and liabilities corresponding to "private flows" in the terminology of Forbes and Warnock (2012) and Forbes and Warnock (2021): portfolio debt, portfolio equity, and banking flows. I exclude FDI and reserves. The data come from the International Financial Statistics database provided by the IMF.

The ratio $A(w)/L(w)$ is important because it shows the retrenchment capacity of the local agents. The size of their foreign assets relative to their country's external liabilities determines how effectively they can replace foreign investors in a capital flight event. This is the main mechanism

Table 2: Model parameters.

parameter	value	meaning
regular countries		
ρ	0.0720	discount rate
ν	0.0383	output rate
σ	0.1168	output volatility
λ	0.0436	wealth emigration rate
special country		
$\hat{q}$	0.1437	asset stock
γ	1.8806	risk-taking capacity
$\hat{\lambda}$	0.0892	wealth emigration rate

behind the heterogeneous responses to global shocks. Table 1 shows that, while not exact, the match between the model and the data on the five moments of A/L is close. Table 2 shows the parameter values. Appendix A provides additional details.

Turning to aggregate shocks, I postulate Ornstein-Uhlenbeck processes for the global risk-taking capacity $\gamma(t)$ and global output $\nu(t)$. Given persistence $\{\mu_\gamma, \mu_\nu\}$ and loadings $\{\sigma_\gamma, \sigma_\nu\}$,

$$d\gamma(t) = \mu_\gamma(\gamma - \gamma(t))dt + \sigma_\gamma \cdot dW(t) \tag{9}$$

$$d\nu(t) = \mu_\nu(\nu - \nu(t))dt + \sigma_\nu \cdot dW(t) \tag{10}$$

Here $dW(t)$ is a two-dimensional standard Brownian motion and σ_γ and σ_ν are two-dimensional vectors. Without loss of generality, I take $\sigma_{\nu 1} = 0$, which leaves me with five parameters to estimate: $\{\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2}\}$.

I use two global series from the model for estimation: total external assets $A(t)$ and the average risky asset price $P(t)$:

$$A(t) = \int (1 - \theta(w,t))w dG(w,t)$$

$$P(t) = \int p(w,t) dG(w,t)$$

I linearize the model with respect to aggregate shocks and use the second-order approximation around $\sigma = 0$, which turns out to be good in the third order: error terms are of order $O(\sigma^4)$. Having obtained the linear perfect-foresight responses $\Delta A(t)$ and $\Delta P(t)$, I normalize them by the steady-state values and use the moments of $a_t = \Delta A(t)/A$ and $p_t = \Delta P(t)/P$. Here A and P correspond to $A(t)$ and $P(t)$ computed at the steady-state functions $\theta(w)$, $p(w)$, and $g(w)$. I then work with the quarterly frequency of a_t and p_t. Table 3a lists the five quarterly moments I use: autocorrelations of a_t and p_t, their standard deviations, and the contemporaneous correlation between the two. Appendix A provides details of the estimation procedure.

5.1 Data

To construct the data analog for outward flows, I take the Balance of Payments and International Investment Positions data from the IMF. For a country i in quarter t, the quantity $F_{it}^{\text{acq(raw)}}$ denotes net purchases of foreign assets measured in dollars. This is a flow variable that measures transactions, leaving out changes in positions due to valuation. Following Forbes and Warnock (2012) and Forbes and Warnock (2021), I take a smooth version:

$$F_{it}^{\text{acq}} = \sum_{t-3}^{t} F_{is}^{\text{acq(raw)}} - \sum_{t-7}^{t-4} F_{is}^{\text{acq(raw)}}$$

I restrict attention to portfolio debt, portfolio equity, and "other" assets, the latter corresponding to banking flows. The empirical analog of a_t is

$$\tilde{a}_t = \frac{\sum_i F_{it}^{\text{acq}}}{\sum_i A_{i,t-1}}$$

where $A_{i,t-1}$ is the dollar stock of these assets one quarter before in country i. See Appendix C for details of data construction and sample statistics.

I construct the analog of p_t from the MCSI asset price index that excludes the US. Denote the quarterly version of this index by Q_t. The analog of p_t is quarterly returns smoothed over the four-quarter window: $\tilde{p}_t = \sum_{t-3}^{t} Q_s/Q_{s-1}$.

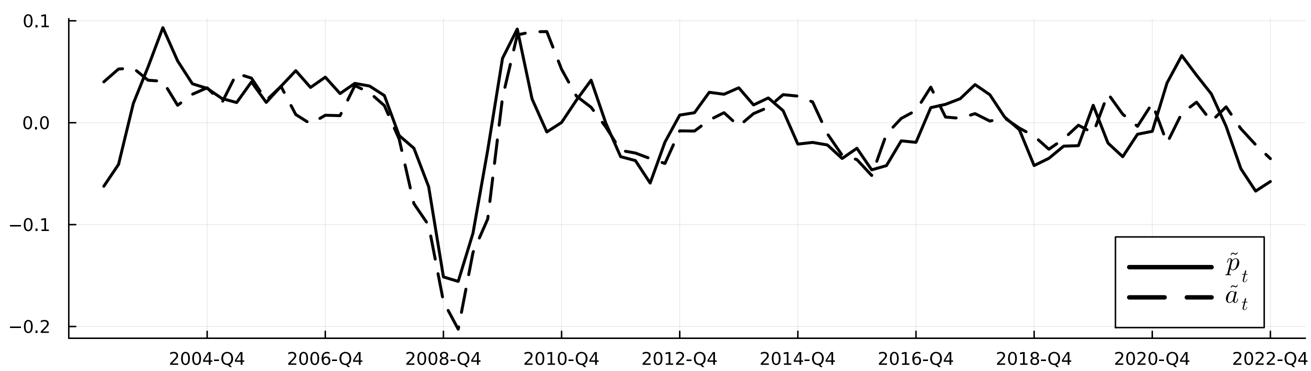


Figure 6: Constructed data series $\tilde{p}(t)$ and $\tilde{a}(t)$.

Figure 6 shows the data. The sample is 8 quarters long, starting in Q1 of 2003 and ending in Q4 of 2022. Table 3b shows estimated parameters. Table 3c shows the parameters of the discrete-time processes corresponding to equation (9) and equation (10). Taking τ to be the length of one time

(a) targeted moments.

	std(p_t)	std(a_t)	corr(p_t, a_t)	corr(p_t, p_{t-1})	corr(a_t, a_{t-1})
data	0.046	0.048	0.725	0.776	0.843
model	0.046	0.049	0.721	0.776	0.843

(b) Estimation results.

μ_γ	μ_ν	$\sigma_{\gamma 1}$	$\sigma_{\gamma 2}$	$\sigma_{\nu 2}$
0.0966	0.0394	0.1422	0.5344	1.0141
(0.0002)	(0.0002)	(0.0002)	(0.0006)	(0.0003)

(c) Finite-time parameters of the shock processes.

	ρ_γ	ρ_ν	$\varsigma_{\gamma 1}$	$\varsigma_{\gamma 2}$	$\varsigma_{\nu 2}$
quarterly	0.875	0.776	$0.024 \cdot \gamma$	$0.010 \cdot \gamma$	$0.022 \cdot \nu$
annual	0.586	0.363	$0.040 \cdot \gamma$	$0.016 \cdot \gamma$	$0.032 \cdot \nu$

Table 3: estimation.

period, equation (9) and equation (10) generate the following discrete-time analogs:

$$\gamma_{t+1} - \gamma = \rho_\gamma(\gamma_t - \gamma) + \varsigma_{\gamma 1}\varepsilon_{1,t+1} + \varsigma_{\gamma 2}\varepsilon_{2,t+1}$$
$$\nu_{t+1} - \nu = \rho_\nu(\nu_t - \nu) + \varsigma_{\nu 2}\varepsilon_{2,t+1}$$

Here $\{\varepsilon_{i,t+1}\}_{i=1,2}$ are standard normals, and $\{\rho_\gamma, \rho_\nu, \varsigma_{\gamma 1}, \varsigma_{\gamma 2}, \varsigma_{\nu 2}\}$ depend on $\{\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2}\}$ and the length of the time period τ. Table 3c shows the results for $\tau = 0.25$ and $\tau = 1$.

5.2 Untargeted moments

The model is able to match the five parameters of the aggregate series exactly. I next show that it also qualitatively matches the differences between advanced economies and emerging markets. For this exercise, I put the boundary between advanced economies (AE) and emerging markets (EM) at the point $\overline{w}$ where the first-order response of asset prices $\Delta p(w,t)$ to a risk-taking capacity shock $\Delta\gamma(t)$ changes the sign. To the right of the boundary lay rich countries, where asset prices increase after a negative shock to $\gamma(t)$. To the left are emerging markets. To compute group-level aggregates, I take integrals with conditional distributions for $[0, \overline{w}]$ and $[\overline{w}, \infty)$ respectively.

In the data, I use the IMF classification to designate advanced economies and emerging markets for capital flow series.[2] For risky asset prices, I use two different equity price indices from MSCI: its EM[3] index and its developed markets EAFE[4] index.

[2] https://www.imf.org/en/Publications/WEO/weo-database/2023/April/groups-and-aggregates
[3] https://www.msci.com/documents/10199/c0db0a48-01f2-4ba9-ad01-226fd5678111
[4] https://www.msci.com/documents/10199/822e3d18-16fb-4d23-9295-11bc9e07b8ba

Table 4: untargeted moments.

	std(p_t)		std(a_t)		std(b_t)	
	data	**model**	**data**	**model**	**data**	**model**
advanced economies	0.046	0.037	0.050	0.026	0.040	0.031
emerging markets	0.060	0.042	0.059	0.048	0.025	0.019

In addition to the series $a_t = \Delta A(t)/A$ and $p_t = \Delta P(t)/P$ described above, I also use the ratio of outward flows to external liabilities $b_t = \Delta A(t)/L$. Here

$$L = \int p(w)\hat{h}(w)dG(w)$$

In the model, this moment shows how much retrenchment happens in both country groups relative to the stock of the intermediary's group-level holdings. As discussed in Section 4, at the level of individual countries, this variable is an important determinant of how much adjustment to external shocks happens through prices and how much happens through quantities. In advanced economies, retrenchment should be more active as measured relative to outstanding stock of liabilities, and this is what stabilizes their local risk premia.

Table 4 shows standard deviations of asset prices, outward flows normalized by external assets, and outward flows normalized by external liabilities in the two country groups. While not hitting the moments exactly, the model reproduces the sign of the difference in all three. Asset prices are more volatile in emerging markets. Outward flows relative to foreign assets are more volatile in emerging markets too, reflecting a smaller denominator. Importantly, outward flows relative to foreign liabilities are more volatile in advanced economies, reflecting the fact that local agents retrench in times of negative global shocks, taking over their domestic asset markets and stabilizing price movements.

The second set of untargeted moments that the model reproduces qualitatively describes the cyclicality of asset prices and the relative performance of advanced economies compared to emerging markets. Given the series $\{\tilde{p}_t, \tilde{p}_t^{AE}, \tilde{p}_t^{EM}, \tilde{a}_t\}$, I regress $\tilde{p}_t$ and $\tilde{p}_t^{AE} - \tilde{p}_t^{EM}$ on $\tilde{a}_t$:

$$\tilde{p}_t = \alpha + \beta\tilde{a}_t + u_t \tag{11}$$

$$\tilde{p}_t^{AE} - \tilde{p}_t^{EM} = \underline{\alpha} + \underline{\beta}\tilde{a}_t + v_t \tag{12}$$

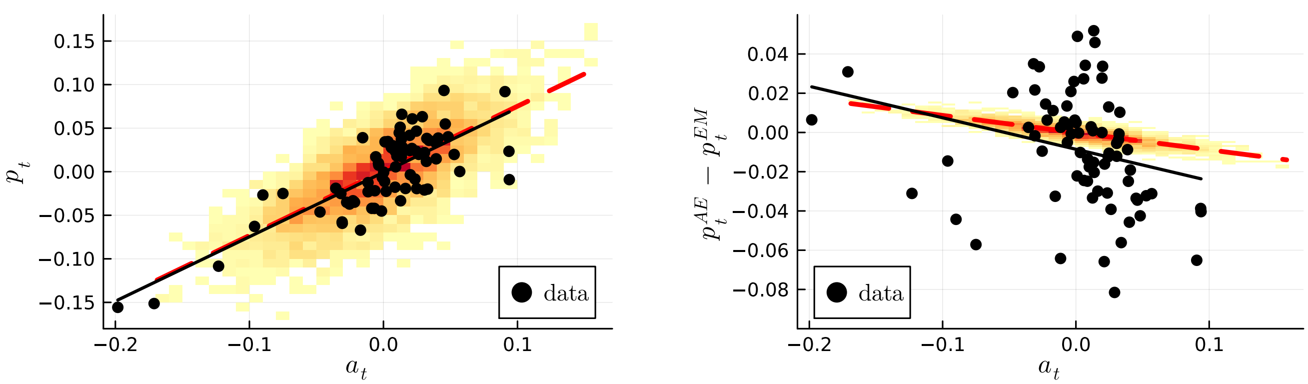
I then run the same regressions on one thousand years of simulated data in the model. Table 5 presents the results.

The good fit on the first regression is due to the fact that $\text{cor}(\tilde{a}_t, \tilde{p}_t)$ is one of the estimation targets. The coefficient in the second regression is not targeted. The model reproduces its sign correctly, showing that risky assets in advanced economies typically outperform those in emerging

Table 5: asset price cyclicality regressions in equation (11) and equation (12).

	$\tilde{p}_t$		$\tilde{p}_t^{AE} - \tilde{p}_t^{EM}$	
	data	**model**	**data**	**model**
$\tilde{a}_t$	0.739	0.700	-0.160	-0.084
	(0.069)	(0.010)	(0.066)	(0.001)
R^2	0.60	0.55	0.07	0.74
N	77	4000	77	4000

markets when capital flows recede, both in the model and the data. The size of the coefficient is about half of that in the data, showing that the model can generate about half of the variation in the relative performance of advanced economies compared to emerging markets out of the same variation in global capital flows. The total variation in the relative performance of equities is considerably larger in data, however, owing to other factors not captured by global capital flows.



(a) Average risky asset returns against total normalized outward flows.

(b) Relative risky asset returns in AE in EM against total normalized outward flows.

Figure 7: Data and model simulations for equation (11) and equation (12).

6 Additional Empirical Results

I now present additional evidence on the heterogeneous responses of countries to global capital flight events and interpret them through the lens of the model. I use two ways to classify countries. First, I designate them as advanced economies or emerging markets according to the IMF breakdown. Second, I use the ratio of private foreign assets (portfolio debt and equity plus "other" assets, which stands for bank claims) to GDP as a measure of a country's financial development. This variable is a direct counterpart to wealth in the model since the model keeps expected output constant across countries, implicitly making the same normalization. The correlation between the two measures, excluding offshore financial centers, is 69%. Asset stock data come from the IMF IFS database, and GDP numbers come from the World Bank.

As country-specific outcomes, I first take a panel of government bond yields y_{it} from the IMF International Financial Statistics database to complement equity price indices provided by MSCI. I then add financial account flows from the IMF Balance of Payments database to construct the following panels. First, I take total acquisition of foreign private assets $F_{it}^{\text{acq(raw)}}$ and total incurrence of external private liabilities $F_{it}^{\text{inc(raw)}}$ by country-quarter and apply the smoothing procedure from Forbes and Warnock (2012) and Forbes and Warnock (2021):

$$F_{it}^{\text{acq}} = \sum_{t-3}^{t} F_{is}^{\text{acq(raw)}} - \sum_{t-7}^{t-4} F_{is}^{\text{acq(raw)}}$$
$$F_{it}^{\text{inc}} = \sum_{t-3}^{t} F_{is}^{\text{inc(raw)}} - \sum_{t-7}^{t-4} F_{is}^{\text{inc(raw)}}$$

I then produce panels of normalized asset acquisition and incurrence:

$$a_{it} = \frac{F_{it}^{\text{acq}}}{A_{i,t-1}}$$
$$b_{it} = \frac{F_{it}^{\text{acq}}}{L_{i,t-1}}$$
$$l_{it} = \frac{F_{it}^{\text{inc}}}{L_{i,t-1}}$$

Here a_{it} is acquisition normalized by assets one quarter before, b_{it} is acquisition normalized by liabilities one quarter before, and l_{it} is incurrence normalized by liabilities one quarter before. The interpretation of a_{it} is the intensity of portfolio rebalancing: how much of their foreign holdings investors sell when they retrench in crises or how much they increase their foreign holdings in booms. The interpretation of b_{it} is the scale of their retrenchment relative to potential sales by foreign investors in a capital flight event. Large negative numbers here mean that local agents are actively replacing foreign investors. Through the lens of the model, this indicates that a country adjusts to foreign demand shocks through quantities rather than prices. The variable l_{it} shows how much of their holdings foreign investors sell.

I extract the principal components f_t^{acq} and f_t^{inc} from the acquisition and incurrence panels $\{F_{it}^{\text{acq}}\}$ and $\{F_{it}^{\text{it}}\}$, choosing the largest balanced panel in the sample: 78 quarters from Q4 of 2003 to Q1 of 2023, which includes 85 and 83 countries for acquisition and incurrence respectively. These principal components serve as a measure of the global cycle in capital flows. Figure 8 plots the two series. Capital flight events around the Global Financial Crisis in 2008-2009, the European debt crisis in 2011-2012, the stock market crash in developing countries in 2016, and the trade war instability of 2018 are clearly visible on the figure. The acquisition and incurrence principal

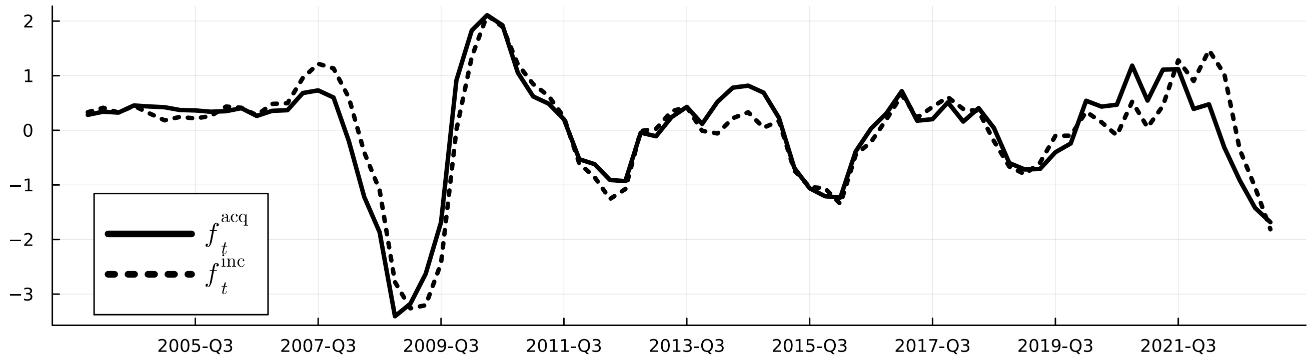


Figure 8: principal components of asset acquisition and liability incurrence.

components are closely related: the correlation between them is 92%. This is in line with existing evidence on the strong co-movement of inward and outward gross flows found by Forbes and Warnock (2012), Forbes and Warnock (2021), Barrot and Serven (2018), Cerutti, Claessens, and Puy (2019), Davis, Valente, and Van Wincoop (2021), and Miranda-Agrippino and Rey (2022).

I also use two price-based aggregates as a measure of the global financial cycle: CBOE VIX and the excess bond premium (EBP) of Gilchrist and Zakrajšek (2012). The sample covers Q4 of 2003 to Q1 of 2023. Table 6 shows their correlations with the principal components of gross flows. Unsurprisingly, high VIX and excess bond premium that signal rising uncertainty and tightening financial conditions are associated with receding capital flows.

Table 6: correlations between the principal components of gross flows and VIX and EBP.

	VIX	EBP
f_t^{acq}	-0.48	-0.74
f_t^{inc}	-0.42	-0.66

I next take the six panels $\{F_{it}^{\mathrm{acq}}, F_{it}^{\mathrm{acq}}, y_{it}, a_{it}, b_{it}, l_{it}\}$ and compute country-specific loadings on all four aggregates $\{f_t^{\mathrm{acq}}, f_t^{\mathrm{inc}}, \mathrm{VIX}_t, \mathrm{EBP}_t\}$. The goal is to see if the loadings of advanced economies are systematically different from those of emerging markets, showing heterogeneity in exposure, and whether this heterogeneity is consistent with the model predictions. Formally, let $\beta_i(x, z)$ be country i's loading of the outcome x on the aggregate z:

$$x_{it} = \alpha_i(x, z) + \beta_i(x, z) z_t + \varepsilon_{it}(x, z)$$

I then estimate $\eta^{AE}(x, z)$ and $\eta^{G}(x, z)$ in

$$\beta_i(x, z) = \xi^{AE}(x, z) + \eta^{AE}(x, z)\mathbb{1}\{i \in AE\} + \varepsilon_i^{AE} \tag{13}$$

$$\beta_i(x, z) = \xi^{G}(x, z) + \eta^{G}(x, z) G_i + \varepsilon_i^{G} \tag{14}$$

Table 7: loadings of asset acquisition F_{it}^{acq}, liability incurrence F_{it}^{inc}, and bond yields y_{it} on the global factors regressed on the advanced economy status and the assets-to-GDP ratio. The table shows the coefficients $\{\eta^{AE}(x,z), \eta^{G}(x,z)\}$ from regressions in equation (13) and equation (14) for aggregates z listed as columns and outcomes x listed as rows.

	loadings on f_t^{acq}		loadings on f_t^{inc}		loadings on VIX_t		loadings on EBP_t	
	$\mathbb{1}\{i \in AE\}$	G_i	$\mathbb{1}\{i \in AE\}$	G_i	$\mathbb{1}\{i \in AE\}$	G_i	$\mathbb{1}\{i \in AE\}$	G_i
F_{it}^{acq}	0.397	0.193	0.371	0.207	-0.216	-0.131	-0.296	-0.174
	(0.054)	(0.038)	(0.052)	(0.036)	(0.039)	(0.026)	(0.044)	(0.030)
F_{it}^{inc}	0.289	0.136	0.262	0.130	-0.195	-0.111	-0.266	-0.134
	(0.057)	(0.039)	(0.061)	(0.041)	(0.043)	(0.029)	(0.051)	(0.034)
y_{it}	0.962	0.552	0.919	0.551	-0.623	-0.371	-0.644	-0.386
	(0.559)	(0.310)	(0.555)	(0.307)	(0.497)	(0.276)	(0.412)	(0.228)

Here G_i is a country i's average assets-to-GDP ratio in 2014-2020. I estimate $\{\eta^{AE}(x,z), \eta^{G}(x,z)\}$ for all outcomes x (acquisition, incurrence, bond yields, acquisition normalized by assets and by liabilities, and incurrence normalized by liabilities) and all aggregates z (principal components of acquisition and incurrences, VIX, and EBP). Appendix C provides details for all regressions.

Table 7 shows the results for the first three outcomes: total private foreign asset acquisition, total private external liability incurrence, and government bond yields. The acquisition and incurrence series are normalized to have a unit standard deviation for each country. All aggregate series are normalized as well. The coefficients in this table read as follows. Take, for example, the first column and the first row. In advanced economies, a one standard deviation increase in f_t^{acq} is associated with 0.397 more standard deviations of foreign asset acquisition than in emerging markets. Conversely, when f_t^{acq} falls by one standard deviation, advanced economies sell 0.397 standard deviations more of their foreign assets than emerging markets.

The row marked $\{F_{it}^{\text{inc}}\}$ refers to the incurrence of liabilities, which stands for asset sales and purchases of foreign investors in country i. Foreign agents sell 0.289 standard deviations more of their holdings in advanced economies than in emerging markets when f_t^{acq} decreases by one standard deviation. This might go against conventional wisdom that holds emerging markets to be more exposed to capital flight. However, it is consistent with the mechanism in my model: even though asset sales are targeting emerging markets, advanced economies have larger and more elastic investors who stand ready to buy back their domestic assets in bad times. This results in larger transaction volumes that indicate adjustment through quantities rather than prices.

The last rows are devoted to bond yields. The coefficients have the right sign but are not precisely estimated. They should be interpreted as suggestive. A standard increase in f_t^{acq} induces government bond yields to rise by 1pp more in advanced economies (or rather fall by 1pp less, since f_t^{acq} rises in booms when yields are falling). This means that emerging markets reap larger benefits in terms of borrowing costs in good times. In bad times, their bond yields rise more.

Table 8: loadings of asset acquisition normalized by assets a_{it} and liabilities b_{it} and liability incurrence normalized by liabilities l_{it} on the global factors regressed on the advanced economy status and the assets-to-GDP ratio. The table shows the coefficients $\{\eta^{AE}(x,z), \eta^{G}(x,z)\}$ from equation (13) and equation (14) for aggregates z listed as columns and outcomes x listed as rows.

	loadings on f_t^{acq}		loadings on f_t^{inc}		loadings on VIX_t		loadings on EBP_t	
	$\mathbb{1}\{i \in AE\}$	G_i	$\mathbb{1}\{i \in AE\}$	G_i	$\mathbb{1}\{i \in AE\}$	G_i	$\mathbb{1}\{i \in AE\}$	G_i
a_{it}	0.027	0.010	0.025	0.010	-0.016	-0.009	-0.018	-0.009
	(0.010)	(0.006)	(0.010)	(0.006)	(0.008)	(0.005)	(0.009)	(0.006)
b_{it}	**0.022**	**0.009**	**0.019**	**0.012**	**-0.002**	**-0.015**	**-0.012**	**-0.014**
	(0.007)	(0.004)	(0.006)	(0.004)	(0.013)	(0.008)	(0.006)	(0.004)
l_{it}	0.020	0.007	0.018	0.006	-0.014	-0.006	-0.020	-0.008
	(0.006)	(0.004)	(0.006)	(0.004)	(0.005)	(0.003)	(0.006)	(0.004)

The signs are negative for VIX and EBP since these factors rise in crises. The results stand for assets-to-GDP as a measure of financial development instead of the advanced economy indicator, which is also in line with the model predictions.

Table 8 does the same exercise for gross flows normalized by stocks: acquisition as a share of assets a_{it}, acquisition as a share of liabilities b_{it}, and incurrence as a share of liabilities l_{it}. These series are not normalized. When f_t^{acq} falls by one standard deviation, investors from advanced economies sell 2.7 percentage points more of their foreign holdings. They also sell 2.2 percentage point more of their country's external liabilities. Finally, foreign investors sell 2 percentage points more of their holdings in advanced economies. The latter two numbers confirm that investors in advanced economies retrench more actively, replacing the shortfall in foreign demand. This pattern is largely robust to the choice of the global factor and stands when economies are classified into developed and developing using their assets-to-GDP ratios.

7 Conclusion

This paper develops a heterogeneous-country model of the world economy with symmetric fundamentals and one state variable per country. While simple enough to show the main results analytically in a small-shock approximation, the model captures the main dimensions of heterogeneous responses to global financial shocks: asset prices react more in emerging markets, while outflows of foreign investors are larger in advanced economies. Given its ability to encompass the entire cross-section of risk premia and generate persistent wealth differences out of symmetric fundamentals, in future work, this framework could prove useful for studying the causes and consequences of persistent differences in asset prices and interest rates. It could also help study the global equilibrium implications of monetary and fiscal shocks in the dominant country, commodity price shocks, and shocks to financial and trade openness.

A Details of calibration and estimation.

A.1 Calibration

To calibrate the model, I construct the empirical distribution of assets-to-liability ratios A/L. I take private assets and liabilities: portfolio debt and equity and "other" claims. My sample starts in Q1 of 2003 and extends to 2024. I exclude the US to account for the fact it is a special country in the model. To adjust for the fact that not all countries are present in the sample throughout the period, I assign weights $\{\omega_i\}$ to all observations of a country i, where ω_i is the inverse of the share of the quarters the country i appears in the panel. I solve for the steady state of the model in full, not using the second-order approximation.

A.2 Estimation

Formally, given a sample path of $\{W(s)\}$ and the corresponding sample paths of $\{\Delta\gamma(s), \Delta\nu(s)\}$, I take the first-order approximation of $\{A(s), P(s)\}$ around $\Delta\gamma(s) \equiv \Delta\nu(s) \equiv 0$. In addition, at every t, the agents perceive the sample paths of $\{\Delta\gamma(s), \Delta\nu(s)\}_{s>t}$ in the future as follows. They do not know the future realizations of $\{W(s)\}_{s>t}$ and assume that $\{\Delta\gamma(s), \Delta\nu(s)\}_{s>t}$ will simply decay to their long-run values: for all $s > t$, $\Delta\gamma(s) = e^{-\mu_\gamma(s-t)}\Delta\gamma(t)$ and $\Delta\nu(s) = e^{-\mu_\nu(s-t)}\Delta\nu(t)$. This corresponds to setting $\sigma_\gamma = \sigma_\nu = (0, 0)$ and solving the model with perfect foresight of the aggregate shocks. To simplify computations, I additionally rely on the second-order approximation of the model around $\sigma = 0$, which is really a third-order approximation because error terms are $O(\sigma^4)$. I normalize the first-order deviations $\Delta A(t)$ and $\Delta P(t)$ by the steady-state value: $a_t = \Delta A(t)/A$ and $p_t = \Delta P(t)/P$. The following proposition establishes the main technical result.

Proposition 5. *Consider a first-order perfect-foresight approximation of the model around the steady state. Assume that output shocks are small, $\sigma_\nu = \sigma^2 \boldsymbol{\sigma}_\nu$. Suppose the linearized economy converges to the steady state at infinity for any unanticipated transitory shock to $\gamma(t)$ and $\nu(t)$. Then, given a sample path of $\{W(s)\}$,*

$$\Delta a(t) = J_{a,\gamma} \int_{s=0}^{\infty} e^{-\mu_\gamma s} \sigma_\gamma \cdot dW(t-s) + J_{a,\nu} \int_{s=0}^{\infty} e^{-\mu_\nu s} \sigma_\nu \cdot dW(t-s) + O(\sigma^4)$$
$$\Delta p(t) = J_{p,\gamma} \int_{s=0}^{\infty} e^{-\mu_\gamma s} \sigma_\gamma \cdot dW(t-s) + J_{p,\nu} \int_{s=0}^{\infty} e^{-\mu_\nu s} \sigma_\nu \cdot dW(t-s) + O(\sigma^4)$$

The numbers $\{J_{a,\gamma}, J_{a,\nu}, J_{p,\gamma}, J_{p,\nu}\}$ only depend on the steady-state objects and parameters and μ_γ.

I provide the functional forms of $\{J_{a,\gamma}, J_{a,\nu}, J_{p,\gamma}, J_{p,\nu}\}$ in the proof. This result is similar in spirit to Proposition 4. The technical significance is that the weights on past realizations only have to account for the exponential decay rate. For a sequence-space Jacobian analogy, the Jacobian only has zeros under the main diagonal, and to the right of the main diagonal, all rows are translated copies of each other, with entries decaying exponentially as the time index moves to the past.

The expressions in Proposition 5 allow for explicit characterization of the following moments:

$$\begin{aligned}
\mathbb{E}[\Delta a(t)\Delta a(t-\tau)] &= \frac{e^{-\mu_\gamma \tau} J_{a,\gamma}^2(\sigma_{\gamma 1}^2 + \sigma_{\gamma 2}^2)}{2\mu_\gamma} + \frac{e^{-\mu_\nu \tau} J_{a,\nu}^2 \sigma_{\nu 2}^2}{2\mu_\nu} + \frac{(e^{-\mu_\gamma \tau} + e^{-\mu_\nu \tau}) J_{a,\gamma} J_{a,\nu} \sigma_{\gamma 2}\sigma_{\nu 2}}{(\mu_\gamma + \mu_\nu)} \\
\mathbb{E}[\Delta p(t)\Delta p(t-\tau)] &= \frac{e^{-\mu_\gamma \tau} J_{p,\gamma}^2(\sigma_{\gamma 1}^2 + \sigma_{\gamma 2}^2)}{2\mu_\gamma} + \frac{e^{-\mu_\nu \tau} J_{p,\nu}^2 \sigma_{\nu 2}^2}{2\mu_\nu} + \frac{(e^{-\mu_\gamma \tau} + e^{-\mu_\nu \tau}) J_{p,\gamma} J_{p,\nu} \sigma_{\gamma 2}\sigma_{\nu 2}}{(\mu_\gamma + \mu_\nu)} \\
\mathbb{E}[\Delta a(t)\Delta p(t)] &= \frac{J_{a,\gamma} J_{p,\gamma}(\sigma_{\gamma 1}^2 + \sigma \gamma 2^2)}{2\mu_\gamma} + \frac{J_{a,\nu} J_{p,\nu} \sigma_{\nu 2}^2}{2\mu_\nu} + \frac{(J_{a,\gamma} J_{p,\nu} + J_{p,\gamma} J_{a,\nu}) \sigma_{\gamma 2}\sigma_{\nu 2}}{\mu_\gamma + \mu_\nu} \\
\mathbb{E}[\Delta a(t)^2] &= \frac{J_{a,\gamma}^2(\sigma_{\gamma 1}^2 + \sigma_{\gamma 2}^2)}{2\mu_\gamma} + \frac{J_{a,\nu}^2 \sigma_{\nu 2}^2}{2\mu_\nu} + \frac{2 J_{a,\gamma} J_{a,\nu} \sigma_{\gamma 2}\sigma_{\nu 2}}{\mu_\gamma + \mu_\nu} \\
\mathbb{E}[\Delta p(t)^2] &= \frac{J_{p,\gamma}^2(\sigma_{\gamma 1}^2 + \sigma_{\gamma 2}^2)}{2\mu_\gamma} + \frac{J_{p,\nu}^2 \sigma_{\nu 2}^2}{2\mu_\nu} + \frac{2 J_{p,\gamma} J_{p,\nu} \sigma_{\gamma 2}\sigma_{\nu 2}}{\mu_\gamma + \mu_\nu}
\end{aligned}$$

To get the mapping from the continuous-time parameters to their discrete analogs, solve the stochastic differential equation (9) and equation (10):

$$\begin{aligned}
\gamma(t) - \gamma &= e^{-\mu_\gamma \tau}(\gamma(t-\tau) - \gamma) + \int_0^\tau e^{-\mu_\gamma s} \sigma_{\gamma 1} dW_1(t-s) + \int_0^\tau e^{-\mu_\gamma s} \sigma_{\gamma 2} dW_2(t-s) \\
\nu(t) - \nu &= e^{-\mu_\nu \tau}(\nu(t-\tau) - \nu) + \int_0^\tau e^{-\mu_\nu s} \sigma_{\gamma 2} dW_2(t-s)
\end{aligned}$$

Substituting the stochastic integrals random variables and defining parameters,

$$\begin{aligned}
\gamma_{t+1} - \gamma &= \rho_\gamma(\gamma_t - \gamma) + \varsigma_{\gamma 1}\varepsilon_{1,t+1} + \varsigma_{\gamma 2}\varepsilon_{2,t+1} \\
\nu_{t+1} - \nu &= \rho_\nu(\nu_t - \nu) + \varsigma_{\nu 2}\varepsilon_{2,t+1}
\end{aligned}$$

Here the persistence parameters are $\rho_\gamma = e^{-\mu_\gamma \tau}$, $\rho_\nu = e^{-\mu_\nu \tau}$, random variables $(\varepsilon_{1,t+1}, \varepsilon_{2,t+1})$ are independent standard normals, and the volatilities of innovations are

$$\varsigma_{\gamma 1} = \sigma_{\gamma 1}\sqrt{\mathbb{V}\left[\int_0^\tau e^{-\mu_\gamma s} dW_1(s)\right]} = \sigma_{\gamma 1}\sqrt{\mathbb{E}\left[\int_0^\tau e^{-2\mu_\gamma s} ds\right]} = \sigma_{\gamma 1}\sqrt{\frac{1 - e^{-2\mu_\gamma \tau}}{2\mu_\gamma}}$$

Similarly,

$$\begin{aligned}
\varsigma_{\gamma 2} &= \sigma_{\gamma 2}\sqrt{\frac{1 - e^{-2\mu_\gamma \tau}}{2\mu_\gamma}} \\
\varsigma_{\nu 2} &= \sigma_{\nu 2}\sqrt{\frac{1 - e^{-2\mu_\nu \tau}}{2\mu_\nu}}
\end{aligned}$$

B Proofs.

Proof of Lemma 1. Start with local agents. Denote by w their individual wealth and let x be all other state variables. Excess returns are

$$dR = \mu_R(x,t)dt + \sigma_R(x,t)dZ$$

Since aggregate dynamics are deterministic, x only loads on the same diffusion dZ. Let $\mu_X(x,t)$ and $\sigma_X(x,t)$ be the drift and loading vectors of x:

$$dx = \mu_X(x,t)dt + \sigma_X(x,t)dZ$$

The HJB equation for the local agent's value $V(w,x,t)$ is

$$\begin{aligned}\rho V(w,x,t) - \partial_t V(w,x,t) = \max_{c,\theta}\ & \rho\log(c) + (r(t)w - c + \theta\mu_R(x,t)w)\partial_w V(w,x,t) \\ & + \frac{\theta^2\sigma_R(x,t)^2w^2}{2}\partial^2_{ww}V(w,x,t) + \mu_X(x,t)\cdot\partial_x V(w,x,t) \\ & + \frac{1}{2}\mathrm{tr}[\sigma_X(x,t)'\partial^2_{xx}V(w,x,t)\sigma_X(x,t)] + \sigma_X(x,t)\cdot\partial^2_{wx}V(w,x,t)\theta\sigma_R(x,t)w\end{aligned}$$

Guess $V(w,x,t) = \log(w) + \eta(x,t)$. Under this and $\sigma_R(x,t) > 0$, $\partial^2_{wx}V(w,x,t) = 0$, $c^* = \rho w$, and

$$\theta^* = \frac{\mu_R(x,t)}{\sigma_R(x,t)^2}$$

Plugging this into the HJB and cancelling terms,

$$\rho\eta(x,t) - \partial_t\eta(x,t) = r(t) - \rho + \frac{\mu_R(x,t)^2}{2\sigma_R(x,t)^2} + \mu_X(x,t)\cdot\partial_x\eta(x,t) + \frac{1}{2}\mathrm{tr}[\sigma_X(x,t)'\partial^2_{xx}\eta(x,t)\sigma_X(x,t)]$$

This verifies the conjecture that $V(w,x,t)$ is separable over w and (x,t).

Continue with the intermediary. Its state variables are its own wealth $\hat{w}$ and time: aggregate dynamics are deterministic, so the special country's aggregate wealth $\underline{\hat{w}}$ and the wealth distribution of the regular countries $G(\cdot,t)$ are functions of time only. The intermediary's value function $\hat{V}(\hat{w},t)$ solves the following HJB:

$$\begin{aligned}\rho\hat{V}(\hat{w},t) - \partial_t\hat{V}(\hat{w},t) = \max_{\hat{c},\theta(\cdot),\hat{\theta}}\ & \rho\log(\hat{c}) \\ & + \left(r(t)\hat{w} - c + \int \mu_R(w,t)\hat{\theta}(w)\hat{w}dG(w,t) + \hat{\mu}_R(t)\hat{\theta}\hat{w}\right)\partial_{\hat{w}}\hat{V}(\hat{w},t) \\ \text{s.t.}\ & \int \sigma_R(w,t)^2\theta(w)^2dG(w,t) \le \gamma\int \mu_R(w,t)\theta(w)dG(w,t) \qquad \text{(A.1)}\end{aligned}$$

Conjecture that $\hat{V}(\hat{w},t) = \log(\hat{w}) + \hat{\eta}(t)$. Under this and $\sigma_R(w,t) > 0$, $\hat{c} = \rho\hat{w}$ and

$$\theta^*(w) = \frac{1+\gamma\xi(\hat{w},t)}{2\xi(\hat{w},t)} \cdot \frac{\mu_R(w,t)}{\sigma_R(w,t)^2}$$

Here $\xi(\hat{w},t)$ is the multiplier on the value-at-risk constraint in equation (A.1). In general, the multiplier can depend on all states.

Observe that the constraint binds whenever $\mu_R(w,t) > 0$ for a positive measure of x. Other cases are not interesting since the value-at-risk constrain is only satisfied if $\theta^*(w) = 0$ for all w, but with $\mu_R(w,t) = 0$, local agents do not hold any assets either, so asset markets do not clear.

Plugging $\theta^*(w)$ into equation (A.1),

$$\left(\frac{(1+\gamma\xi(\hat{w},t))^2}{4\xi(\hat{w},t)^2} - \gamma\frac{1+\gamma\xi(\hat{w},t)}{2\xi(\hat{w},t)}\right) \cdot \int \frac{\mu_R(w,t)^2}{\sigma_R(w,t)^2} dG(w,t) = 0$$

Since $\mu_R(w,t) > 0$ for a positive measure of x, $\gamma\xi(\hat{w},t) = 1$, and hence

$$\theta^*(w) = \gamma \cdot \frac{\mu_R(w,t)}{\sigma_R(w,t)^2}$$

Plugging this and $\hat{c} = \rho\hat{w}$ back into the HJB and cancelling terms,

$$\rho\hat{\eta}(t) - \hat{\eta}'(t) = r(t) - \rho + \gamma \int \frac{\mu_R(w,t)^2}{\sigma_R(w,t)^2} dG(w,t)$$

This verifies the conjecture that $\hat{V}(\hat{w},t)$ is separable over $\hat{w}$ and t and completes the proof.

Finally, I describe potential pathological equilibria with $\sigma_R(w,t) = 0$. Since in this case it has to be that $\mu_R(w,t) = 0$,

$$dp = (r(t)p - \nu(t))dt - \sigma dZ$$

Asset prices are not guaranteed to be positive and do not have a stationary distribution even if aggregates are constant. The drift of the price is increasing in the price itself, and the volatility is constant, leading to stochastic instability. Asset prices are also divorced from the fundamentals, not being a function of local wealth or the intermediary's wealth or risk-taking capacity. Finally, since $\mu_R(w,t) = \sigma_R(w,t) = 0$, asset positions are indeterminate, and the presence of risky assets does not affect wealth dynamics. Wealth is solely determined by the initial conditions and the perpetual youth terms. The condition $\sigma_R(w,t) \neq 0$ rules these economically uninteresting equilibria out. $\square$

Proof of Proposition 1. Portfolio shares in Lemma 1, the fact that $h_{it} = \theta_{it}w_{it}/p_{it}$ and $\hat{h}_{it} =$

$\hat{\theta}_{it}\hat{w}_t/p_{it}$, and market clearing for each country's tree ($h_{it}+\hat{h}_{it}=1$) lead to the following expression:

$$p_{it} = (w_{it} + \gamma_t \hat{w}_t) \cdot \frac{\mu^R_{it}}{(\sigma^R_{it})^2}$$

From this expression, it follows that $\theta_{it} = p_{it}/(w_{it}+\gamma_t\hat{w}_t)$ and $\hat{\theta}_{it} = \gamma_t p_{it}/(w_{it}+\gamma_t\hat{w}_t)$m which leads to $h_{it} = w_{it}/(w_{it}+\gamma_t\hat{w}_t)$ and $\hat{h}_{it} = \gamma_t\hat{w}_t/(w_{it}+\gamma_t\hat{w}_t)$. Finally, equation (4) follows from applying the relation between $(\mu^R_{it}, \sigma^R_{it})$ and $(\mu^p_{it}, \sigma^p_{it})$. $\square$

Proof of Proposition 2. Applying Itô's lemma to $p(w,t)$,

$$\begin{aligned}
\mu_p(w,t) &= \partial_t p(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww}p(w,t)\\
\sigma_p(w,t) &= \sigma_w(w,t)\partial_w p(w,t)
\end{aligned}$$

Now plug this into equation (4):

$$r(t)p(w,t) - \partial_t p(w,t) = \nu(t) + \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww}p(w,t) - \frac{(\sigma_w(w,t)\partial_w p(w,t)+\sigma)^2}{w+\gamma(t)\hat{w}(t)}$$

Plug the optimal consumption and portfolio choice into the local agent's budget constraint:

$$\begin{aligned}
\mu_w(w,t) &= (r(t)-\rho-\lambda)w + \hat{\lambda}\hat{w}(t) + \frac{\mu_R(w,t)^2}{\sigma_R(w,t)^2}w\\
\sigma_w(w,t) &= \frac{\mu_R(w,t)}{\sigma_R(w,t)}w
\end{aligned}$$

Eliminating $\mu_R(w,t)$ and $\sigma_R(w,t)$ from the expression for $\mu_w(w,t)$,

$$\mu_w(w,t) = (r(t)-\rho-\lambda)w + \hat{\lambda}\hat{w}(t) + \frac{\sigma_w(w,t)^2}{w}$$

Next, using the fact that $\sigma_w(w,t) = \theta(w)\sigma_R(w,t)w$, the definition $\sigma_R(w,t) = (\sigma+\sigma_p(w,t))/p(w,t)$, and $\theta(w) = p(w)/(w+\gamma\hat{w}(t))$, which follows from market clearing,

$$\sigma_w(w,t) = \frac{(\sigma+\sigma_p(w,t))w}{w+\gamma(t)\hat{w}(t)} = \frac{(\sigma+\sigma_w(w,t)\partial_w p(w,t))w}{w+\gamma(t)\hat{w}(t)} = \frac{\sigma w}{w+\gamma(t)\hat{w}(t) - w\partial_w p(w,t)}$$

With this, the pricing equation can be transformed to

$$\begin{aligned}
r(t)p(w,t) - \partial_t p(w,t) = \nu(t) &- \frac{\sigma^2(w+\gamma(t)\hat{w}(t))}{[w+\gamma(t)\hat{w}(t) - w\partial_w p(w,t)]^2}\\
&+ \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial^2_{ww}p(w,t)
\end{aligned}$$

Define the risk compensation

$$\pi(w,t) = \frac{w + \gamma(t)\hat{w}(t)}{[w + \gamma(t)\hat{w}(t) - w\partial_w p(w,t)]^2}$$

This completes the description of $p(w,t)$, $\pi(w,t)$, $\mu_w(w,t)$, and $\sigma_w(w,t)$. To derive the expression for the interest rate, multiply equation (6) by the density $g(w,t)$ and integrate:

$$\begin{aligned} r(t)\int p(w,t)g(w,t)dw - \int \partial_t p(w,t)g(w,t)dw &= \nu(t) - \sigma^2 \int \pi(w,t)g(w,t)dw \\ &+ \int \mu_w(w,t)g(w,t)\partial_w p(w,t)dw + \int \frac{\sigma_w(w,t)^2 g(w,t)}{2}\partial^2_{ww}p(w,t)dw \end{aligned}$$

Integrate the last two terms by parts to get

$$\begin{aligned} \int r(t)p(w,t)g(w,t)dw - \int \partial_t p(w,t)g(w,t)dw &= \nu(t) - \int \sigma^2\pi(w,t)g(w,t)dw \\ &- \int \partial_w(\mu_w(w,t)g(w,t))p(w,t)dw + \int \frac{\partial^2_{ww}(\sigma_w(w,t)^2 g(w,t))}{2}p(w,t)dw \end{aligned}$$

Using equation (5), replace the last two terms

$$r(t)\int p(w,t)g(w,t)dw - \int \partial_t p(w,t)g(w,t)dw = \nu(t) - \sigma^2 \int \pi(w,t)g(w,t)dw + \int \partial_t g(w,t)p(w,t)dw$$

This implies

$$r(t)\int p(w,t)g(w,t)dw - \int \partial_t(p(w,t)g(w,t))dw = \nu(t) - \sigma^2 \int \pi(w,t)g(w,t)dw \tag{A.2}$$

Now use the fact that expected returns on the special country's tree are zero:

$$r(t)\hat{p}(t) - \hat{p}'(t) = \nu(t)$$

Multiplying this equation by $\hat{q}$ and adding to equation (A.2),

$$\begin{aligned} r(t)\left(\int p(w,t)g(w,t)dw + \hat{q}\hat{p}(t)\right) - \partial_t\left(\int p(w,t)g(w,t)dw + \hat{q}\hat{p}(t)\right) &= (1+\hat{q})\nu(t) \\ &- \sigma^2 \int \pi(w,t)g(w,t)dw \end{aligned}$$

Consumption market clearing and $c(w,t) = \rho w$ and $\hat{c}(t) = \rho\hat{w}(t)$ imply

$$\int p(w,t)g(w,t)dw + \hat{q}\hat{p}(t) = \int wg(w,t)dw + \hat{w}(t) = \frac{(1+\hat{q})\nu(t)}{\rho}$$

Hence,

$$\frac{(1+\hat{q})r(t)\nu(t)}{\rho} - \frac{(1+\hat{q})\nu'(t)}{\rho} = (1+\hat{q})\nu(t) - \sigma^2 \int \pi(w,t)g(w,t)dw$$

Reorganizing this,

$$r(t) = \rho + \frac{\nu'(t)}{\nu(t)} - \frac{\rho\sigma^2}{(1+\hat{q})\nu(t)} \int \pi(w,t)g(w,t)dw$$

This completes the proof. □

Proof of Corollary 1. In the steady state, $d\hat{w}(t) = 0$. Using the intermediary's budget constraint and setting the perpetual youth terms to zero, $\lambda = \hat{\lambda} = 0$,

$$\begin{aligned}
\hat{c} = r\hat{w} + \int \hat{w}\hat{\theta}(w)\mu_R(w)dG(w) &= r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b} + \hat{q}\hat{p}\right) + \int \hat{w}\hat{\theta}(w)\theta(w)\sigma_R(w)^2 dG(w) \\
&= \hat{q}\nu + r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \int \frac{\gamma\hat{w}(\sigma_w(w)p'(w)+\sigma)^2}{(w+\gamma\hat{w})^2} dG(w) \\
&= \hat{q}\nu + r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \int \frac{\gamma\hat{w}\sigma^2}{[w+\gamma\hat{w}-wp'(w)]^2} dG(w) \\
&= \hat{q}\nu + r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \int \sigma^2\hat{h}(w)\pi(w)dG(w)
\end{aligned}$$

The firs equality here uses the definition of wealth $\hat{w}$ and fact that $\theta(w) = mu_R(w)/\sigma_R(w)^2$, the second equality uses $\hat{p} = \nu/r$ and plugs in $\sigma_R(w) = (\sigma_p(w)+\sigma)/p(w) = (\sigma_w(w)p'(w)+\sigma)/p(w)$, the third one plugs in $\sigma_w(w)$, and the last one uses the expressions for $\pi(w)$ and $\hat{h}(w)$. This proves the first part.

On the other hand,

$$\begin{aligned}
&\hat{c} = r\hat{w} + \int \hat{w}\hat{\theta}(w)\mu_R(w)dG(w) = r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b} + \hat{q}\hat{p}\right) \\
&+ \int \hat{w}\hat{\theta}(w)\frac{\mu_p(w)+\nu-rp(w)}{p(w)} dG(w) \\
&= \hat{q}\nu + r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \int \gamma\hat{w}\frac{\mu_p(w)+\nu-rp(w)}{w+\gamma\hat{w}} dG(w) \\
&= \hat{q}\nu + r\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \int \hat{h}(w)(\nu - rp(w))dG(w) + \int \hat{h}(w)\mu_p(w)dG(w)
\end{aligned}$$

Here the first equality uses the definition of wealth $\hat{w}$ and expected excess returns $\mu_R(w)$, the second equality uses $\hat{p} = \nu/r$ and $\hat{\theta}(w) = \gamma\theta(w) = \gamma p(w)/(w+\gamma\hat{w})$, and the third one uses the expression for $\hat{h}(w)$. This proves the second part. □

Proof of Corollary 2. Consider the volatility of wealth first. Since $p'(w) \longrightarrow 0$ as $w \longrightarrow \infty$,

$$\sigma_w(w) = \frac{\sigma}{1 + \hat{w}/w - p'(w)} \longrightarrow \sigma$$

From this, it immediately follows that

$$\frac{\mu_w(w)}{w} = r - \rho - \lambda + \frac{\hat{\lambda}\hat{w}}{w} + \frac{\sigma_w(w)^2}{w^2} \longrightarrow r - \rho - \lambda$$

The risk price $\pi(w)$ converges to zero as well:

$$\pi(w) = \frac{w + \gamma\hat{w}}{[w + \gamma\hat{w} - wp'(w)]^2} = \frac{1}{w} \cdot \frac{1 + \gamma\hat{w}/w}{[1 + \gamma/w - p'(w)]^2} \longrightarrow 0$$

Now consider the pricing equation

$$rp(w) = \nu - \sigma^2\pi(w) + \mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w)$$

The claim is that $p(w) \longrightarrow \nu/r$ as $w \longrightarrow \infty$. Suppose, toward a contradiction, that there exists $\varepsilon > 0$ such that for all $W > 0$ there is a $w > W$ such that $|p(w) - \nu/r| > \varepsilon$. Then, since $p''(w) \longrightarrow 0$ and $\sigma_w(w)0$ as $w \longrightarrow \infty$, it must be that there exists $\varepsilon > 0$ such that for all $W > 0$ there is a $w > W$ such that $|\mu_w(w)p'(w)| > \varepsilon$. This last inequality can be rewritten as

$$\left|\frac{\mu_w(w)}{w} \cdot p'(w)w\right| > \varepsilon \Longrightarrow \left|\frac{\mu_w(w)}{w}\right| \cdot |p'(w)w| > \varepsilon$$

But since $\mu_w(w)/w \longrightarrow r - \rho - \lambda$, there exists a $\overline{W} > 0$ such that for all $w > \overline{W}$, it holds that $|\mu_w(w)/w| < 2|r - \rho - \lambda|$. Hence, there exists a $\overline{W} > 0$ such that for all $w > \overline{W}$,

$$|p'(w)|w > |p'(w)w| > \frac{\varepsilon}{2|r - \rho - \lambda|}$$

This implies that $p'(w)$ does not change its sign on $(\overline{W}, \infty)$. Hence, on all of the $(\overline{W}, \infty)$,

$$\text{either } p'(w) > \frac{\varepsilon}{2w|r - \rho - \lambda|} \text{ or } p'(w) < -\frac{\varepsilon}{2w|r - \rho - \lambda|}$$

Integrating $p'(w)$ from $\overline{W}$ to $(1 + \delta)\overline{W}$,

$$\text{either } p((1 + \delta)\overline{W}) > p(\overline{W}) + \frac{\log(1 + \delta)\varepsilon}{2|r - \rho - \lambda|} \text{ or } p((1 + \delta)\overline{W}) < p(\overline{W}) - \frac{\log(1 + \delta)\varepsilon}{2|r - \rho - \lambda|}$$

This contradicts $p(\cdot)$ having a finite limit at infinity. Hence, $p(w) \longrightarrow \nu/r$ as $w \longrightarrow \infty$. $\square$

Proof of Proposition 3. Start with the pricing equation:

$$rp(w) = \nu - \sigma^2\pi(w) + \mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w)$$

Take the interest rate:

$$r = \rho - \frac{\rho\sigma^2}{(1+\hat{q})\nu}\int \pi(x)dG(x) = \rho + O(\sigma^2)$$

This implies

$$\begin{aligned}\mu_w(w) &= (r - \rho - \boldsymbol{\lambda}\sigma^2)w + \hat{\boldsymbol{\lambda}}\hat{\sigma}^2\hat{w} + \frac{\sigma_w(w)^2}{w} \\ &= \hat{\boldsymbol{\lambda}}\hat{\sigma}^2\hat{w} - \left(\frac{\rho}{(1+\hat{q})\nu}\int \pi(x)dG(x) + \boldsymbol{\lambda}\right)\sigma^2 w + \frac{\sigma^2 w}{[w + \gamma\hat{w} - wp'(w)]^2} = O(\sigma^2)\end{aligned}$$

Since $\mu_w(w) = O(\sigma^2)$, $\sigma_w(w)^2 = O(\sigma^2)$, and $r = \rho + O(\sigma^2)$, the pricing equation can be cast as

$$\rho p(w) = \nu + O(\sigma^2) \Longrightarrow p(w) = \frac{\nu}{\rho} + O(\sigma^2)$$

This immediately implies $p'(w) = O(\sigma^2)$ and $p''(w) = O(\sigma^2)$, leading to

$$\pi(w) = \frac{1}{w + \gamma\hat{w}} + O(\sigma^2) \equiv \boldsymbol{\pi}(w) + O(\sigma^2)$$

Hence,

$$r = \rho - \frac{\rho\sigma^2}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)dG(x) + O(\sigma^4)$$

This establishes the approximation for r. Using this and $\mu_w(w)p'(w) + \sigma_w(w)^2p''(w)/2 = O(\sigma^4)$,

$$\left(\rho - \frac{\rho\sigma^2}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)dG(x)\right)p(w) = \nu - \sigma^2\boldsymbol{\pi}(w) + O(\sigma^4)$$

Reorganizing,

$$p(w) = \frac{\nu}{\rho} + \frac{\sigma^2}{\rho}\left[\frac{1}{1+\hat{q}}\int \boldsymbol{\pi}(x)dG(x) - \boldsymbol{\pi}(w)\right] + O(\sigma^4)$$

Similarly, $r\hat{p} = \nu$ implies

$$\hat{p} = \frac{\nu}{\rho} + \frac{\sigma^2}{(1+\hat{q})\rho}\int \boldsymbol{\pi}(x)dG(x) + O(\sigma^4)$$

This establishes the approximation for asset prices. Finally, wealth dynamics are

$$
\begin{aligned}
\sigma_w(w) &= \frac{\sigma w}{w+\gamma\hat{w}} + O(\sigma^3) \equiv \sigma \boldsymbol{s}(w) + O(\sigma^3) \\
\mu_w(w) &= \sigma^2 \left[\hat{\boldsymbol{\lambda}}\hat{w} + \left(\boldsymbol{\pi}(w)^2 - \frac{\rho}{(1+\hat{q})\nu} \int \boldsymbol{\pi}(x) dG(x) - \boldsymbol{\lambda} \right) w \right] + O(\sigma^4) \equiv \sigma^2 \boldsymbol{m}(w) + O(\sigma^4)
\end{aligned}
$$

Take the Kolmogorov forward equation,

$$
(\mu_w(w)g(w))' = \frac{1}{2}(\sigma_w(w)^2 g(w))''
$$

Define $\boldsymbol{g}(\cdot)$ by

$$
(\boldsymbol{m}(w)\boldsymbol{g}(w))' = \frac{1}{2}(\boldsymbol{s}(w)\boldsymbol{g}(w))''
$$

Expanding the drift and volatility,

$$
(\boldsymbol{m}(w)g(w))' - \frac{1}{2}(\boldsymbol{s}(w)g(w))'' = (\boldsymbol{m}(w)[g(w)-\boldsymbol{g}(w)])' - \frac{1}{2}(\boldsymbol{s}(w)[g(w)-\boldsymbol{g}(w)])'' = O(\sigma^2)
$$

Hence, $g(w) = \boldsymbol{g}(w) + O(\sigma^2)$. This completes the proof. $\square$

Proof of Proposition 4. First, consider a constant $\nu(t \equiv \nu)$. Take the pricing equation (6) and linearize it with respect to $\Delta\gamma(t) \equiv \gamma(t) - \gamma$. Take the first-order deviations $\Delta r(t)$, $\Delta\hat{w}(t)$, $\Delta p(w,t)$, $\Delta\pi(w,t)$, $\Delta\mu_w(w,t)$ and $\Delta\sigma_w(w,t)$:

$$
\begin{aligned}
\Delta r(t)p(w) + r\Delta p(w,t) - \Delta p_t(w,t) = &-\sigma^2\Delta\pi(w,t) + \Delta\mu_w(w,t)p'(w) + \mu_w(w)\partial_w\Delta p(w,t) \\
&+ \Delta\sigma_w(w,t)\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial^2_{ww}\Delta p(w,t)
\end{aligned}
$$

Doing the same with the equation for $\hat{p}(t)$,

$$
\Delta r(t)\hat{p} + r\Delta\hat{p}(t) - \Delta\hat{p}'(t) = 0
$$

The expression for $r(t)$ expands as

$$
\Delta r(t) = \frac{\sigma^2\rho}{(1+\hat{q})\nu}\int \Delta\pi(x,t)dG(x) - \frac{\sigma^2\rho}{(1+\hat{q})\nu}\int \pi(x)\Delta g(x,t)dx
$$

Here $\Delta g(x,t)$ is the first-order deviation of $g(w,t)$ from $g(w)$ satisfying

$$
\Delta g_t(w,t) = -\partial_w(\mu_w(w)\Delta g(w,t) + \Delta\mu_w(w,t)g(w)) + \partial^2_{ww}\left[\frac{\sigma_w(w)^2\Delta g(w,t)}{2} + 2\Delta\sigma_w(w,t)\sigma_w(w)g(w)\right]
$$

The first-order deviations of the drift and volatility are

$$\Delta\mu_w(w,t) = \Delta r(t)w + \hat{\lambda}\Delta w(t) + \frac{2\Delta\sigma_w(w,t)\sigma_w(w)}{w}$$
$$\Delta\sigma_w(w,t) = \frac{\sigma w[\Delta p_w(w,t)w - \Delta\gamma(t)\hat{w} - \gamma\Delta\hat{w}(t)]}{[w+\gamma\hat{w}-wp'(w)]^2}$$

The firs-order deviation of the risk compensation is

$$\Delta\pi(w,t) = \frac{\Delta\gamma(t)\hat{w} + \gamma\Delta\hat{w}(t)}{[w+\gamma\hat{w}-wp'(w)]^2} - \frac{2\pi(w)(\Delta\gamma(t)\hat{w} + \gamma\Delta w(t) - w\Delta p_w(w,t))}{w+\gamma\hat{w}-wp'(w)}$$

The first-order deviation of the special country's wealth is

$$\Delta w(t) = -\int x\Delta g(x,t)dx$$

Now consider the second-order approximation around $\sigma = 0$. Let $\hat{\lambda} = \hat{\boldsymbol{\lambda}}\sigma^2$ and $\lambda = \boldsymbol{\lambda}\sigma^2$. First, since $\Delta r(t) = O(\sigma^2)$, $\Delta\mu_w(w,t) = O(\sigma^2)$ as well. Together with $\Delta\sigma_w(w,t) = O(\sigma)$, $\mu_w(w) = O(\sigma^2)$, and $\sigma_w(w) = O(\sigma)$, this implies

$$r\Delta p(w,t) - \partial_t\Delta p(w,t) = O(\sigma^2)$$

This means $\Delta p(w,t) = O(\sigma^2)$, leading to $\Delta p_w(w,t) = O(\sigma^2)$ and $\Delta p_{ww}(w,t) = O(\sigma^2)$. Next, $\Delta\mu_w(w) = O(\sigma^2)$, $\Delta\sigma_w(w,t) = O(\sigma)$, $\mu_w(w) = O(\sigma^2)$, and $\sigma_w(w) = O(\sigma)$ imply $\Delta g_t(w,t) = O(\sigma^2)$ and hence $\Delta g(w,t) = O(\sigma^2)$. This means $\Delta w(t) = O(\sigma^2)$, leading to

$$\Delta\pi(w,t) = -\Delta\gamma(t)\hat{w}\boldsymbol{\pi}(w)^2 + O(\sigma^2)$$

Plugging this into the expression for $\Delta r(t)$,

$$\Delta r(t) = \Delta\gamma(t)\sigma^2 \cdot \frac{\rho\hat{w}}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)^2 dG(x) + O(\sigma^4) = \Delta\gamma(t)\sigma^2 \cdot \frac{\rho\hat{w}}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4)$$

Using this in the differential equation for $\Delta\hat{p}(t)$,

$$\rho\Delta\hat{p}(t) - \Delta\hat{p}'(t) = \Delta\gamma(t)\sigma^2 \cdot \frac{\hat{w}}{1+\hat{q}}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4)$$

Since $\Delta\gamma(t) = \delta e^{-\mu_\gamma t}$ and the terminal condition $\Delta\hat{p}(t) \longrightarrow 0$ as $t \longrightarrow 0$, this easily integrates:

$$\Delta\hat{p}(t) = -\Delta\gamma\sigma^2 \cdot \frac{\hat{w}}{\rho+\mu_\gamma} \cdot \frac{1}{1+\hat{q}}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) + O(\sigma^4)$$

The last remaining piece is the pricing equation for $\Delta p(w,t)$. Using the fact that $\Delta\mu_w(w,t)p'(w) = O(\sigma^4)$, $\mu_w(w)\Delta p_w(w,t) = O(\sigma^4)$, $\Delta\sigma_w(w,t)\sigma_w(w)p''(w) = O(\sigma^4)$, and $\sigma_w(w)^2\Delta p_{ww}(w,t) = O(\sigma^4)$,

$$\rho\Delta p(w,t) - \partial_t \Delta p(w,t) = \Delta\gamma(t)\sigma^2 \cdot \hat{w} \cdot \left[\frac{1}{1+\hat{q}}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4)$$

With $\Delta\gamma(t) = \delta e^{-\mu_\gamma t}$ and the terminal condition $\Delta p(w,t) \longrightarrow 0$ as $t \longrightarrow 0$, this integrates to

$$\Delta p(w,t) = -\Delta\gamma(t)\sigma^2 \cdot \frac{\hat{w}}{\rho+\mu_\gamma} \cdot \left[\frac{1}{1+\hat{q}}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - \boldsymbol{\pi}(w)^2\right] + O(\sigma^4)$$

This completes the proof. $\square$

Proof of Corollary 3. The impact revaluation of the intermediary's wealth is, up to $o(\sigma^3)$,

$$\begin{aligned}
\Delta\hat{w}(0) &= \int \Delta p(w,0)\hat{h}(w)d\mathcal{G}(w) + \Delta\hat{p}(0)\hat{q} \\
&= -\frac{\Delta\gamma(0)\sigma^2}{\rho+\mu_\gamma}\left[\frac{1}{1+\hat{q}}\int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\left(\int \hat{h}(w)d\mathcal{G}(w) + \hat{q}\right) - \int \hat{h}(w)\boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\right] \\
&= -\frac{\Delta\gamma(0)\sigma^2}{\rho+\mu_\gamma}\left[\int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w)\int \hat{h}(w)d\mathcal{G}(w) - \int \boldsymbol{\pi}(w)^2\hat{h}(w)d\mathcal{G}(w) + \frac{\hat{q}}{1+\hat{q}}\Omega(\hat{w})\right] \\
&= \frac{\Delta\gamma(0)\sigma^2}{\rho+\mu_\gamma}\left[\mathbb{C}\left[\hat{h}(w), \boldsymbol{\pi}(w)^2\right] - \frac{\hat{q}}{1+\hat{q}}\Omega(\hat{w})\right]
\end{aligned}$$

Here $\Omega(\hat{w})$ is given by

$$\Omega(\hat{w}) = \left[1 - \int \hat{h}(w)d\mathcal{G}(w)\right]\int \boldsymbol{\pi}(w)^2 d\mathcal{G}(w) \le \int \frac{1}{(w+\gamma\hat{w})^2}d\mathcal{G}(w) \le \frac{\rho^2}{((1+\hat{q})\nu - (1-\gamma)\rho\hat{w})^2}$$

Here the first inequality uses $0 \le \hat{h}(\cdot) \le 1$ and $\boldsymbol{\pi}(w) = (w+\gamma\hat{w})^{-2}$, and the second uses Jensen's inequality: given $\hat{w}$, $\mathbb{E}[w] = (1+\hat{q})\nu/\rho - \hat{w}$, and $(w+\gamma\hat{w})^{-2}$ is a convex positive function of x, so $\mathbb{E}[(w+\gamma\hat{w})^{-2}] \le (\mathbb{E}[w]+\gamma\hat{w})^{-2} = ((1+\hat{q})\nu/\rho - (1-\gamma)\hat{w})^2$. Since $\Omega(\hat{w})$ is a bounded positive function, for small enough $\hat{q}$,

$$\mathbb{C}\left[\hat{h}(w), \boldsymbol{\pi}(w)^2\right] - \frac{\hat{q}}{1+\hat{q}}\Omega(\hat{w}) > 0$$

This follows from the fact that $\hat{h}(w) = \gamma\hat{w}\boldsymbol{\pi}(w)$ is a positive decreasing function, which is hence positively correlated with another positive decreasing function $\boldsymbol{\pi}(w)^2$. This fact means that for small enough $\hat{q}$, $\Delta\hat{w}(0)$ has the same sign as $\Delta\gamma(0)$. This completes the proof. $\square$

Proof of Proposition 5. Start with $\gamma(t)$. Using the derivations in the proof of Proposition 4,

$$\begin{aligned}
\Delta\pi(w,t) &= -\Delta\gamma(t)\hat{w}\boldsymbol{\pi}(w) + O(\sigma^2) \\
\Delta\sigma_w(w,t) &= -\Delta\gamma(t)\sigma\hat{w}w\boldsymbol{\pi}(w)^2 + O(\sigma^3) \\
\Delta\mu_w(w,t) &= \Delta r(t)w + \frac{2\Delta\sigma_w(w,t)\sigma\boldsymbol{s}(w)}{w} + O(\sigma^4) \\
&= \Delta\gamma(t)\sigma^2 \cdot \left[\frac{\rho\hat{w}w}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - 2\hat{w}w\boldsymbol{\pi}(w)^3\right] + O(\sigma^4)
\end{aligned}$$

Expanding the Kolmogorov forward equation,

$$\begin{aligned}
\Delta g_t(w,t) = &- \sigma^2\partial_w(\boldsymbol{m}(x)\Delta g(w,t)) - \partial_w(\Delta\mu_w(w,t)g(w)) + \frac{\sigma^2}{2}\partial^2_{ww}(\boldsymbol{s}^2\Delta g(w,t)) \\
&+ \sigma\partial^2_{ww}(\Delta\sigma_w(w,t)\boldsymbol{s}(w)g(w)) + O(\sigma^4)
\end{aligned}$$

This uses the fact that $\sigma_w(w) = \sigma\boldsymbol{s}(w) + O(\sigma^3)$ and $\mu_w(w) = \sigma^2\boldsymbol{m}(w) + O(\sigma^4)$. From $\Delta\mu_w(w,t) = O(\sigma^2)$ and $\Delta\sigma_w(w,t) = O(\sigma^2)$, it follows that $\Delta g_t(w,t) = O(\sigma^2)$, meaning that $\Delta g(w,t) = O(\sigma^2)$ and hence $\Delta\partial_w g(w,t) = O(\sigma^2)$ and $\Delta\partial^2_{ww}g(w,t) = O(\sigma^2)$. With this and $g(w) = \boldsymbol{g}(w) + O(\sigma^2)$,

$$\begin{aligned}
\Delta g_t(w,t) &= -\partial_w(\Delta\mu_w(w,t)\boldsymbol{g}(w)) + \sigma\partial^2_{ww}(\Delta\sigma_w(w,t)\boldsymbol{s}(w)\boldsymbol{g}(w)) + O(\sigma^4) \\
&= \Delta\gamma(t)\sigma^2\hat{w}\left[2(w\boldsymbol{\pi}(w)^3\boldsymbol{g}(w))' - \frac{\rho(w\boldsymbol{g}(w))'}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - (w^2\boldsymbol{\pi}(w)^3\boldsymbol{g}(w))''\right] + O(\sigma^4)
\end{aligned}$$

Since I assume that the economy converges to the steady state in the long run, the limit of $\Delta g(w,t)$ as $t \longrightarrow \infty$ is zero. Using the functional form of $\Delta(t)$ and integrating between t and infinity,

$$\Delta g(w,t) = \Delta\gamma(t)\frac{\sigma^2\hat{w}}{\mu_\gamma}\left[(w^2\boldsymbol{\pi}(w)^3\boldsymbol{g}(w))'' + \frac{\rho(w\boldsymbol{g}(w))'}{(1+\hat{q})\nu}\int \boldsymbol{\pi}(x)^2 d\mathcal{G}(x) - 2(w\boldsymbol{\pi}(w)^3\boldsymbol{g}(w))'\right] + O(\sigma^4)$$

This defines $J_{g,\gamma}$ in $\Delta g(w,t) \equiv J_{g,\gamma}(w)\Delta\gamma(t)$. Given $\Delta p(w,t) = J_{p,\gamma}(w)\Delta\gamma(t)$ from Proposition 4, I can turn to p_t and a_t:

$$\begin{aligned}
p_t &= \frac{1}{P}\int \Delta p(w,t)dG(w) + \frac{1}{P}\int p(w)\Delta g(w,t)dw \\
&= \frac{\Delta\gamma(t)}{P}\left[\int J_{p,\gamma}(w)d\mathcal{G}(w) + \int p(w)J_{g,\gamma}(w)dw\right] + O(\sigma^4) \\
a_t &= -\frac{1}{A}\int \Delta\theta(w,t)wdG(w) + \frac{1}{A}\int(1-\theta(w))w\Delta g(w,t)dw \\
&= \frac{\Delta\gamma(t)}{A}\left[\int(\hat{w}p(w)\boldsymbol{\pi}(w)^2 w - J_{p,\gamma}(w)\boldsymbol{\pi}(w)w)d\mathcal{G}(w)\right] \\
&+ \frac{\Delta\gamma(t)}{A}\int\left(1-\theta(w) - \gamma\int p(x)x\boldsymbol{\pi}(x)^2 d\mathcal{G}(x)\right)wJ_{g,\gamma}(w)dw + O(\sigma^4)
\end{aligned}$$

The last equality here uses the fact that

$$
\begin{aligned}
\Delta\theta(w,t) &= \frac{\Delta p(w,t)}{w+\gamma\hat{w}} - \frac{(\Delta\gamma(t)\hat{w}+\gamma\Delta\hat{w}(t))p(w)}{(w+\gamma\hat{w})^2} + O(\sigma^4) \\
&= \Delta p(w,t)\boldsymbol{\pi}(w) - p(w)\boldsymbol{\pi}(w)^2(\Delta\gamma(t)\hat{w}+\gamma\Delta\hat{w}(t)) + O(\sigma^4) \\
&= \Delta\gamma(t)\left[J_{p,\gamma}(w)\boldsymbol{\pi}(w) - \hat{w}p(w)\boldsymbol{\pi}(w)^2\right] + \gamma p(w)\boldsymbol{\pi}(w)^2\int x\Delta g(x,t)dx + O(\sigma^4) \\
&= \Delta\gamma(t)\left[J_{p,\gamma}(w)\boldsymbol{\pi}(w) - \hat{w}p(w)\boldsymbol{\pi}(w)^2 + \gamma p(w)\boldsymbol{\pi}(w)^2\int xJ_{g,\gamma}(x)dx\right] + O(\sigma^4)
\end{aligned}
$$

Simplify the expression for a_t by using the fact that $\theta(w) = p(w)\boldsymbol{\pi}(w)$ and $h(w) = w\boldsymbol{\pi}(w)$:

$$
\begin{aligned}
a_t = &\frac{\Delta\gamma(t)}{A}\int(\hat{w}\theta(w) - J_{p,\gamma}(w))h(w)d\mathcal{G}(w) \\
&+ \frac{\Delta\gamma(t)}{A}\int\left[1-\theta(w)-\gamma\int h(x)\theta(x)d\mathcal{G}(x)\right]wJ_{g,\gamma}(w)dw + O(\sigma^4)
\end{aligned}
$$

Now consider a shock to $\Delta\nu(t)$. The assumption is that $\sigma_\nu = \boldsymbol{\sigma}_\nu\sigma^2$, meaning that the shock $\Delta\nu(t)$ itself is of the second order. Take the first-order deviations $\Delta r(t)$, $\Delta\hat{w}(t)$, $\Delta p(w,t)$, $\Delta\pi(w,t)$, $\Delta\mu_w(w,t)$ and $\Delta\sigma_w(w,t)$:

$$
\begin{aligned}
\Delta r(t)p(w) + r\Delta p(w,t) - \Delta p_t(w,t) = &\Delta\nu(t) - \sigma^2\Delta\pi(w,t) + \Delta\mu_w(w,t)p'(w) + \mu_w(w)\partial_w\Delta p(w,t) \\
&+ \Delta\sigma_w(w,t)\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial^2_{ww}\Delta p(w,t)
\end{aligned} \tag{A.3}
$$

Expanding the equation for $\hat{p}(t)$,

$$
\Delta r(t)\hat{p} + r\Delta\hat{p}(t) - \Delta\hat{p}'(t) = \Delta\nu(t)
$$

The expression for $r(t)$ expands as

$$
\begin{aligned}
\Delta r(t) = &\frac{\Delta\nu'(t)}{\nu} + \frac{\rho\sigma^2\Delta\nu(t)}{(1+\hat{q})\nu^2}\int\pi(x)dG(x) \\
&- \frac{\rho\sigma^2}{(1+\hat{q})\nu}\left[\int\Delta\pi(x,t)dG(x) + \int\pi(x)\Delta g(x,t)dx\right]
\end{aligned} \tag{A.4}
$$

Since $\Delta\nu'(t) = O(\sigma^2)$, the interest rate deivation is of the same order: $\Delta r(t) = O(\sigma^2)$. The first-order deviation of the wealth density $\Delta g(x,t)$ solves

$$
\begin{aligned}
\Delta g_t(w,t) = &-\partial_w(\mu_w(w)\Delta g(w,t) + \Delta\mu_w(w,t)g(w)) \\
&+ \partial^2_{ww}\left[\frac{\sigma_w(w)^2\Delta g(w,t)}{2} + 2\Delta\sigma_w(w,t)\sigma_w(w)g(w)\right]
\end{aligned} \tag{A.5}
$$

The first-order deviations of the drift and volatility are

$$\Delta\mu_w(w,t) = \Delta r(t)w + \hat{\boldsymbol{\lambda}}\sigma^2\Delta w(t) + \frac{2\Delta\sigma_w(w,t)\sigma_w(w)}{w}$$
$$\Delta\sigma_w(w,t) = \frac{\sigma w[\Delta p_w(w,t)w - \gamma\Delta\hat{w}(t)]}{[w+\gamma\hat{w}-wp'(w)]^2} = \sigma w\boldsymbol{\pi}(w)[\Delta p_w(w,t)w - \gamma\Delta\hat{w}(t)] + O(\sigma^3)$$

Since $\Delta\mu_w(w,t) = O(\sigma^2)$, all terms on the right in the pricing equation (A.3) are $O(\sigma^2)$. Hence

$$r\Delta p(w,t) - \Delta p_t(w,t) = O(\sigma^2)$$

This implies $\Delta p(w,t) = O(\sigma^2)$, $\Delta p_w(w,t) = O(\sigma^2)$, and $\Delta p_{ww}(w,t) = O(\sigma^2)$. Hence, all terms on the right in equation (A.3) are of order $O(\sigma^4)$, except for $\Delta\nu(t) - \sigma^2\Delta\pi(w,t)$, where

$$\Delta\pi(w,t) = \frac{\gamma\Delta\hat{w}(t)}{[w+\gamma\hat{w}-wp'(w)]^2} - \frac{2\pi(w)(\gamma\Delta w(t) - w\Delta p_w(w,t))}{w+\gamma\hat{w}-wp'(w)} = O(\sigma^2)$$

The equality follows from the fact that the first-order deviation of the special country's wealth is

$$\Delta w(t) = \frac{(1+\hat{q})\Delta\nu(t)}{\rho} - \int x\Delta g(x,t)dx$$

But since $\Delta\mu_w(w,t) = O(\sigma^2)$, $\mu_w(w) = O(\sigma^2)$, $\Delta\sigma_w(w,t) = O(\sigma)$, and $\sigma_w(w)$, the expanded Kolmogorov forward equation (A.5) implies $\Delta g_t(w,t) = O(\sigma^2)$ and hence $\Delta g(w,t) = O(\sigma^2)$. This leads to $\Delta\hat{w}(t) = O(\sigma^2)$, which implies $\Delta\pi(w,t) = O(\sigma^2)$ because $\Delta p_w(w,t) = O(\sigma^2)$ too.

Coming back to the expression for the interest rate in equation (A.4),

$$\Delta r(t) = \frac{\Delta\nu'(t)}{\nu} + O(\sigma^4)$$

The interest rate only reacts to consumption growth expectations in the second order. Since all terms on the right in equation (A.3) are of order $O(\sigma^4)$ except for $\Delta nu(t)$,

$$\rho\Delta p(w,t) - \Delta p_t(w,t) = \Delta\nu(t) - \frac{\nu}{\rho}\cdot\frac{\Delta\nu'(t)}{\nu} + O(\sigma^4) = \Delta\nu(t)\cdot\frac{\mu_\nu+\rho}{\rho} + O(\sigma^4)$$

This uses $\Delta\nu(t) = \delta_\nu e^{-\mu_\nu t}$. Integrating,

$$\Delta p(t) = \frac{\Delta\nu(t)}{\rho} \equiv J_{p,\nu}\Delta\nu(t)$$

In the second order, prices only react to the expected discounted dividend changes.

Returning to $\Delta g(w,t)$, $\Delta\sigma_w(w,t) = O(\sigma^3)$ and $\Delta g(w,t) = O(\sigma^2)$ imply

$$\begin{aligned}\Delta g_t(w,t) &= -\partial_w(\Delta\mu_w(w,t)\boldsymbol{g}(w)) + O(\sigma^4) = -\Delta r(t)(w\boldsymbol{g}(w))' + O(\sigma^4)\\ &= \frac{\mu_\nu}{\nu}\Delta\nu(t)(w\boldsymbol{g}(w))' + O(\sigma^4)\end{aligned}$$

Integrating between t and infinity and using the assumption on convergence,

$$\Delta g(w,t) = -\frac{(w\boldsymbol{g}(w))'}{\nu}\Delta\nu(t) + O(\sigma^4) \equiv J_{g,\nu}(w)\Delta\nu(t) + O(\sigma^4)$$

The expressions for $\Delta\theta(w,t)$ is now

$$\begin{aligned}\Delta\theta(w,t) &= \frac{\Delta p(w,t)}{w+\gamma\hat{w}} - \frac{\gamma\Delta\hat{w}(t)p(w)}{(w+\gamma\hat{w})^2} + O(\sigma^4)\\ &= \Delta p(w,t)\boldsymbol{\pi}(w) - p(w)\boldsymbol{\pi}(w)^2\gamma\Delta\hat{w}(t) + O(\sigma^4)\\ &= \Delta\nu(t)\left[J_{p,\nu}(w)\boldsymbol{\pi}(w) - p(w)\boldsymbol{\pi}(w)^2\frac{\gamma(1+\hat{q})}{\rho}\right] + \gamma p(w)\boldsymbol{\pi}(w)^2\int x\Delta g(x,t)dx + O(\sigma^4)\\ &= \Delta\nu(t)\left[J_{p,\nu}(w)\boldsymbol{\pi}(w) - p(w)\boldsymbol{\pi}(w)^2\frac{\gamma(1+\hat{q})}{\rho} + \gamma p(w)\boldsymbol{\pi}(w)^2\int xJ_{g,\nu}(x)dx\right] + O(\sigma^4)\end{aligned}$$

Plugging this into a_t and replacing $\theta(w) = p(w)\boldsymbol{\pi}(w)$ and $h(w) = w\boldsymbol{\pi}(w)$,

$$\begin{aligned}a_t &= -\frac{\Delta\nu(t)}{A}\int\left[J_{p,\nu}(w)\boldsymbol{\pi}(w) - p(w)\boldsymbol{\pi}(w)^2\frac{\gamma(1+\hat{q})}{\rho}\right]wd\mathcal{G}(w)\\ &\quad + \frac{\Delta\nu(t)}{A}\int\left[1-\theta(w) - \gamma\int p(x)x\boldsymbol{\pi}(x)^2d\mathcal{G}(x)\right]wJ_{g,\nu}(w)dw + O(\sigma^4)\\ &= \frac{\Delta\nu(t)}{A}\int\left[\theta(w)\frac{\gamma(1+\hat{q})}{\rho} - J_{p,\nu}(w)\right]h(w)d\mathcal{G}(w)\\ &\quad + \frac{\Delta\nu(t)}{A}\int\left[1-\theta(w) - \gamma\int\theta(x)h(x)d\mathcal{G}(x)\right]wJ_{g,\nu}(w)dw + O(\sigma^4)\end{aligned}$$

The expression for p_t is

$$p_t = \frac{\Delta\nu(t)}{P}\left[\int J_{p,\nu}(w)d\mathcal{G}(w) + \int p(w)J_{g,\nu}(w)dw\right] + O(\sigma^4)$$

Collecting the results for the two shocks,

$$\begin{aligned}a_t &= J_{a,\gamma}\Delta\gamma(t) + J_{a,\nu}\Delta\nu(t) + O(\sigma^4)\\ p_t &= J_{p,\gamma}\Delta\gamma(t) + J_{p,\nu}\Delta\nu(t) + O(\sigma^4)\end{aligned}$$

Here

$$
\begin{aligned}
J_{a,\gamma} &= \int (\theta(w)\hat{w} - J_{p,\gamma}(w))\frac{h(w)}{A}d\mathcal{G}(w) + \int \left[1 - \theta(w) - \gamma \int h(x)\theta(x)d\mathcal{G}(x)\right]\frac{w}{A}J_{g,\gamma}(w)dw \\
J_{a,\nu} &= \int \frac{J_{p,\gamma}}{A}(w)d\mathcal{G}(w) + \int \frac{p(w)}{A}J_{g,\gamma}(w)dw \\
J_{p,\gamma} &= \int \left[\theta(w)\frac{\gamma(1+\hat{q})}{\rho} - J_{p,\nu}(w)\right]\frac{h(w)}{A}d\mathcal{G}(w) + \int \left[1 - \theta(w) - \gamma \int \theta(x)h(x)d\mathcal{G}(x)\right]\frac{w}{A}J_{g,\nu}(w)dw \\
J_{p,\nu} &= \int \frac{J_{p,\nu}}{P}(w)d\mathcal{G}(w) + \int \frac{p(w)}{P}J_{g,\nu}(w)dw
\end{aligned}
$$

The statement of the proposition follows from solving the stochastic differential equations for the shocks:

$$
\begin{aligned}
d\gamma(t) &= \mu_\gamma(\gamma - \gamma(t))dt + \sigma_\gamma \cdot dW(t) \\
d\nu(t) &= \mu_\nu(\nu - \nu(t))dt + \sigma_\nu \cdot dW(t)
\end{aligned}
$$

These imply

$$
\begin{aligned}
\gamma(t) &= (1 - e^{-\mu_\gamma \tau})\gamma + e^{-\mu_\gamma \tau}\gamma(t-\tau) + \int_0^\tau e^{-\mu_\gamma s}\sigma_\gamma \cdot dW(t-s) \\
\nu(t) &= (1 - e^{-\mu_\nu \tau})\nu + e^{-\mu_\nu \tau}\gamma(t-\tau) + \int_0^\tau e^{-\mu_\nu s}\sigma_\nu \cdot dW(t-s)
\end{aligned}
$$

Taking the limit $\tau \longrightarrow \infty$ completes the proof. $\square$

C Additional details for empirics.

I use the following panels provided by the IMF:

- Balance of Payments dataset:
  - `BFPA_BP6_USD`: portfolio asset acquisition
  - `BFPL_BP6_USD`: portfolio liability incurrence
  - `BFOA_BP6_USD`: other asset acquisition
  - `BFOL_BP6_USD`: other liability incurrence
- International Financial Statistics dataset:
  - `IAP_BP6_USD`: portfolio assets
  - `ILP_BP6_USD`: portfolio liabilities
  - `IAO_BP6_USD`: other assets
  - `ILOFR_BP6_USD`: other liabilities
  - `FIGB_PA`: government bond yields

Figure A.1 shows the number of countries for which these data are available each year. Flow data from the Balance of Payments dataset are generally available for a larger number of countries, hence the lower numbers for the datasets of flows normalized by stocks.

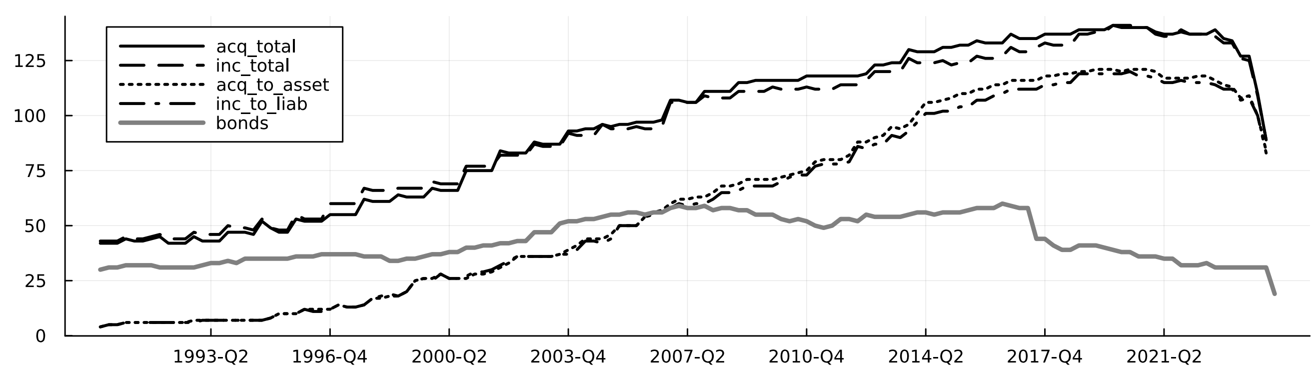


Figure A.1: number of countries present in the sample of asset acquisition, liability incurrence, asset acquisition over asset stock, liability incurrence over liability stock, and bond yields.

I combine portfolio and other assets into total private assets, and do the same for liabilities, asset acquisition, and liability incurrence. I then apply the deseasoning procedure to the data and compute the ratios acquisition to assets a_{it}, acquisition to liabilities b_{it}, and incurrence to liabilities l_{it}. To extract the principal components from acquisition and incurrence panels, I choose the largest balanced panels. I then extract the principal components using two methods: by

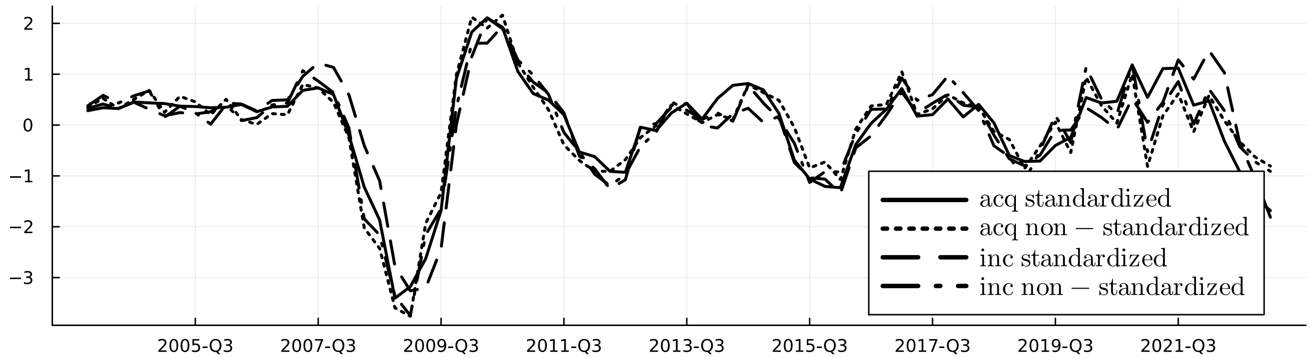


Figure A.2: Principal components of asset acquisition and liability incurrence extracted from standardized and non-standardized panels.

standardizing each country's serie and raw. The rationale for standardizing is that the principal component will naturally be putting higher weights on countries with larger dollar flows, excluding smaller and less developed countries with smaller asset positions. I use the principal components of the standardized panels throughout. However, as Figure A.2 shows, the difference between the two methods is not substantial.

Table 9: number of observations and R^2 in all second-stage regressions: R^2_G refers to regressions on the assets-to-GDP ratio, and R^2_{AE} refers to regressions on the advanced economy dummies.

	N	R^2_G	R^2_{AE}		N	R^2_G	R^2_{AE}
loadings on f_t^{acq}				**loadings on f_t^{inc}**			
F_{it}^{acq}	108	0.39	0.15	F_{it}^{acq}	108	0.49	0.14
F_{it}^{inc}	105	0.21	0.08	F_{it}^{inc}	105	0.18	0.06
y_{it}	47	0.08	0.02	y_{it}	47	0.08	0.02
a_{it}	106	0.04	0.03	a_{it}	106	0.05	0.03
b_{it}	107	0.08	0.04	b_{it}	107	0.18	0.04
l_{it}	103	0.06	0.04	l_{it}	103	0.05	0.03
loadings on VIX$_t$				**loadings on EBP$_t$**			
F_{it}^{acq}	108	0.40	0.10	F_{it}^{acq}	108	0.49	0.13
F_{it}^{inc}	105	0.25	0.07	F_{it}^{inc}	105	0.25	0.09
y_{it}	47	0.05	0.01	y_{it}	47	0.07	0.02
a_{it}	106	0.06	0.02	a_{it}	106	0.04	0.01
b_{it}	107	0.06	0.00	b_{it}	107	0.22	0.01
l_{it}	103	0.06	0.03	l_{it}	103	0.07	0.04

I then complement the acquisition and incurrence principal components f_t^{acq} and f_t^{inc} with VIX$_t$ and EBP$_t$. These four series are my global factors. With six panels $\{F_{it}^{\text{acq}}\}$, $\{F_{it}^{\text{inc}}\}$, $\{y_{it}\}$, $\{a_{it}\}$, $\{b_{it}\}$, and $\{l_{it}\}$, I run regressions in two stages. First, I find country-specific loadings of each of the panel variables on each of the factors. I obtain $6 \times 4 = 24$ loadings for every country in total, keeping only countries for which the sample is long enough (20 or more quarters). This gives me 24 cross-sections of loadings. I complement these 24 cross-sections with average assets-to-GDP ratios over the period 2015-2020, where the GDP data come from the World Bank, and the cross-section of advanced economy dummies from the IMF.

I then run 24 second-stage regressions of country-specific loadings on their assets-to-GDP ratios, and another 24 regressions of country-specific loadings on their advanced economy dummies. In the main text, I present the 48 coefficients and the associated standard errors in Table 7 and Table 8. Table 9 presents the number of observations and R^2 for these regressions.

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