A Proofs

Proof. (of Proposition 1) Using the definition of $\mathcal{W}(b^T, b^N)$,

$$\mathcal{W} = \max \phi \left((C_0^w)^{1-\zeta} + \beta_w \mathbb{E}[(C_1^w)^{1-\sigma}]^{\frac{1-\zeta}{1-\sigma}} \right) + (1-\phi) \left((C_0^e)^{1-\zeta_e} + \beta_e \mathbb{E}[(C_1^e)^{1-\sigma_e}]^{\frac{1-\zeta_e}{1-\sigma_e}} \right) \tag{58}$$

$$\text{s.t. } p_0^\alpha C_0^w = e_0^{w,T} + p_0 e_0^{N,w} - p_0 q^N b^N - q^T b^T \tag{59}$$

$$p_0^\alpha C_0^e = e_0^{e,T} + p_0 e_0^{e,N} + p_0 q^N b^N + q^T b^T + \tilde{q}\tilde{b} \tag{60}$$

$$p_1^\alpha C_1^w = e_1^{w,T} + p_1 e_1^{w,N} + w_1 l + b^T + p_1 b^N \tag{61}$$

$$p_1^\alpha C_1^e = e_1^{e,T} + p_1 e_1^{e,N} + f(z_1, l) - w_1 l - z_1 - b^T - p_1 b^N - \tilde{b} \tag{62}$$

$$q^T = \beta_w \mathbb{E}\left[\frac{p_0^\alpha}{p_1^\alpha} \frac{(C_1^w)^{-\sigma}}{(C_0^w)^{-\sigma}} \right] \mathbb{E}\left[\frac{(C_1^w)^{1-\sigma}}{(C_0^w)^{1-\sigma}} \right]^{\frac{\sigma-\zeta}{1-\sigma}} \tag{63}$$

$$q^N = \beta_w \mathbb{E}\left[\frac{p_0^{1-\alpha}}{p_1^{1-\alpha}} \frac{(C_1^w)^{-\sigma}}{(C_0^w)^{-\sigma}} \right] \mathbb{E}\left[\frac{(C_1^w)^{1-\sigma}}{(C_0^w)^{1-\sigma}} \right]^{\frac{\sigma-\zeta}{1-\sigma}} \tag{64}$$

$$(1-\tau^T) q^T = \beta_e \mathbb{E}\left[\frac{p_0^\alpha}{p_1^\alpha} \frac{(C_1^e)^{-\sigma_e}}{(C_0^e)^{-\sigma_e}} \cdot (1 + \theta^{-1}\delta_1(f_z(z_1, l) - 1)) \right] \mathbb{E}\left[\frac{(C_1^e)^{1-\sigma_e}}{(C_0^e)^{1-\sigma_e}} \right]^{\frac{\sigma_e-\zeta_e}{1-\sigma_e}} \tag{65}$$

$$(1-\tau^N) q^N = \beta_e \mathbb{E}\left[\frac{p_0^{1-\alpha}}{p_1^{1-\alpha}} \frac{(C_1^e)^{-\sigma_e}}{(C_0^e)^{-\sigma_e}} \cdot (1 + \theta^{-1}\delta_1(f_z(z_1, l) - 1)) \right] \mathbb{E}\left[\frac{(C_1^e)^{1-\sigma_e}}{(C_0^e)^{1-\sigma_e}} \right]^{\frac{\sigma_e-\zeta_e}{1-\sigma_e}} \tag{66}$$

$$(1-\tau^T) q^T = (1-\tilde{\tau})\tilde{q} \tag{67}$$

$$p_1 = \frac{\alpha}{1-\alpha} \frac{f(z_1, l) - z_1 + e_1^{w,T} + e_1^{e,T} - \tilde{b}}{e_1^{w,N} + e_1^{e,N}} \tag{68}$$

$$z_1 = \min\left\{ \bar{z}, \theta^{-1}(p_1(\bar{b} - b^N) - b^T - \tilde{b}) \right\} \tag{69}$$

$$w_1 = f_l(z_1, l) \tag{70}$$

Maximization is over $\{C_0^w, C_0^e, q^T, q^N, \tau^T, \tau^N, \tilde{\tau}\}$ and $\{C_1^w, C_1^e, p_1, z_1, w_1\}$ for any realization of the traded endowments $\epsilon = (e_1^{w,T}, e_1^{e,T})$. Let the multipliers on the budget constraints (59), (60), (61), and (62) be λ_0^w, λ_0^e, λ_1^w, and λ_1^e. Let the multipliers on the asset price constraints (63), (64), (65), and (66) be $\mu^{T,w}$, $\mu^{N,w}$, $\mu^{T,e}$, and $\mu^{N,e}$. Finally, denote the functions in the right-hand side of (63), (64), (65), and (66) by $Q^{T,w}$, $Q^{N,w}$, $Q^{T,e}$, and $Q^{N,e}$.

Notice that the taxes $(\tau^T, \tau^N, \tilde{\tau})$ can be set residually so that the constraints in (65), (66), and (67) are always satisfied. The multipliers on these constraints are hence zero. In particular, $\mu^{T,e} = \mu^{N,e} = 0$. Taking the derivatives with respect to C_0^w, C_0^e, C_1^w, and C_1^e (the latter two variables really mean their realizations in every state of the shock ϵ),

$$\phi(1-\zeta)(C_0^w)^{-\zeta} = p_0^\alpha \lambda_0^w + \mu^{T,w}\frac{\partial Q^{T,w}}{\partial C_0^w} + \mu^{N,w}\frac{\partial Q^{N,w}}{\partial C_0^w} \tag{71}$$

$$(1-\phi)(1-\zeta_e)(C_0^e)^{-\zeta_e} = p_0^\alpha \lambda_0^e \tag{72}$$

$$\pi_1(\epsilon)\cdot\beta^w\phi(1-\zeta)C_1^w(\epsilon)^{-\sigma}\mathbb{E}\left[C_1^w(\epsilon)^{1-\sigma}\right]^{\frac{\sigma-\zeta}{1-\sigma}} = p_1^\alpha\lambda_1^w(\epsilon) + \mu^{T,w}\frac{\partial Q^{T,w}}{\partial C_1^w(\epsilon)} + \mu^{N,w}\frac{\partial Q^{N,w}}{\partial C_1^w(\epsilon)} \tag{73}$$

$$\pi_1(\epsilon)\cdot\beta^e(1-\phi)(1-\zeta_e)C_1^e(\epsilon)^{-\sigma_e}\mathbb{E}\left[C_1^e(\epsilon)^{1-\sigma_e}\right]^{\frac{\sigma_e-\zeta_e}{1-\sigma_e}} = p_1^\alpha\lambda_1^e(\epsilon) \tag{74}$$

Here $\pi_1(\epsilon)$ is the probability of the realization ϵ. Taking the derivatives with respect to q^T and q^N,

$$\mu^{T,w} = b^T(\lambda_0^w - \lambda_0^e) \tag{75}$$

$$\mu^{N,w} = p_0 b^N(\lambda_0^w - \lambda_0^e) \tag{76}$$

Changes in these prices have a purely redistributive effect. Combining these equations with (71) and (72) and using the assumption that the weights satisfy $\phi(1-\zeta)(C_0^w)^{-\zeta} = (1-\phi)(1-\zeta_e)(C_0^w)^{-\zeta_e}$,

$$\begin{aligned} 0 &= (\lambda_0^w - \lambda_0^e)\left(p_0^\alpha + b^T\frac{\partial Q^{T,w}}{\partial C_0^w} + p_0 b^N\frac{\partial Q^{N,w}}{\partial C_0^w}\right) \\ &= (\lambda_0^w - \lambda_0^e)\frac{p_0^\alpha C_0^w + \zeta(q^T b^T + p_0 b^N q^N)}{C_0^w} \end{aligned} \tag{77}$$

The numerator of the fraction is not zero. If $q^T b^T + p_0 b^N q^N \geq 0$ then it is positive, and if $q^T b^T + p_0 b^N q^N < 0$, then

$$p_0^\alpha C_0^w + \zeta(q^T b^T + p_0 b^N q^N) > p_0^\alpha C_0^w + q^T b^T + p_0 b^N q^N = e_0^{T,w} + p_0 e_0^{N,w} > 0 \tag{78}$$

This implies $\lambda_0^w = \lambda_0^e$, meaning that, to the planner, the marginal value of additional dollar allocated to an agent is the same across agents. From (75) and (76) this implies that $\mu^{T,w} = \mu^{N,w} = 0$. Then, using the definitions in the text of the proposition, $\pi_1(\epsilon)\Lambda^e(\epsilon) = \lambda_1^w(\epsilon)/\lambda_0^w$, $\pi_1(\epsilon)\Lambda^e(\epsilon) = \lambda_1^e(\epsilon)/\lambda_0^e$, and $\mathcal{U}_0 = \lambda_0^w = \lambda_0^e$.

Next, denote by $Z(b^T, b^N, \epsilon)$, $P(z_1, \epsilon)$, and $W(z_1)$ the solution of the system of (68), (69), and (70), where z_1 is expressed as a function of the shock ϵ and debt (b^T, b^N), p_1 is expressed as a function of z_1 and ϵ, and w_1 as a function of z_1. By the envelope theorem, the derivatives of $\mathcal{W}(b^T, b^N)$ are

$$\begin{aligned} \frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^T} &= \mathbb{E}\left[\frac{\partial P}{\partial z_1}\cdot\frac{\partial Z}{\partial b^T}\left(\Lambda^w(b^N + e_1^{N,w} - \alpha p_1^{\alpha-1}C_1^w) + \Lambda^e(e_1^{N,e} - b^N - \alpha p_1^{\alpha-1}C_1^e)\right)\right] \\ &+ \mathbb{E}\left[\frac{\partial W}{\partial z_1}l\cdot\frac{\partial Z}{\partial b^T}(\Lambda^w - \Lambda^e)\right] + \mathbb{E}\left[\frac{\partial Z}{\partial b^T}\Lambda^e(f_z(z_1,l)-1)\right] + \mathbb{E}[\Lambda^w - \Lambda^e] \end{aligned} \tag{79}$$

$$\begin{aligned} \frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^N} &= \mathbb{E}\left[\frac{\partial P}{\partial z_1}\cdot\frac{\partial Z}{\partial b^N}\left(\Lambda^w(b^N + e_1^{N,w} - \alpha p_1^{\alpha-1}C_1^w) + \Lambda^e(e_1^{N,e} - b^N - \alpha p_1^{\alpha-1}C_1^e)\right)\right] \\ &+ \mathbb{E}\left[\frac{\partial W}{\partial z_1}l\cdot\frac{\partial Z}{\partial b^N}(\Lambda^w - \Lambda^e)\right] + \mathbb{E}\left[\frac{\partial Z}{\partial b^N}\Lambda^e(f_z(z_1,l)-1)\right] + \mathbb{E}[p_1\cdot(\Lambda^w - \Lambda^e)] \end{aligned} \tag{80}$$

Using the fact that $\alpha p_1^\alpha C_1^w = p_1 c_1^{N,w}$, $\alpha p_1^\alpha C_1^e = p_1 c_1^{N,e}$, and $c_1^{N,w} + c_1^{T,w} = e_1^{N,w} + e_1^{T,w}$,

$$\begin{aligned}\frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^T} &= \mathbb{E}\left[(\Lambda^w - \Lambda^e)\mathcal{Z}_1\mathcal{D}_1^p m_1^w\right] + \mathbb{E}\left[(\Lambda^w - \Lambda^e)\mathcal{Z}_1\mathcal{D}_1^w l\right] \\ &+ \mathbb{E}\left[\Lambda^e \mathcal{Z}_1(f_z(z_1, l) - 1)\right] + \mathbb{E}[\Lambda^w - \Lambda^e] \qquad (81)\\ \frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^N} &= \mathbb{E}\left[p_1 \cdot (\Lambda^w - \Lambda^e)\mathcal{Z}_1\mathcal{D}_1^p m_1^w\right] + \mathbb{E}\left[p_1 \cdot (\Lambda^w - \Lambda^e)\mathcal{Z}_1\mathcal{D}_1^w l\right] \\ &+ \mathbb{E}\left[p_1 \cdot \Lambda^e \mathcal{Z}_1(f_z(z_1, l) - 1)\right] + \mathbb{E}[p_1 \cdot (\Lambda^w - \Lambda^e)] \qquad (82)\end{aligned}$$

Here $m_1^w = b^N + e_1^{N,w} - c_1^{N,w}$ and $(\mathcal{Z}_1, \mathcal{D}_1^w, \mathcal{D}_1^p)$ is the notation for the derivatives of (Z, W, P).

Now recall that

$$\begin{aligned} q^T &= \mathbb{E}[\Lambda^w] \qquad (83)\\ q^N &= \mathbb{E}[p_1\Lambda^w] \qquad (84)\\ (1-\tau^T)q^T &= \mathbb{E}[\Lambda^e(1 + \delta_1\theta^{-1}(f_z(z_1, l) - 1))] \qquad (85)\\ (1-\tau^N)q^N &= \mathbb{E}[p_1\Lambda^e(1 + \delta_1\theta^{-1}(f_z(z_1, l) - 1))] \qquad (86)\end{aligned}$$

Plugging these,

$$\begin{aligned}\frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^T} &= \mathbb{E}\left[(\Lambda^w - \Lambda^e)(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\mathcal{Z}_1\right] + \mathbb{E}\left[\Lambda^e(\mathcal{Z}_1 + \delta_1\theta^{-1})(f_z(z_1, l) - 1)\right] + \tau^T q^T \qquad (87)\\ \frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^N} &= \mathbb{E}\left[p_1 \cdot (\Lambda^w - \Lambda^e)(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\mathcal{Z}_1\right] + \mathbb{E}\left[p_1 \cdot \Lambda^e(\mathcal{Z}_1 + \delta_1\theta^{-1})(f_z(z_1, l) - 1)\right] \\ &+ \tau^N q^N p_0 \qquad (88)\end{aligned}$$

This completes the proof. □

Proof. (of Proposition 2) Using Proposition 1 and denoting $s_1 = p_1/p_0$,

$$\begin{aligned}\frac{1}{p_0\mathcal{U}_0}\left(\frac{\partial \mathcal{W}}{\partial b^N} - \mathbb{E}[p_1]\frac{\partial \mathcal{W}}{\partial b^T}\right) &= \mathbb{E}\left[s_1 \cdot (\Lambda^w - \Lambda^e)(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\mathcal{Z}_1\right] \\ &- \mathbb{E}[s_1] \cdot \mathbb{E}\left[(\Lambda^w - \Lambda^e)(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\mathcal{Z}_1\right] \\ &+ \mathbb{E}\left[s_1 \cdot \Lambda^e(\mathcal{Z}_1 + \delta_1\theta^{-1})(f_z(z_1, l) - 1)\right] \\ &- \mathbb{E}[s_1] \cdot \mathbb{E}\left[\Lambda^e(\mathcal{Z}_1 + \delta_1\theta^{-1})(f_z(z_1, l) - 1)\right] \\ &+ (q^N - (1-\tau^N)q^N) - (q^T - (1-\tau^T)q^T)\mathbb{E}[s_1] \\ &= \mathbb{C}\left[s_1, (\Lambda^w - \Lambda^e)(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\mathcal{Z}_1\right] \\ &+ \mathbb{C}\left[s_1, \Lambda^e(\mathcal{Z}_1 + \delta_1\theta^{-1})(f_z(z_1, l) - 1)\right] \\ &- (q^T\mathbb{E}[s_1] - q^N) + ((1-\tau^T)q^T\mathbb{E}[s_1] - (1-\tau^N)q^N) \qquad (89)\end{aligned}$$

Using the definitions of the UIP violations $\Delta_{UIP}^w = q^T\mathbb{E}[s_1] - q^N$ and $\Delta_{UIP}^e = (1-\tau^T)q^T\mathbb{E}[s_1] - (1-\tau^N)q^N$ leads to the result in the proposition. □

Proof. (of Corollary 1) This corollary follows directly from Proposition 1 when the derivatives of $\mathcal{W}(b^T, b^N)$ are set to zero. □

B Alternative setups

In this section, we discuss extensions of the model that incorporate foreign savings, alternative specifications of the borrowing constraint, capital controls, and an alternative specification of taxes.

B.1 Foreign savings

Suppose that workers are allowed to buy claims to traded goods abroad. Denote their holdings by $\tilde{B}$. Suppose that they buy these claims at the same price $\tilde{q}$ at which entrepreneurs borrow from abroad. The problem of the workers becomes

$$\mathcal{V}^w = \max \; \mathcal{C}\big(c_0^{N,w}, c_0^{T,w}\big)^{1-\zeta} + \beta_w \mathbb{E}\Big[\mathcal{C}\big(c_1^{N,w}, c_1^{T,w}\big)^{1-\sigma}\Big]^{\frac{1-\zeta}{1-\sigma}} \tag{90}$$

$$\text{s.t. } p_0 c_0^{N,w} + c_0^{T,w} + q^T b^T + \tilde{q}\tilde{B} + p_0 q^N b^N \le w_0 l_0 + e_0^{T,w} + p_0 e_0^{N,w} \tag{91}$$

$$p_1 c_1^{N,w} + c_1^{T,w} \le w_1 l_1 + e_1^{T,w} + p_1 e_1^{N,w} + b^T + \tilde{B} + p_1 b^N \tag{92}$$

The balance of payments identity changes to

$$c_0^{T,w} + c_0^{T,e} = f(z_0, l_0) - z_0 + e_0^{T,w} + e_0^{T,e} + \tilde{q}(\tilde{b} - \tilde{B})$$
$$c_1^{T,w} + c_1^{T,e} = f(z_1, l_1) - z_1 + e_1^{T,w} + e_1^{T,e} - (\tilde{b} - \tilde{B})$$

This means that the equilibrium exchange rate is given by

$$p_0 = \frac{\alpha}{1-\alpha} \cdot \frac{c_0^{T,w} + c_0^{T,e}}{c_0^{N,w} + c_0^{N,e}} = \frac{\alpha}{1-\alpha} \cdot \frac{f(z_0, l_0) - z_0 + e_0^{T,w} + e_0^{T,e} + \tilde{q}(\tilde{b} - \tilde{B})}{e_0^{N,w} + e_0^{N,e}} \tag{93}$$

$$p_1 = \frac{\alpha}{1-\alpha} \cdot \frac{c_1^{T,w} + c_1^{T,e}}{c_1^{N,w} + c_1^{N,e}} = \frac{\alpha}{1-\alpha} \cdot \frac{f(z_1, l_1) - z_1 + e_1^{T,w} + e_1^{T,e} + (\tilde{B} - \tilde{b})}{e_1^{N,w} + e_1^{N,e}} \tag{94}$$

Suppose that, on the supply side, the price of the cross-border claims only depends on the net flows, $\tilde{q} = Q(\tilde{b} - \tilde{B})$, and that there are no taxes.

Observe that, for any $x \in [0, \tilde{B}]$, an equilibrium of this model with $(b^T, \tilde{b}, \tilde{B})$ is equivalent to an equilibrium with $(b^T + x, \tilde{b} - x, \tilde{B} - x)$. The budget constraints across the two equilibria are the same, since workers save $b^T + \tilde{B}$ and entrepreneurs borrow $\tilde{b}$ in foreign currency in both cases. The expression for the exchange rate are identical across the two cases. Finally, the borrowing constraint of the entrepreneurs is the same in the two allocations, since it only features the total amount borrowed in foreign currency. In the extreme case $x = \tilde{B}$, workers do not save abroad like in the baseline model.

The upshot is that we can restrict savings abroad to be zero without loss of generality provided that the foreign interest rate only depends on the net foreign asset position. Marginal effects $\mathcal{D}_1^p$, $\mathcal{D}_1^w$, and $\mathcal{Z}_1$ are also the same as in the baseline model. This means that the results from Section 3 can be extended to this richer version upon using a suitable tax system. Specifically, the planner would need to impose the same capital control tax $\tilde{\tau}$ on the savers and reimburse the proceeds $\tilde{\tau}\tilde{q}\tilde{B}$ to them. This insures that both savers and borrowers are indifferent between domestic and cross-border foreign currency flow, and the NFA remains fixed throughout the exercise.

B.2 Alternative borrowing constraints

Wage bill in the constraint. Consider the following modification to the entrepreneur's problem:

$$\mathcal{V}^e = \max \mathcal{C}(c_0^{e,N}, c_0^{e,T})^{1-\zeta_e} + \beta_e \mathbb{E}[\mathcal{C}(c_1^{e,N}, c_1^{e,T})^{1-\sigma_e}]^{\frac{1-\zeta_e}{1-\sigma_e}} \tag{95}$$

$$\text{s.t. } p_0 c_0^{N,e} + c_0^{T,e} \leq f(z_0, l_0) - w_0 l_0 - z_0 + y_0^{T,e} + p_0 y_0^{N,e} \tag{96}$$
$$+ (1-\tilde{\tau})\tilde{q}\tilde{b} + (1-\tau^T) q^T b^T + (1-\tau^N) p_0 q^N b^N + T$$

$$p_1 c_1^{N,e} + c_1^{T,e} + \tilde{b} + b^T + p_1 b^N \leq f(z_1, l_1) - w_1 l_1 - z_1 + y_1^{T,e} + p_1 y_1^{N,e} \tag{97}$$

$$\theta_z z_1 + \theta_l w_1 l_1 \leq p_1 (\bar{b} - b^N) - b^T - \tilde{b} \tag{98}$$

The new element here is the portion $\theta_l w_1 l_1$ of the wage bill that has to be pre-funded alongside the portion of tradable inputs $\theta_z z_1$. Assume first $\theta_z > 0$ and $\theta_l > 0$. The first-order conditions with respect to l_1 and z_1 are modified to

$$f_z(z_1, l_1) = 1 + \mu\theta_z \tag{99}$$

$$f_l(z_1, l_1) = w_1(1 + \mu\theta_l) \tag{100}$$

Here μ is the Lagrange multiplier on (98). Since labor is supplied inelastically at l, adjustment in (100) happens through w_1. Wages are now directly impacted by debt through the borrowing constraint in addition to the effect through z_1. We can express the wage as

$$w_1 = \frac{\theta_z f_l(z_1, l)}{\theta_z + \theta_l(f_z(z_1, l) - 1)} \tag{101}$$

The borrowing constraint can be rewritten as

$$\theta_z \left(z_1 + \frac{\theta_l f_l(z_1, l) l}{\theta_z + \theta_l(f_z(z_1, l) - 1)} \right) \leq p_1 (\bar{b} - b^N) - b^T - \tilde{b} \tag{102}$$

The market-clearing condition for the non-traded good, and hence the expression for the exchange rate, does not change. The planner's problem can still be solved as in the baseline version of the model: isolating $z_1 = Z(b^T, b^N, \epsilon)$, $p_1 = P(Z(b^T, b^N, \epsilon), \epsilon)$, and $w_1 = W(Z(b^T, b^N, \epsilon))$, then computing marginal effects of debt. The marginal effect of z_1 on p_1 is the same as before:

$$\mathcal{D}_1^p = \frac{\alpha}{(1-\alpha) y_1^N} \cdot (f_z(z_1, l) - 1) \tag{103}$$

The marginal effects on the wage and input use now take a more involved form:

$$\mathcal{D}_1^w = \frac{\theta_z f_{zl}(z_1, l)(\theta_z + \theta_l(f_z(z_1, l) - 1)) - \theta_z \theta_l f_l(z_1, l) f_{zz}(z_1, l)}{(\theta_z + \theta_l(f_z(z_1, l) - 1))^2} \tag{104}$$

$$\mathcal{Z}_1 = -\frac{\delta_1}{\theta_z + \delta_1 \theta_l \mathcal{D}_1^w - \delta_1 \mathcal{D}_1^p (\bar{b} - b^N)} \tag{105}$$

In the denominator of $\mathcal{Z}_1$, there is now an additional positive term $\delta_1 \theta_l \mathcal{D}_1^w$ that attenuates the magnitude of $\mathcal{Z}_1$. When the borrowing constraint becomes binding in this setup, part of the adjustment happens trough w_1 and less through z_1. The expressions for the marginal benefits of increasing debt and de-dollarization remain the same up to these changes in $\mathcal{D}_1^w$ and $\mathcal{Z}_1$.

Now assume $\theta_l > 0$ but $\theta_z = 0$. In this case, (99) means that $f_z(z_1, l) = 1$ for any realization of the shock ϵ and any debt level. The exchange rate is hence determined by ϵ only. There is no amplification through the exchange rate and no uninternalized effects besides those on wages. The derivatives of the wage with respect to b^T and b^N have to be computed directly:

$$\frac{dw_1}{db^T} = -\frac{1}{\theta_l l} \tag{106}$$

$$\frac{dw_1}{db^N} = -\frac{p_1}{\theta_l l} \tag{107}$$

This is the only effect that the planner takes into account and the agents do not. The benefits of increasing debt and de-dollarizing the portfolio now only come from the wage bill and are purely redistributive. Analogously to Proposition 1 and Proposition 2, we can express them as

$$\frac{1}{\mathcal{U}_0}\frac{d\mathcal{W}}{db^T} = -\frac{1}{\theta_l l}\mathbb{E}[\Lambda^w - \Lambda^e] + \tau^T q^T \tag{108}$$

$$\frac{1}{\mathcal{U}_0}\frac{d\mathcal{W}}{db^T} = -\frac{1}{\theta_l l}\mathbb{E}[p_1(\Lambda^w - \Lambda^e)] + p_0\tau^N q^N \tag{109}$$

$$\Delta = \frac{1}{\theta_l l}\mathbb{C}\left[\Lambda^w - \Lambda^e, \frac{p_1}{p_0}\right] - (\Delta^w_{UIP} - \Delta^e_{UIP}) \tag{110}$$

From the workers' perspective, gains from de-dollarization come from making the constraint on wages less tight in times of depreciation, which decreases the correlation between wages and the exchange rate. This is a benefit if depreciation is associated with a high marginal utility Λ^w. For entrepreneurs, it is the opposite because they pay wages rather than receive them.

Revenue in the borrowing constraint. Suppose now a part of revenue $\theta_f f(z_1, l_1)$ for $\theta_f < \theta_z$ could be pledged as collateral. The borrowing constraint changes to

$$\theta_z z_1 \leq \theta_f f(z_1, l_1) + p_1\left(\bar{b} - b^N\right) - b^T - \tilde{b} \tag{111}$$

The first-order conditions with respect to z_1 and l_1 are modified to

$$f_z(z_1, l_1)(1 + \mu\theta_f) = 1 + \mu\theta_z \tag{112}$$

$$f_l(z_1, l_1)(1 + \mu\theta_f) = w_1 \tag{113}$$

Rearranging and plugging $l_1 = l$,

$$w_1 = f_l(z_1, l)\frac{\theta_z + \theta_f}{\theta_z - \theta_f f_z(z_1, l)} \tag{114}$$

We can again solve the planner's problem by isolating $z_1 = Z(b^T, b^N, \epsilon)$, $p_1 = P(Z(b^T, b^N, \epsilon), \epsilon)$, and $w_1 = W(Z(b^T, b^N, \epsilon))$, and then computing marginal effects of debt. The marginal effect of z_1 on p_1 does not change. The marginal effect of z_1 on w_1 changes to

$$\mathcal{D}_1^w = \frac{(\theta_z + \theta_f)(f_{zl}(z_1, l)(\theta_z - \theta_f f_z(z_1, l)) + f_l(z_1, l) f_{zz}(z_1, l)\theta_f)}{(\theta_z - \theta_f f_z(z_1, l))^2} \tag{115}$$

The marginal effect of debt on z_1 changes to

$$\mathcal{Z}_1 = -\frac{\delta_1}{\theta_z - \delta_1 f_z(z_1, l) - \delta_1 \mathcal{D}_1^p\big(\bar{b} - b^N\big)} \tag{116}$$

There is a negative term $-\delta_1 f_z(z_1, l)$ in the denominator that is new relative to the baseline. Additional amplification comes from the fact that lower input use tightens the borrowing constraint by reducing pledgeable revenue. The expressions for the marginal benefits of increasing debt and de-dollarization remain the same up to the changes in $\mathcal{D}_1^w$ and $\mathcal{Z}_1$ above.

B.3 Varying cross-border debt $\tilde{b}$

The function $\mathcal{W}(b^T, b^N)$ takes $\tilde{b}$ as a fixed parameter. Using the envelope theorem, we can describe the impact of $\tilde{b}$ on welfare by taking the derivative analogous to the ones in Proposition 1.

To do this, we first need to see how (p_1, z_1, w_1) depend on $\tilde{b}$ when the system of (31), (32), and (33) is solved to produce the functions $p_1 = \tilde{P}(z_1, \tilde{b}, \epsilon)$, $z_1 = \tilde{Z}(b^T, b^N, \tilde{b}, \epsilon)$, and $w_1 = W(z_1)$. The difference between $P(\cdot)$ and $\tilde{P}(\cdot)$ is that the latter incorporates $\tilde{b}$ as a variables as it acknowledges the direct effect of $\tilde{b}$ on p_1 through the numerator of the right-hand side:

$$p_1 = \frac{\alpha}{1-\alpha} \frac{f(z_1, l) - z_1 + e_1^{w,T} + e_1^{e,T} - \tilde{b}}{e_1^{w,N} + e_1^{e,N}} \tag{117}$$

Similarly, the difference between $Z(\cdot)$ and $\tilde{Z}(\cdot)$ is that the latter incorporates $\tilde{b}$ as an argument in

$$z_1 = \min\left\{\bar{z}, \theta^{-1}(p_1(\bar{b} - b^N) - b^T - \tilde{b})\right\} \tag{118}$$

Accordingly, the marginal effect of $\tilde{b}$ incorporates the partial derivative of $\tilde{P}(\cdot)$ with respect to $\tilde{b}$:

$$\begin{aligned} \tilde{\mathcal{Z}}_1 = \frac{d\tilde{Z}}{d\tilde{b}} &= \frac{d\tilde{P}}{d\tilde{b}}\theta^{-1}\delta_1(\bar{b} - b^N) - \theta^{-1}\delta_1 = \left(\mathcal{D}_1^p \tilde{\mathcal{Z}}_1 + \frac{\partial \tilde{P}}{\partial \tilde{b}}\right)\theta^{-1}\delta_1(\bar{b} - b^N) - \theta^{-1}\delta_1 \\ &= \frac{\delta_1}{\theta - \mathcal{D}_1^p \delta_1(\bar{b} - b^N)}\left(\frac{\partial \tilde{P}}{\partial \tilde{b}}(\bar{b} - b^N) - 1\right) = -\frac{\delta_1(1 + \tilde{\Delta}(\bar{b} - b^N))}{\theta - \mathcal{D}_1^p \delta_1(\bar{b} - b^N)} \end{aligned} \tag{119}$$

Here $\tilde{\Delta}$ is given by

$$\tilde{\Delta} = -\frac{\partial \tilde{P}}{\partial \tilde{b}} = \frac{\alpha}{(1-\alpha)(e_1^{N,w} + e_1^{N,e})} \tag{120}$$

We can now take the derivative of $\mathcal{W}$, going through all the steps in the proof of Proposition 1:

$$\begin{aligned} \frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial \tilde{b}} = &\ \mathbb{E}\left[(\Lambda^w - \Lambda^e)\left(\frac{\partial \tilde{P}}{\partial z_1}\frac{d\tilde{Z}}{d\tilde{b}} + \frac{\partial \tilde{P}}{\partial \tilde{b}}\right)m_1^w\right] + \mathbb{E}\left[(\Lambda^w - \Lambda^e)\frac{d\tilde{Z}}{d\tilde{b}}\frac{\partial W}{\partial z_1}l\right] \\ &+ \mathbb{E}\left[\frac{d\tilde{Z}}{d\tilde{b}}\Lambda^e(f_z(z_1, l) - 1)\right] + \tilde{q} + Q'(\tilde{b})\tilde{b} - \mathbb{E}[\Lambda^e] \end{aligned} \tag{121}$$

Here $Q'(\tilde{b})\tilde{b}$ reflects the fact that the interest rate on foreign loans changes because their supply is not perfectly elastic. Note that changing $\tilde{b}$ also changes p_0, the exchange rate at $t = 0$. This effect does not appear in the welfare calculation because it only redistributes through a revaluation of non-tradables at $t = 0$, and the weight ϕ makes this redistribution contribute exactly zero to $\mathcal{W}$ on the margin.

The private first-order condition of the entrepreneurs with respect to $\tilde{b}$ is

$$(1 - \tilde{\tau})\tilde{q} = \mathbb{E}[\Lambda^e(1 + \theta^{-1}(f_z(z_1, l) - 1))] \tag{122}$$

Plugging this and using the notation for the marginal effects,

$$\frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial \tilde{b}} = \mathbb{E}\left[(\Lambda^w - \Lambda^e)(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\tilde{\mathcal{Z}}_1\right] + \mathbb{E}\left[\Lambda^e(\tilde{\mathcal{Z}}_1 + \delta_1\theta^{-1})(f_z(z_1, l) - 1)\right] + \tilde{\tau}\tilde{q}$$
$$+ Q'(\tilde{b})\tilde{b} - \mathbb{E}\left[(\Lambda^w - \Lambda^e)\tilde{\Delta} m_1^w\right] \tag{123}$$

There are three differences between (123) and (38), its analog for b^T. First, the marginal effect of $\tilde{b}$ on z_1 is stronger than that of b^T. This is because $\tilde{b}$ has a direct effect on the exchange rate. A higher $\tilde{b}$ lowers the exchange rate and puts additional pressure on z_1 through the borrowing constraint. Second, the last term in (123) captures the redistributive consequences of this direct effect of $\tilde{b}$ on p_1. A change in the exchange rate leads to a revaluation of the non-traded endowments. This term reflects the planner's incentives to manage the exchange rate by choosing the net foreign asset position, akin to those in Farhi and Werning (2012) and Farhi and Werning (2016). Third, the planner realizes that foreign supply of loans is not perfectly elastic, and the price $\tilde{q}$ can be manipulated. This is a standard monopsonistic effect.

B.4 Alternative tax rebates

In the baseline model redistribution motives at $t = 0$ are shut down by the choice of weight ϕ. Another way to shut it down is to directly compensate agents for changes in asset prices. This is enough to shut down redistribution at $t = 0$ since taxes do not change any other prices in this period. Below we show how analysis from Section 3 could be repeated in this setup to reach the same conclusions.

A tax system is a tuple $\mathcal{T} = \{\tau^N, \tau^T, \tilde{\tau}, T^e, T^w\}$. Let $\hat{q}^N$ and $\hat{q}^N$ be the debt prices corresponding to the unregulated equilibrium, the one with no intervention: $\tau^N = \tau^T = \tilde{\tau} = T^w = T^e = 0$. The constraint we impose on the lump-sum transfers is that, for any tax system $\mathcal{T} = \{\tau^N, \tau^T, \tilde{\tau}, T^e, T^w\}$,

$$T^w = p_0 b^N\left[q^N(\mathcal{T}) - \hat{q}^N\right] + b^T\left[q^T(\mathcal{T}) - \hat{q}^T\right] \tag{124}$$
$$T^e = p_0 b^N\left[\hat{q}^N - (1 - \tau^N)q^N(\mathcal{T})\right] + b^T\left[\hat{q}^T - (1 - \tau^T)q^T(\mathcal{T})\right] + \tilde{b}\left[\tilde{q} - (1 - \tilde{\tau})\tilde{q}\right] \tag{125}$$

In words, the planner reimburses agents with the innovations to asset prices that taxes introduce. Since agents do not factor the effect of their decisions on rebates taxes still induce substitution effects. However, since price differences are compensated taxes don't induce income effects. The consumption-saving bundles chosen at $t = 0$ in the unregulated equilibrium are still available under any tax system $\mathcal{T}$. The budget is balanced:

$$T^w + T^e = p_0 b^N \tau^N q^N + b^T \tau^T q^T + \tilde{\tau}\tilde{q} \tag{126}$$

The planner's problem becomes

$$\max \phi\left((C_0^w)^{1-\zeta}+\beta_w\mathbb{E}[(C_1^w)^{1-\sigma}]^{\frac{1-\zeta}{1-\sigma}}\right)+(1-\phi)\left((C_0^e)^{1-\zeta_e}+\beta_e\mathbb{E}[(C_1^e)^{1-\sigma_e}]^{\frac{1-\zeta_e}{1-\sigma_e}}\right) \tag{127}$$

$$\text{s.t. } p_0^\alpha C_0^w = e_0^{w,T}+p_0e_0^{N,w}-p_0\hat{q}^Nb^N-\hat{q}^Tb^T \tag{128}$$

$$p_0^\alpha C_0^e = e_0^{e,T}+p_0e_0^{e,N}+p_0\hat{q}^Nb^N+\hat{q}^Tb^T+\tilde{q}\tilde{b} \tag{129}$$

$$p_1^\alpha C_1^w = e_1^{w,T}+p_1e_1^{w,N}+w_1l+b^T+p_1b^N \tag{130}$$

$$p_1^\alpha C_1^e = e_1^{e,T}+p_1e_1^{e,N}+f(z_1,l)-w_1l-z_1-b^T-p_1b^N-\tilde{b} \tag{131}$$

$$q^T = \beta_w\mathbb{E}\left[\frac{p_0^\alpha}{p_1^\alpha}\frac{(C_1^w)^{-\sigma}}{(C_0^w)^{-\sigma}}\right]\mathbb{E}\left[\frac{(C_1^w)^{1-\sigma}}{(C_0^w)^{1-\sigma}}\right]^{\frac{\sigma-\zeta}{1-\sigma}} \tag{132}$$

$$q^N = \beta_w\mathbb{E}\left[\frac{p_0^{1-\alpha}}{p_1^{1-\alpha}}\frac{(C_1^w)^{-\sigma}}{(C_0^w)^{-\sigma}}\right]\mathbb{E}\left[\frac{(C_1^w)^{1-\sigma}}{(C_0^w)^{1-\sigma}}\right]^{\frac{\sigma-\zeta}{1-\sigma}} \tag{133}$$

$$(1-\tau^T)q^T = \beta_e\mathbb{E}\left[\frac{p_0^\alpha}{p_1^\alpha}\frac{(C_1^e)^{-\sigma_e}}{(C_0^e)^{-\sigma_e}}\cdot(1+\theta^{-1}\delta_1(f_z(z_1,l_1)-1))\right]\mathbb{E}\left[\frac{(C_1^e)^{1-\sigma_e}}{(C_0^e)^{1-\sigma_e}}\right]^{\frac{\sigma_e-\zeta_e}{1-\sigma_e}} \tag{134}$$

$$(1-\tau^N)q^N = \beta_e\mathbb{E}\left[\frac{p_0^{1-\alpha}}{p_1^{1-\alpha}}\frac{(C_1^e)^{-\sigma_e}}{(C_0^e)^{-\sigma_e}}\cdot(1+\theta^{-1}\delta_1(f_z(z_1,l_1)-1))\right]\mathbb{E}\left[\frac{(C_1^e)^{1-\sigma_e}}{(C_0^e)^{1-\sigma_e}}\right]^{\frac{\sigma_e-\zeta_e}{1-\sigma_e}} \tag{135}$$

$$(1-\tau^T)q^T = (1-\tilde{\tau})\tilde{q} \tag{136}$$

$$p_1 = \frac{\alpha}{1-\alpha}\frac{f(z_1,l)-z_1+e_1^{w,T}+e_1^{e,T}-\tilde{b}}{e_1^{w,N}+e_1^{e,N}} \tag{137}$$

$$z_1 = \min\left\{\bar{z},\theta^{-1}(p_1(\bar{b}-b^N)-b^T-\tilde{b})\right\} \tag{138}$$

$$w_1 = f_l(z_1,l) \tag{139}$$

The only difference with the baseline problem is that now the budget constraints in (128), (129), (130), and (131) include $\hat{q}^T$ and $\hat{q}^N$ instead of q^T and q^N. This difference is very important, as $(q^T,q^N,\tau^T,\tau^N,\tilde{\tau})$ now only appear in (132), (133), (134), (135), and (136). We can solve the problem without regard for $(q^T,q^N,\tau^T,\tau^N,\tilde{\tau})$ first, and then set them residually using these equations. The other block of the problem solution, the variables (p_1,z_1,w_1), can be treated exactly the same way as in the baseline.

The advantage of this approach is that we can characterize the marginal benefits of increasing debt for any $\phi\in[0,1]$ without having to track the change in asset prices. The disadvantage is that marginal benefits will depend on the benchmark asset prices $(\hat{q}^T,\hat{q}^N)$.

The next proposition computes these benefits analogously to Proposition 1:

Proposition 3. The net marginal benefits from increasing debt in tradables are given by

$$\frac{\partial\mathcal{W}}{\partial b^T} = \underbrace{\mathbb{E}[\mathcal{U}_1^e\cdot(\mathcal{Z}_1+\theta^{-1}\delta_1)(f_z(z_1,l)-1)]}_{\text{Fisherian amplification}}+\underbrace{\mathbb{E}[(\mathcal{U}_1^w-\mathcal{U}_1^e)\cdot\mathcal{Z}_1(\mathcal{D}_1^pm_1^w+\mathcal{D}_1^wl_1)]}_{\text{endowment revaluation}}$$
$$+\underbrace{[\mathcal{U}_0^w\cdot(q^T-\hat{q}^T)+\mathcal{U}_0^e\cdot(\hat{q}^T-(1-\tau^T)q^T)]}_{\text{portfolio distortion}} \tag{140}$$

The net marginal benefits from increasing debt in non-tradables are given by

$$\frac{\partial \mathcal{W}}{\partial b^N} = \mathbb{E}[\mathcal{U}_1^e \cdot p_1(\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1, l) - 1)] + \mathbb{E}[(\mathcal{U}_1^w - \mathcal{U}_1^e) \cdot p_1 \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l_1)] + p_0[\mathcal{U}_0^w \cdot (q^N - \hat{q}^N) + \mathcal{U}_0^e \cdot (\hat{q}^N - (1 - \tau^N) q^N)] \tag{141}$$

Here $m_1^w = b^N + e_1^{N,w} - c_1^{N,w}$, and marginal utilities are

$$\mathcal{U}_0^w = \phi(1 - \zeta) \frac{(C_0^w)^{-\zeta}}{p_0^\alpha} \tag{142}$$

$$\mathcal{U}_1^w = \beta^w \phi(1 - \zeta) \frac{(C_1^w)^{-\sigma}}{p_1^\alpha} \mathbb{E}\left[(C_1^w)^{1-\sigma}\right]^{\frac{\sigma - \zeta}{1 - \sigma}} \tag{143}$$

$$\mathcal{U}_0^e = (1 - \phi)(1 - \zeta_e) \frac{(C_0^e)^{-\zeta_e}}{p_0^\alpha} \tag{144}$$

$$\mathcal{U}_1^e = \beta^e (1 - \phi)(1 - \zeta_e) \frac{(C_1^w)^{-\sigma_e}}{p_1^\alpha} \mathbb{E}\left[(C_1^w)^{1-\sigma_e}\right]^{\frac{\sigma_e - \zeta_e}{1 - \sigma_e}} \tag{145}$$

The first terms in (140) and (141) are positive if the constraint binds with positive probability. If the weight ϕ is such that $\mathcal{U}_0^w = \mathcal{U}_0^e$, then (140) and (141) collapse to (38) and (40) in Proposition 1.

The main difference between (140) and its analog (38) in Proposition 1 is the last term. Instead of measuring the gap between the private and social value of transferring resources from $t = 0$ to $t = 1$, it now captures the cost of suppressing savings as measured by the change in the market interest rates. If $q^T > \hat{q}^T$, lower interest rates show that there is under-saving from the private perspective of the workers. If $\hat{q}^T > (1 - \tau^T) q^T$, higher after-tax interest rates show that there is under-borrowing from the private perspective of the borrowers.

We now use the expressions for marginal costs of deleveraging to describe the constrained-efficient allocation under $\phi = 1$, with a full focus on the workers, and under $\phi = 0$, focusing on the entrepreneurs. These special case allows for a clear explanation of the economics behind the intervention. Define the appreciation of the domestic currency as $s_1 \equiv p_1/p_0$.

Corollary 2. Under $\phi = 1$, the constrained-efficient allocation satisfies

$$\hat{q}^T - q^T = \mathbb{E}\left[\Lambda^w \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l_1)\right] \tag{146}$$

$$\hat{q}^N - q^N = \mathbb{E}\left[\Lambda^w \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l_1) \cdot s_1\right] \tag{147}$$

Here Λ^w is the pricing kernel of the workers, $\Lambda^w = \mathcal{U}_1^w / \mathcal{U}_0^w$. Under $\phi = 0$, the constrained-efficient allocation satisfies

$$(1 - \tau^T) q^T - \hat{q}^T = \mathbb{E}[\Lambda^e((\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1, l) - 1) - \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l_1))] \tag{148}$$

$$(1 - \tau^N) q^N - \hat{q}^N = \mathbb{E}[\Lambda^e((\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1, l) - 1) - \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l_1)) \cdot s_1] \tag{149}$$

Here Λ^e is the pricing kernel of the entrepreneurs, $\Lambda^e = \mathcal{U}_1^e / \mathcal{U}_0^e$. If the weight ϕ is such that $\mathcal{U}_0^w = \mathcal{U}_0^e$, then the social optimum satisfies (46) and (47) in Corollary 1.

These expressions equate the marginal costs of distorting portfolio choice to the marginal benefits of clearing the balance sheets of excess debt. The marginal costs of portfolio distortion are captured by the difference between the unregulated interest rate and the interest rate the workers ask for at the new debt levels. Lower interest rates, for instance, mean that they under-save from their private perspective. The marginal benefit of intervention is captured by the balance sheet impact of debt on their non-financial income that the workers do not internalize. To see the intuition behind this, consider, as in the baseline model, the true payoff of a foreign currency claim at $t = 1$:

$$\left(\text{true payoff}\right)_{t=1} = \left(\text{claim payout}\right)_{t=1} + \underbrace{\mathcal{Z}_1 \mathcal{D}_1^w l_1 + \mathcal{Z}_1 \mathcal{D}_1^p m_1^w}_{\text{not internalized}} \tag{150}$$

Workers do not take into account the effect of their savings on the wage bill they will receive, as in Bocola and Lorenzoni (2020b). This term is negative when the constraint binds, so workers overestimate the payoff of having a claim maturing in these states. They also do not realize that the input use affects the exchange rate and revalues their net trading position in non-tradables, receipts less expenditures.

If the last terms in (150) are negative and the workers overestimate the payoff of their assets, the planner faces them with lower interest rates and forces them to internalize the balance sheet effects. The difference between the interest rates is exactly the expected discounted marginal effect of debt, as in Korinek (2018). This logic is the same for debt denominated in traded and non-traded goods, although the payoff of the latter is correlated with the exchange rate and loses value in times of depreciation.

The logic is exactly the same in the case $\phi = 0$. The planner changes the interest rates that the entrepreneurs face in accordance with the strength of the effects they do not internalize. If the expressions on the right-hand sides of (148) and (148) are negative, the planner makes sure that the after-tax asset prices $(1 - \tau^T)q^T$ and $(1 - \tau^N)q^N$ are lower than those in the unregulated equilibrium. This means that borrowing is more expensive and the entrepreneurs are forced to incur less liabilities.

Marginal de-dollarization. We now study the benefits of de-dollarization, again setting $\phi = 1$ or $\phi = 0$ for tractability. As in the baseline model, we consider the following perturbation: we increase the holdings of claims to non-traded goods b^N and decrease the holdings of claims to traded goods b^T by the same amount scaled by $\mathbb{E}[p_1]$. This increases the payouts in proportion to p_1 in each state but takes away a non-contingent portion (because foreign currency payouts are constant across states). Moreover, this non-contingent decrease in payouts due to lower b^T is equal to the expected increase due to raising b^N.

Formally, the marginal change in welfare we compute is

$$\Delta^i = \frac{1}{p_0 \mathcal{U}_0^i} \left(\frac{d\mathcal{W}}{db^N} - \mathbb{E}[p_1] \frac{d\mathcal{W}}{db^T} \right), \; i \in \{w, e\} \tag{151}$$

The following proposition is the analog of Proposition 2:

PROPOSITION 4. Suppose $\phi = 1$. Then,

$$\Delta^w = \underbrace{\mathbb{C}\left[\Lambda^w \mathcal{Z}_1 (\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l_1), s_1\right]}_{\text{removing contagion}} - \underbrace{\left[\Delta_{UIP}^w - \hat{\Delta}_{UIP}\right]}_{\text{insurance loss}} + \underbrace{\mathbb{E}\left[s_1 - \hat{s}_1\right] \hat{q}^T}_{\text{revaluation}} \tag{152}$$

Here $\hat{s}_1 = \hat{p}_1/p_0$ stands for appreciation of the domestic currency in the unregulated equilibrium, $\Delta^w_{UIP} = q^T\mathbb{E}[s_1] - q^N$ and $\hat{\Delta}_{UIP} = \hat{q}^T\mathbb{E}[\hat{s}_1] - \hat{q}^N$. Under $\phi = 0$,

$$\Delta^e = \mathbb{C}[\Lambda^e((\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1, l) - 1) - \mathcal{Z}_1(\mathcal{D}^p_1 m^w_1 + \mathcal{D}^w_1 l_1)), s_1] + [\Delta^e_{UIP} - \hat{\Delta}_{UIP}] \\ - \mathbb{E}[s_1 - \hat{s}_1]\hat{q}^T \tag{153}$$

Here $\Delta^e_{UIP} = (1-\tau^T)q^T\mathbb{E}[s_1] - (1-\tau^N)q^N$. If the weight ϕ is such that $\mathcal{U}^w_0 = \mathcal{U}^e_0$, then $\Delta^e = \Delta^w$ and they are both given by (43) in Proposition 2.

The main difference between this result and Proposition 2 is the last term in (152) and (153). This is an artifact of the tax system that effectively transfers resources between $t = 0$ and $t = 1$ at benchmark asset prices $\hat{q}^T$ and $\hat{q}^N$. They do not change to reflect the changes in the distribution of p_1, which creates relative mispricing.

As a result, savers benefit from the strengthening of domestic currency if that is what macroprudential policy leads to.

All other takeaways from Proposition 4 are the same as those from Proposition 2. Benefits to dedollarization stem from making the uninternalized effects of debt less correlated with the exchange rate, which increase welfare if marginal utility is high in times of depreciation. The insurance losses (in the case of workers) and benefits (in the case of entrepreneurs) can be measured by the change in the UIP violation relative to the unregulated equilibrium. Insurance terms in both (152) and (153) are zero when taxes are zero, so this effect is of second order.

B.5 Details on example from Section 3

Recall that the setup for the example assumes that $\theta = 1$ and that $f(z, l)$ is separable over z and l. We also take the following limits: $l \to 0$, $\alpha \to 0$, $e^{N,w}_1 + e^{N,e}_1 \to 0$, $\alpha/(e^{N,w}_1 + e^{N,e}_1) \to 1$. This leads to the following expressions for p_1 and z_1:

$$p_1 = f(z_1, 0) - z_1 + y^T_1 - \tilde{b} \tag{154}$$

$$z_1 = (f(z_1, 0) - z_1 + y^T_1 - \tilde{b})(\bar{b} - b^N) - b^T - \tilde{b} \tag{155}$$

The marginal effect $\mathcal{Z}_1$ and $\mathcal{D}^p_1$ are given by

$$\mathcal{D}^1_p = f_z(z_1, 0) - 1 = \gamma \tag{156}$$

$$\mathcal{Z}_1 = \mathcal{Z}_1(f_z(z_1, 0) - 1)(\bar{b} - b^N) - 1 = -\frac{1}{1 - (f_z(z_1, 0) - 1)(\bar{b} - b^N)} = -\frac{1}{1 - \gamma(\bar{b} - b^N)} \tag{157}$$

The total derivatives of z_1 and p_1 with respect to y^T_1 are

$$\frac{dz_1}{dy_1} = \frac{dz_1}{dy_1}(f_z(z_1, 0) - 1)(\bar{b} - b^N) + (\bar{b} - b^N) = \frac{\bar{b} - b^N}{1 - \gamma(\bar{b} - b^N)} \tag{158}$$

$$\frac{dp_1}{dy_1} = \frac{dz_1}{dy_1}(f_z(z_1, 0) - 1) + 1 = \frac{1}{1 - \gamma(\bar{b} - b^N)} \tag{159}$$

For both of these derivatives to be positive, it is sufficient that $b^N \in \big(\bar{b} - 1/\gamma, \bar{b}\big)$ for all realizations of γ. This can be ensured by setting suitable endowments.

Numerical example. We now show a specific parameterization.

Table 2: Parameters for a numerical example

	Description	Value
σ	risk-aversion of the workers	1
ζ	inverse IES of the workers	1
β_w	discount factor of the workers	1
σ_e	risk-aversion of the entrepreneurs	0
ζ_e	inverse IES of the entrepreneurs	0
β_e	discount factor of the entrepreneurs	1
$f(z,l)$	production function	$2\sqrt{z}+2\sqrt{l}$
$e_1^{T,w}$	tradable endowment of the workers at $t=1$	0
$e_0^{T,w}$	tradable endowment of the workers at $t=0$	4
$e_1^{T,e}(\bar{\epsilon})$	tradable endowment of the entrepreneurs at $t=1$ in high state	1
$e_1^{T,e}(\bar{\epsilon})$	tradable endowment of the entrepreneurs at $t=1$ in low state	0.25
$\pi(\bar{\epsilon})$	probability of the high state	0.5
$\tilde{b}$	supply of foreign investment	0
$\bar{b}$	borrowing limit	1.25

Workers have log utility, entrepreneurs have linear utility, and production function is concave. It can be verified that $(b^N, b^T) = (1, 0)$ is an equilibrium portfolio. Start from the distribution of the exchange rate and input use that it generates.

$$p_1(\epsilon) = 2\sqrt{z_1(\epsilon)} - z_1(\epsilon) + e_1^{T,e}(\epsilon) \tag{160}$$

The unconstrained optimal level of input use is $\bar{z} = 1$. In the high state, the constraint is slack, so $p_1(\bar{\epsilon}) = 2$ and $z_1(\bar{\epsilon}) = 1$. In the low state, the constraint binds, and

$$z_1(\underline{\epsilon}) = (2\sqrt{z_1(\underline{\epsilon})} - z_1(\underline{\epsilon}) + 0.25) \cdot (1.25 - 1) - 0 \tag{161}$$

This quadratic equation has a unique positive solution $z_1(\underline{\epsilon}) = 0.25$, leading to $p_1(\underline{\epsilon}) = 1$. The asset prices are equal to $q^T = \mathbb{E}[f_z(z_1, 0)]$ and $p_0 q^N = \mathbb{E}[f_z(z_1, 0)p_1]$. This leads to $q^T = 1.5$ and $p_0 q^N = 2$, and $C_0^w = 2$ follows from $e_0^{T,w} = 4$ and $(b^N, b^T) = (1, 0)$.

Lastly, we need to verify the workers' Euler equations:

$$q^T = 0.5 \cdot \frac{C_0^w}{C_1^w(\bar{\epsilon})} + 0.5 \cdot \frac{C_0^w}{C_1^w(\underline{\epsilon})} \tag{162}$$

$$p_0 q^N = 0.5 \cdot p_1(\bar{\epsilon}) \frac{C_0^w}{C_1^w(\bar{\epsilon})} + 0.5 \cdot p_1(\underline{\epsilon}) \frac{C_0^w}{C_1^w(\underline{\epsilon})} \tag{163}$$

Since $C_1^w(\epsilon) = p_1(\epsilon) \cdot 1$ for both states, these equations hold. In this equilibrium, it also holds that $b^N \in (\bar{b} - 1/\gamma(\epsilon), \bar{b})$ for both ϵ, so the monotonicity condition is satisfied.

B.6 Proofs

Proof. (of Proposition 3) For a fixed pair (b^T, b^N), let $\mathcal{W}$ be the value of the objective in (127) maximized over $(C_0^w, C_0^e, q^T, q^N, \tau^T, \tau^N, \tilde{\tau})$, and $(C_1^w, C_1^e, w_1, p_1, z_1)$ for any realization of the shock ϵ. Let λ_0^w, λ_0^e, λ_1^w, and λ_1^e be the multipliers on the constraints (128), (129), (130), and (131) respectively. Taking the derivative with respect to C_0^w, C_0^e, C_1^w, and C_1^e,

$$\phi(1-\zeta)(C_0^w)^{-\zeta} = p_0^\alpha \lambda_0^w \tag{164}$$

$$(1-\phi)(1-\zeta_e)(C_0^e)^{-\zeta_e} = p_0^\alpha \lambda_0^e \tag{165}$$

$$\pi_1 \cdot \phi(1-\zeta)\beta^w (C_1^w)^{-\sigma} \mathbb{E}\left[(C_1^w)^{1-\sigma}\right]^{\frac{\sigma-\zeta}{1-\sigma}} = p_1^\alpha \lambda_1^w \tag{166}$$

$$\pi_1 \cdot (1-\phi)(1-\zeta_e)\beta^e (C_1^e)^{-\sigma_e} \mathbb{E}\left[(C_1^e)^{1-\sigma_e}\right]^{\frac{\sigma_e-\zeta_e}{1-\sigma_e}} = p_1^\alpha \lambda_1^e \tag{167}$$

Here π_1 is the probability of the specific realization of ϵ. Taking the derivative of the objective with respect to b^T and b^N that are treated as parameters,

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^T} &= \hat{q}^T(\lambda_0^e - \lambda_0^w) + \sum \left[\frac{\partial P}{\partial z_1}\frac{\partial Z}{\partial b^T}(\lambda_1^w(e_1^{N,w} + b^N - \alpha p_1^{\alpha-1}C_1^w) + \lambda_1^e(e_1^{N,e} - b^N - \alpha p_1^{\alpha-1}C_1^e))\right] \\ &+ \sum \left[\frac{\partial W}{\partial z_1}\frac{\partial Z}{\partial b^T} l(\lambda_1^w - \lambda_1^e)\right] + \sum \left[\frac{\partial Z}{\partial b^T}\lambda_1^e(f_z(z_1, l_1) - 1)\right] + \sum[\lambda_1^w - \lambda_1^e] \end{aligned} \tag{168}$$

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^N} &= \hat{q}^N p_0(\lambda_0^e - \lambda_0^w) + \sum \left[\frac{\partial P}{\partial z_1}\frac{\partial Z}{\partial b^N}(\lambda_1^w(e_1^{N,w} + b^N - \alpha p_1^{\alpha-1}C_1^w) + \lambda_1^e(e_1^{N,e} - b^N - \alpha p_1^{\alpha-1}C_1^e))\right] \\ &+ \sum \left[\frac{\partial W}{\partial z_1}\frac{\partial Z}{\partial b^N} l(\lambda_1^w - \lambda_1^e)\right] + \sum \left[\frac{\partial Z}{\partial b^N}\lambda_1^e(f_z(z_1, l_1) - 1)\right] + \sum[(\lambda_1^w - \lambda_1^e)p_1] \end{aligned} \tag{169}$$

Here the functions $Z(b^T, b^N, \epsilon)$, $P(z_1, b^T, b^N, \epsilon)$, and $W(z_1)$ result from solving the system of (137), (139), and (138). Using the notation $\mathcal{Z}_1$, $\mathcal{D}_1^p$, and $\mathcal{D}_1^W$ for their derivatives, plugging (164), (165), (166), and (167), and using the definitions of $\mathcal{U}_0^w$, $\mathcal{U}_0^e$, $\mathcal{U}_1^w$, and $\mathcal{U}_1^e$,

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^T} &= \hat{q}^T(\mathcal{U}_0^e - \mathcal{U}_0^w) + \mathbb{E}\left[\mathcal{Z}_1\mathcal{D}_1^p(\mathcal{U}_1^w(e_1^{N,w} + b^N - \alpha p_1^{\alpha-1}C_1^w) + \mathcal{U}_1^e(e_1^{N,e} - b^N - \alpha p_1^{\alpha-1}C_1^e))\right] \\ &+ \mathbb{E}\left[\mathcal{Z}_1\mathcal{D}_1^w l(\mathcal{U}_1^w - \mathcal{U}_1^e)\right] + \mathbb{E}\left[\mathcal{U}_1^e\mathcal{Z}_1(f_z(z_1, l_1) - 1)\right] + \mathbb{E}\left[\mathcal{U}_1^w - \mathcal{U}_1^e\right] \end{aligned} \tag{170}$$

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^N} &= \hat{q}^N p_0(\mathcal{U}_0^e - \mathcal{U}_0^w) + \mathbb{E}\left[p_1 \cdot \mathcal{Z}_1\mathcal{D}_1^p(\mathcal{U}_1^w(e_1^{N,w} + b^N - \alpha p_1^{\alpha-1}C_1^w) + \mathcal{U}_1^e(e_1^{N,e} - b^N - \alpha p_1^{\alpha-1}C_1^e))\right] \\ &+ \mathbb{E}\left[p_1 \cdot \mathcal{Z}_1\mathcal{D}_1^w l(\mathcal{U}_1^w - \mathcal{U}_1^e)\right] + \mathbb{E}\left[p_1 \cdot \mathcal{Z}_1\mathcal{U}_1^e(f_z(z_1, l_1) - 1)\right] + \mathbb{E}[p_1 \cdot (\mathcal{U}_1^w - \mathcal{U}_1^e)] \end{aligned} \tag{171}$$

Using the fact that $\alpha p_1^{\alpha-1}C_1^e = c_1^e$, $\alpha p_1^{\alpha-1}C_1^w = c_1^w$, exploiting the market-clearing condition $c_1^w + c_1^e = e_1^{N,w} + e_1^{T,w}$, and denoting $m_1^w = e_1^{N,w} + b^N - c_1^w$,

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^T} &= \hat{q}^T(\mathcal{U}_0^e - \mathcal{U}_0^w) + \mathbb{E}\left[\mathcal{Z}_1\mathcal{D}_1^p(\mathcal{U}_1^w - \mathcal{U}_1^e)m_1^w\right] \\ &+ \mathbb{E}\left[\mathcal{Z}_1\mathcal{D}_1^w l(\mathcal{U}_1^w - \mathcal{U}_1^e)\right] + \mathbb{E}\left[\mathcal{U}_1^e\mathcal{Z}_1(f_z(z_1, l_1) - 1)\right] + \mathbb{E}\left[\mathcal{U}_1^w - \mathcal{U}_1^e\right] \end{aligned} \tag{172}$$

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^N} &= \hat{q}^N p_0(\mathcal{U}_0^e - \mathcal{U}_0^w) + \mathbb{E}\left[p_1 \cdot \mathcal{Z}_1\mathcal{D}_1^p(\mathcal{U}_1^w - \mathcal{U}_1^e)m_1^w\right] \\ &+ \mathbb{E}\left[p_1 \cdot \mathcal{Z}_1\mathcal{D}_1^w l(\mathcal{U}_1^w - \mathcal{U}_1^e)\right] + \mathbb{E}\left[p_1 \cdot \mathcal{Z}_1\mathcal{U}_1^e(f_z(z_1, l_1) - 1)\right] + \mathbb{E}[p_1 \cdot (\mathcal{U}_1^w - \mathcal{U}_1^e)] \end{aligned} \tag{173}$$

Using (132), (133), (134), and (135), we can substitute

$$\mathbb{E}\left[\mathcal{U}_1^w\right] = q^T \mathcal{U}_0^w \tag{174}$$

$$\mathbb{E}\left[\mathcal{U}_1^e(1+\theta^{-1}(f_z(z_1,l_1)-1))\right] = (1-\tau^T) q^T \mathcal{U}_0^e \tag{175}$$

$$\mathbb{E}\left[p_1 \mathcal{U}_1^w\right] = q^N p_0 \mathcal{U}_0^w \tag{176}$$

$$\mathbb{E}\left[p_1 \mathcal{U}_1^e(1+\theta^{-1}(f_z(z_1,l_1)-1))\right] = (1-\tau^N) p_0 q^N \mathcal{U}_0^e \tag{177}$$

Plugging this,

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^T} &= \mathcal{U}_0^e(\hat{q}^T - (1-\tau^T)q^T) + \mathcal{U}_0^w(q^T - \hat{q}^T) + \mathbb{E}\left[(\mathcal{U}_1^w - \mathcal{U}_1^e)\mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\right] \\ &\quad + \mathbb{E}\left[\mathcal{U}_1^e(\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l_1)-1)\right] \end{aligned} \tag{178}$$

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b^N} &= p_0\mathcal{U}_0^e(\hat{q}^N - (1-\tau^N)q^N) + p_0\mathcal{U}_0^w(q^N - \hat{q}^N) + \mathbb{E}\left[p_1 \cdot (\mathcal{U}_1^w - \mathcal{U}_1^e)\mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\right] \\ &\quad + \mathbb{E}\left[p_1 \cdot \mathcal{U}_1^e(\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l_1)-1)\right] \end{aligned} \tag{179}$$

Finally, to see that Proposition 1 is nested, assume the weights are chosen such that $\mathcal{U}_0^w = \mathcal{U}_0^e$. Then, there is a number $\mathcal{U}_0 = \mathcal{U}_0^w = \mathcal{U}_0^e$ such that we can write $\Lambda^w = \mathcal{U}_1^w/\mathcal{U}_0$ and $\Lambda^e = \mathcal{U}_1^e/\mathcal{U}_0$. Using this,

$$\begin{aligned}\frac{1}{\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^T} &= \hat{q}^T - (1-\tau^T)q^T + q^T - \hat{q}^T + \mathbb{E}\left[(\Lambda^w - \Lambda^e)\mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\right] \\ &\quad + \mathbb{E}\left[\Lambda^e(\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l_1)-1)\right] \end{aligned} \tag{180}$$

$$\begin{aligned}\frac{1}{p_0\mathcal{U}_0}\frac{\partial \mathcal{W}}{\partial b^N} &= \hat{q}^N - (1-\tau^N)q^N + q^N - \hat{q}^N + \mathbb{E}\left[\frac{p_1}{p_0} \cdot (\Lambda^w - \Lambda^e)\mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\right] \\ &\quad + \mathbb{E}\left[\frac{p_1}{p_0} \cdot \Lambda^e(\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l_1)-1)\right] \end{aligned} \tag{181}$$

Cancelling out $\hat{q}^T$ and $\hat{q}^N$, this recovers (38) and (40) in Proposition 1. □

Proof. (of Corollary 2) First, suppose $\phi = 1$. This means $\mathcal{U}_0^e = \mathcal{U}_1^e = 0$. Plugging this into the expressions in Proposition 3,

$$\hat{q}^T - q^T = \mathbb{E}[\Lambda^w \cdot \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)] \tag{182}$$

$$p_0(\hat{q}^N - q^N) = \mathbb{E}[p_1\Lambda^w \cdot \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)] \tag{183}$$

Here $\Lambda^w = \mathcal{U}_1^w/\mathcal{U}_0^w$. With $p_1/p_0 = s_1$, this leads to the first two expressions in Corollary 2. Now supposing $\phi = 0$ and plugging $\mathcal{U}_0^w = \mathcal{U}_1^w = 0$ instead,

$$(1-\tau^T)q^T - \hat{q}^T = \mathbb{E}[\Lambda^e \cdot (\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l)-1)] - \mathbb{E}[\Lambda^e \cdot \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)] \tag{184}$$

$$p_0((1-\tau^N)q^N - \hat{q}^N) = \mathbb{E}[p_1\Lambda^e \cdot (\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l)-1)] - \mathbb{E}[p_1\Lambda^e \cdot \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)] \tag{185}$$

Here $\Lambda^e = \mathcal{U}_1^e/\mathcal{U}_0^e$. With $p_1/p_0 = s_1$, this leads to the last two expressions in Corollary 2.

Finally, recovering Corollary 1 requires using the fact that the suitable weight ϕ collapses the expressions in Proposition 3 to those in Proposition 1. Corollary 1 follows immediately. □

Proof. (of Proposition 4) With $\phi = 1$, $\mathcal{U}_0^e = \mathcal{U}_1^e = 0$. Plugging this into Proposition 3,

$$\begin{aligned}
\frac{1}{p_0 \mathcal{U}_0^w}\left(\frac{\partial \mathcal{W}}{\partial b^N} - \mathbb{E}[p_1]\frac{\partial \mathcal{W}}{\partial b^T}\right) &= \mathbb{E}[\Lambda^w \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)\cdot s_1] - \mathbb{E}[\Lambda^w \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)]\cdot \mathbb{E}[s_1] \\
&+ (q^N - \mathbb{E}[s_1]q^T) - (\hat{q}^N - \mathbb{E}[s_1]\hat{q}^T) \\
&= \mathbb{C}[\Lambda^w \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l), s_1] - (\mathbb{E}[s_1]q^T - q^N) + (\mathbb{E}[\hat{s}_1]\hat{q}^T - \hat{q}^N) \\
&+ \hat{q}^T(\mathbb{E}[s_1] - \mathbb{E}[\hat{s}_1]) \qquad (186)
\end{aligned}$$

Here $\Lambda^w = \mathcal{U}_1^w/\mathcal{U}_0^w$. Denoting $\Delta_{UIP}^w = \mathbb{E}[s_1]q^T - q^N$ and $\hat{\Delta}_{UIP} = \mathbb{E}[\hat{s}_1]\hat{q}^T - \hat{q}^N$ leads to the first expression in Proposition 4.

Under $\phi = 0$, $\mathcal{U}_0^w = \mathcal{U}_1^w = 0$. Plugging this into the expressions from Proposition 3,

$$\begin{aligned}
\frac{1}{p_0 \mathcal{U}_0^e}\left(\frac{\partial \mathcal{W}}{\partial b^N} - \mathbb{E}[p_1]\frac{\partial \mathcal{W}}{\partial b^T}\right) &= \mathbb{E}[\Lambda^e((\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l) - 1) - \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l))\cdot s_1] \\
&- \mathbb{E}[\Lambda^e((\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l) - 1) - \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l))]\cdot \mathbb{E}[s_1] \\
&- ((1-\tau^N)q^N - \mathbb{E}[s_1](1-\tau^T)q^T) + (\hat{q}^N - \mathbb{E}[s_1]\hat{q}^T) \\
&= \mathbb{C}[\Lambda^e((\mathcal{Z}_1 + \theta^{-1}\delta_1)(f_z(z_1,l) - 1) - \mathcal{Z}_1(\mathcal{D}_1^p m_1^w + \mathcal{D}_1^w l)), s_1] \\
&+ (\mathbb{E}[s_1](1-\tau^T)q^T - (1-\tau^N)q^N) - (\mathbb{E}[\hat{s}_1]\hat{q}^T - \hat{q}^N) \\
&- \hat{q}^T(\mathbb{E}[s_1] - \mathbb{E}[\hat{s}_1]) \qquad (187)
\end{aligned}$$

Here $\Lambda^e = \mathcal{U}_1^e/\mathcal{U}_0^e$. Denoting $\Delta_{UIP}^e = (1-\tau^T)q^T\mathbb{E}[s_1] - (1-\tau^N)q^N$ and $\hat{\Delta}_{UIP} = \mathbb{E}[\hat{s}_1]\hat{q}^T - \hat{q}^N$ leads to the second expression in Proposition 4.

Finally, recovering Proposition 2 requires using the fact that the suitable weight ϕ collapses the expressions in Proposition 3 to those in Proposition 1. Proposition 2 then follows through the step in its proof. □

C Details for numerical illustration

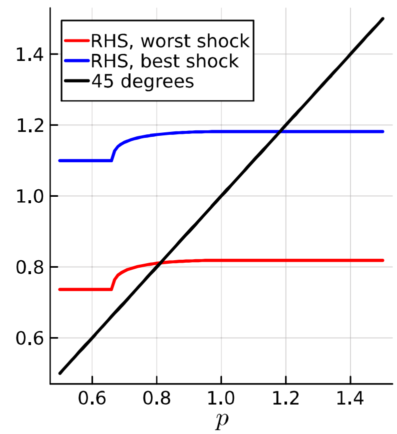
One concern with the quantification is that there could be multiple equilibria at $t = 1$: a low value of p results in low z that confirms that p, but if p had been higher, z would have increased enough to justify that value of p. Mathematically, this would mean that $z_1 = p(z_1, \epsilon)(\bar{b} - b^N) - b^T - \tilde{b}$ has two solutions. Alternatively, if z_1 is treated as a function $\hat{z}(p_1, b^T, b^N, \tilde{b})$,

$$p_1 = \frac{\alpha}{1-\alpha}\frac{f(\hat{z}(p_1, b^T, b^N, \tilde{b}), l) - \hat{z}(p_1, b^T, b^N, \tilde{b}) + e_1^{T,w} + e_1^{T,e} - \tilde{b}}{e_1^{N,w} + e_1^{N,e}} \qquad (188)$$

has multiple solutions p_1.

Figure 4 shows this is not the case in our calibration. Each colored line shows the right hand side of (188) as a function of p_1 given a realization of $\epsilon = (e_1^{N,w}, e_1^{N,e})$. A product market equilibrium is where this line crosses the 45-degree line. The colored lines are increasing as long as the borrowing constraint of firms is binding. The red line shows the right hand side of (188) for a lowe realization of the tradable endowment. Parameters are picked in a way that makes the constraint bind whenever $p_1 < 1$. The right-hand side (RHS) starts bending down to the left of that, since p_1 determines $\hat{z}$, and the line inherits concavity of $f(z, l) - z$.

Figure 4: Uniqueness of the equilibrium



We use the same parameters in Section 4. Table 3 lists them.

Table 3: Parameters for numerical illustration from Section 4

	Description	Value
$f(z,l)$	production function	$\left(\eta z^{1-1/\rho}+(1-\eta)l^{1-1/\rho}\right)^{\frac{\rho}{\rho-1}}$
η	weight on z in production	0.5
ρ	elasticity of substitution between z and l	1.25
σ	risk-aversion of the workers	2.825
ζ	inverse IES of the workers	0.6
β_w	discount factor of the workers	1
σ_e	risk-aversion of the entrepreneurs	0
ζ_e	inverse IES of the entrepreneurs	0
β_e	discount factor of the entrepreneurs	0.824
α	consumption weight on non-tradables	0.5
$e_1^{T,w}$	workers' traded endowment of at $t=1$	0
$e_0^{T,w}$	workers' traded endowment of at $t=0$	4.582
$e_1^{N,w}$	workers' non-traded endowment at $t=1$	4
$e_0^{N,w}$	workers' non-traded endowment at $t=0$	0
$[e_1^{T,e}(\underline{\epsilon}), e_1^{T,e}(\overline{\epsilon})]$	entrepreneurs' traded endowments at $t=1$	[2.278, 3.367]
$e_1^{N,e}$	entrepreneurs' non-traded endowment at $t=1$	0
p_0	exchange rate at $t=0$	0.959
$\tilde{b}$	supply of foreign investment	0.1
$\bar{b}$	borrowing limit	1.301

The distribution of entrepreneurs' endowments at $t=1$ is piecewise uniform. It is a rescaling transformation of the distribution defined on $[0,1]$ with density 1.5 on $[0,0.5]$ and 0.5 on $(0.5,1]$, meaning that the left half of the support has a mass three times larger than the right half.

The choice of parameters is motivated by the following targets:

- $r^T = 1/(1+q^T) = 0.05$, a 5% interest rate on dollar debt
- $(b^T, b^N) = (0.3, 0.7)$, a 30% dollarization rate of savings with a normalization $b^T + b^N = 1$
- $r^N - r^T - \mathbb{E}[p_1/p_0 - 1] = 0.03$, a 3% UIP deviation
- $p_1 = 1$ as the minimal price level under which the constraint is slack
- $p_1 = 1$ as the equilibrium exchange rate when $e_1^{T,e}$ is in the middle of its support
- the constraint binding 75% of the time

The first three targets add realism in the aspects we attach importance to. The last three we choose for convenience.

We pick the lower bound of the support of $e_1^{T,e}(\epsilon)$ so that if the support was 33% wider, at the lowest point there would be exactly two solutions for (188). This is a safety measure ensuring that the economy does not exhibit multiple equilibria even for some debt levels above $(b^N, b^T, \tilde{b}) = (0.7, 0.3, 0.1)$ (the red line in Figure 4 does not move down too much) and our search algorithm always has a unique solution for the exchange rate.

D Borrowing constraint micro-fundation

Seizable endowments. In the main text, we assumed that borrowers are subject to an intra-day borrowing constraint that limits how much of the tradable input they can buy:

$$\theta z_t + \tilde{b}_t + b_t^T + p_t b_t^N \leq p_t \bar{b}$$

This can be derived from the following micro-foundations. Assume that after production is done but before debt repayment and consumption take place, borrowers have the option to default on any part of their debt. In that case, the only cost is that the 'bank' seizes a fixed amount of entrepreneur's non-tradable endowment, $\bar{y}^{N,e}$. It follows that if they default, entrepreneurs would default on the total amount of their debt and that they only consider the impact of defaulting on current income. For them not to default, it has to be that

$$\theta z_t + \tilde{b}_t + b_t^T + p_t b_t^N \leq p_t \bar{y}^N$$

which maps directly into the borrowing constraint we have assumed. What we are trying to capture by assuming that the bank seizes $\bar{y}^N$ is that the bank takes part of the capital goods or real estate in the power of entrepreneurs for a period of time and appropriates the rents associated with it. Because we do not have capital in the model, we focus on the non-tradable part of endowments. This microfoundation requires assuming $\bar{y}^N < y_t^{N,e}$ for all t.

Local capital. Another way to microfound the borrowing constraint is to assume that entrepreneurs operate locally traded capital (land or immovable structures) that the financial intermediaries can seize and turn into non-traded goods. The entrepreneurs' production function is $f(z_t, l_t) = F(z_t, l_t; k_t)$, and the supply of capital is fixed at $k_t = \bar{b}$. If seized, this capital is sold as the non-traded good at price p_t.

Suppose the financial intermediaries guarantee workers' savings. Then, if entrepreneurs default on debt, the losses of intermediaries are $\theta z_t + \tilde{b}_t + b_t^T + p_t b_t^N - p_t \bar{b}$. If the intermediaries cannot accept any possibility of losses, they have to ensure

$$\theta z_t + \tilde{b}_t + b_t^T + p_t b_t^N \leq p_t \bar{b}$$

This formulation puts no restrictions on non-traded endowments of entrepreneurs.

E Decomposition of welfare changes

We conduct the following procedure to decompose welfare changes into efficiency gains, redistribution, and risk-sharing terms. Observe that the worker's value is determined by the following variables: vectors $\boldsymbol{p}$ and $\boldsymbol{w}$ of exchange rate and wage realizations corresponding to realizations of the shock to tradable endowments, taxes $\boldsymbol{\tau} = (\tau^T, \tau^N)$, asset prices $\boldsymbol{q} = (q^N, q^T)$, and portfolio $\boldsymbol{b} = (b^N, b^T)$. The entrepreneur's value is determined by the same vectors and the vector of traded input realizations $\boldsymbol{z}$. Notice that the worker's value only depends on $\boldsymbol{z}$ indirectly through wages and exchange rates. With a slight abuse of notation, let $\mathcal{V}^w(\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{b}, \boldsymbol{q}, \boldsymbol{\tau})$ be the value of the worker given these vectors. Similarly, let $\mathcal{V}^e(\boldsymbol{z}, \boldsymbol{p}, \boldsymbol{w}, \boldsymbol{b}, \boldsymbol{q}, \boldsymbol{\tau})$ be the value of the entrepreneur.

The welfare changes for these agents can be computed as

$$\Delta^w = \mathcal{V}^w(\boldsymbol{p}^{opt}, \boldsymbol{w}^{opt}, \boldsymbol{b}^{opt}, \boldsymbol{q}^{opt}, \boldsymbol{\tau}^{opt}) - \mathcal{V}^w(\boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{189}$$

$$\Delta^e = \mathcal{V}^e(\boldsymbol{z}^{opt}, \boldsymbol{p}^{opt}, \boldsymbol{w}^{opt}, \boldsymbol{b}^{opt}, \boldsymbol{q}^{opt}, \boldsymbol{\tau}^{opt}) - \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{190}$$

Here $\boldsymbol{p}^{eqm}$, $\boldsymbol{w}^{eqm}$, and $\boldsymbol{z}^{eqm}$ are vectors of the exchange rate, wages, and input use in the unregulated equilibrium. The unregulated portfolio and asset prices are $\boldsymbol{b}^{eqm}$ and $\boldsymbol{q}^{eqm}$, and taxes are zero. Similarly, $\boldsymbol{p}^{opt}$, $\boldsymbol{w}^{opt}$, and $\boldsymbol{z}^{opt}$ are vectors of the exchange rate, wages, and input use in the social optimum under taxes $\boldsymbol{\tau}^{opt}$ with portfolio $\boldsymbol{b}^{opt}$ and prices $\boldsymbol{q}^{opt}$.

We compute counterfactual, off-equilibrium changes in welfare that would be caused by each vector changing from the unregulated equilibrium to the optimum separately. We define efficiency gains as

$$\Delta^{\text{efficiency},e} = \mathcal{V}^e(\boldsymbol{z}^{opt}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) - \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{191}$$

Here we hold all distributions except that for input use $\boldsymbol{z}$ at the unregulated equilibrium. This term represents efficiency gains because it reflects changes in resources available to the economy as a whole when the optimal policy is implemented. Similarly, welfare changes associated with redistribution result from changes in $\boldsymbol{p}$ and $\boldsymbol{w}$, since these are prices of goods exchanged internally within the country:

$$\Delta^{\text{wage},w} = \mathcal{V}^w(\boldsymbol{p}^{eqm}, \boldsymbol{w}^{opt}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) - \mathcal{V}^w(\boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{192}$$

$$\Delta^{\text{wage},e} = \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{opt}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) - \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{193}$$

$$\Delta^{\text{exchange rate},w} = \mathcal{V}^w(\boldsymbol{p}^{opt}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) - \mathcal{V}^w(\boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{194}$$

$$\Delta^{\text{exchange rate},e} = \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{opt}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) - \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{195}$$

Changes coming from portfolio distortions illustrate changes in risk-sharing:

$$\Delta^{\text{risk-sharing},w} = \mathcal{V}^w(\boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{opt}, \boldsymbol{q}^{opt}, \boldsymbol{\tau}^{opt}) - \mathcal{V}^w(\boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{196}$$

$$\Delta^{\text{risk-sharing},e} = \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{opt}, \boldsymbol{q}^{opt}, \boldsymbol{\tau}^{opt}) - \mathcal{V}^e(\boldsymbol{z}^{eqm}, \boldsymbol{p}^{eqm}, \boldsymbol{w}^{eqm}, \boldsymbol{b}^{eqm}, \boldsymbol{q}^{eqm}, 0) \tag{197}$$

This decomposition is not exactly additive, as partial welfare changes do not add up to the total Δ^w and Δ^e. The addition discrepancy is only about $3-4\%$ though, which is low enough to make this decomposition useful. The results are in Table 4.

The welfare of the workers increases relative to the unregulated equilibrium, while that of the entrepreneurs decreases: $\Delta^w > 0$ and $\Delta^e < 0$. Welfare gains for workers mostly come from wages, while insurance properties of their portfolio deteriorate relative to the unregulated equilibrium. Welfare losses of entrepreneurs also come from wages and the exchange rates, and efficiency gains offset around a third of those. A less dollarized portfolio allows for more production of tradables. The impact of risk-sharing on entrepreneurs is small because they are risk-neutral in our example.

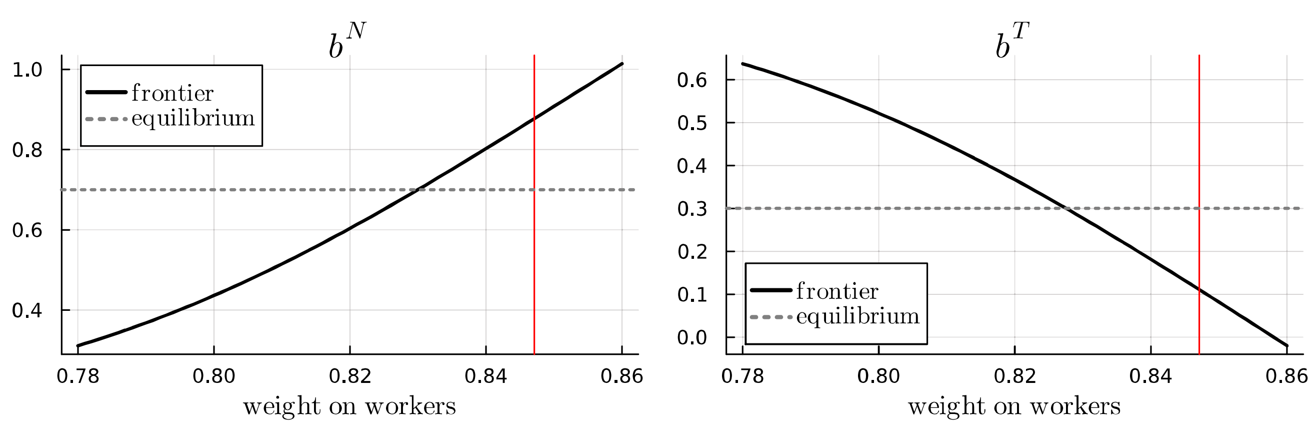
Table 4: decomposition of welfare increase for workers and entrepreneurs.

$\Delta^{\text{efficiency},w}/\Delta^w$	$\Delta^{\text{wage},w}/\Delta^w$	$\Delta^{\text{exchange rate},w}/\Delta^w$	$\Delta^{\text{risk-sharing},w}/\Delta^w$
0	116%	19%	−39%
$\Delta^{\text{efficiency},e}/\Delta^e$	$\Delta^{\text{wage},e}/\Delta^e$	$\Delta^{\text{exchange rate},e}/\Delta^e$	$\Delta^{\text{risk-sharing},e}/\Delta^e$
−34%	111%	17%	3%

F Pareto improvements and Pareto frontier

Pareto frontier can be traced out numerically. The social optimum we characterize in Section 3 is one point on the Pareto frontier that corresponds to a specific weight on workers. Figure 5 shows the optimal portfolio as a function of the weight the planner puts on workers. Dollarization of the savings portfolio decreases in the workers' weight.

Figure 5: Optimal debt as a function of the weight on workers. Red lines show the weight generating the social optimum from Section 3. Dashed lines show debt from the unregulated equilibrium.



We next deal with another point on the Pareto frontier. Specifically, we find the Pareto improvement that maximizes the workers' value. By definition, it leaves entrepreneurs exactly as well-off as the unregulated equilibrium.

We find that the space of Pareto improvements is quite narrow in our model. Dynamic externalities are tightly related to redistribution, Policies that benefit workers do so through increasing wages and strengthening the exchange rate since workers are net sellers of non-tradables. These gains are losses for entrepreneurs.

In the Pareto improvement that we consider, aggregate debt is lower and more dollarized. The expected repayment at $t = 1$ is 0.96 compared to 1.0 in the unregulated equilibrium, and dollarization is 51% compared to 30%. Repeating the analysis from Appendix E, we find that gains to workers come from redistribution through wages and exchange rates. Entrepreneurs take losses on wages and exchange rates, but these losses are compensated by making their portfolios more profitable. Table 5 presents the decomposition of welfare gains and losses analogous to that in Appendix E. The only difference is that here we divide by the value in the unregulated equilibrium to avoid division by zero in the entrepreneurs' case (their welfare change is zero).

Table 5: decomposition of welfare increase for workers and entrepreneurs.

$\Delta^{\text{efficiency},w}/\mathcal{V}^w$	$\Delta^{\text{wage},w}/\mathcal{V}^w$	$\Delta^{\text{exchange rate},w}/\mathcal{V}^w$	$\Delta^{\text{risk-sharing},w}/\mathcal{V}^w$
0	0.026%	0.003%	−0.027%
$\Delta^{\text{efficiency},e}/\mathcal{V}^e$	$\Delta^{\text{wage},e}/\mathcal{V}^e$	$\Delta^{\text{exchange rate},e}/\mathcal{V}^e$	$\Delta^{\text{risk-sharing},e}/\mathcal{V}^e$
0.033%	−0.18%	−0.017%	0.162%

We assess the strength of externalities at this Pareto improvement by computing the marginal benefits of decreasing the two types of debt and de-dollarization as in Section 4. Table 6 presents them alongside the values at the unregulated equilibrium and the social optimum.

Table 6: marginal benefits of intervention (in percentage points).

	$\mathcal{F}^T$	$\mathcal{F}^N$	$\mathcal{F}^\Delta$	$\mathcal{R}^T$	$\mathcal{R}^N$	$\mathcal{R}^\Delta$
unregulated equilibrium	0.77	0.68	0.08	10.03	9.10	0.88
Pareto improvement	0.94	0.82	0.11	8.57	7.69	0.83
social optimum	0.10	0.08	0.01	7.54	6.86	0.64

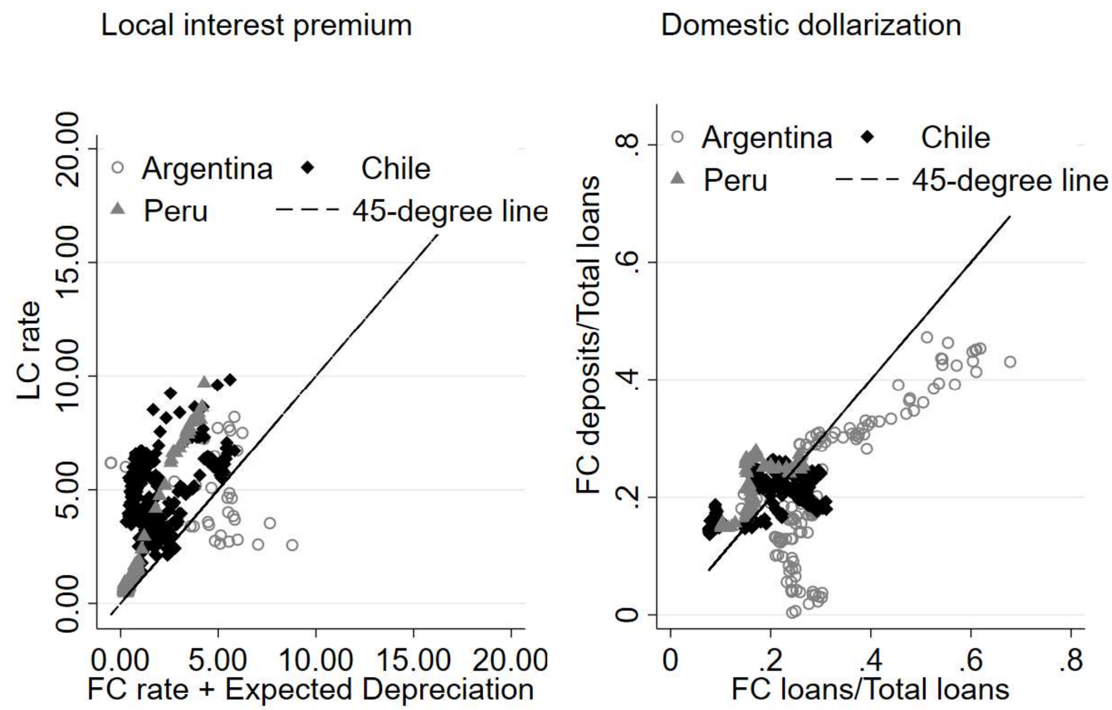
The marginal benefits $\mathcal{R}^T$, $\mathcal{R}^N$, and $\mathcal{R}^\Delta$ coming from redistributive motives are lower in the Pareto improvement case than in the unregulated equilibrium. It means that by implementing this Pareto improvement, the planner goes some way towards the optimum in terms of redistribution. In contrast, the marginal benefits $\mathcal{F}^T$, $\mathcal{F}^N$, and $\mathcal{F}^\Delta$ coming from decreasing amplification are higher than in the unregulated equilibrium. This indicates that the planner has to increase portfolio dollarization to make borrowing cheaper for entrepreneurs as compensation.

G Own calculations of standard facts and data sources

We revisit two well-known stylized facts related to internal financial dollarization. These are quoted in Bocola and Lorenzoni (2020a) and guide their modelling assumptions. We follow a similar approach to theirs, while also highlighting how our third fact appears in our model. These facts also set calibration targets for Section 4.

All our calculations use data for the period 2000-2018, while in some cases the time windows are shorter due to data availability. Data for our first two facts comes from national central banks.

Figure 6: Stylized facts about internal dollarization



Source: National Central Banks.

Fact 1: The domestic interest rate for foreign currency deposits is lower than that for local currency deposits after adjusting for expected depreciation.

The first panel in Figure 6 shows monthly data for passive interest rates for local currency instruments against the interest rate for comparable foreign currency instruments for Argentina, Chile, and Peru. To express everything in local currency we add expected annual depreciation to the interest rate for foreign currency instruments. The main takeaway is that households demand higher interest rates when saving in local currency than when saving in foreign currency, as can be seen from most data points lying above the 45 degree line.

Christiano et al. (2021) perform a similar analysis including more years and countries and arrive at the same conclusion. Compared to their analysis, we focus on saving instruments used by households and not on deposits more generally. We keep instruments with maturities of at least one year. The common reading of this premium is that households are willing to accept lower rates on foreign currency savings because these instruments have insurance properties that local currency savings do not. Gutierrez et al. (2021) provide support for this interpretation using detailed data from Peru. Bocola and Lorenzoni (2020a) also incorporate this channel.

Fact 2: The share of household deposits denominated in foreign currency matches the share of firm liabilities denominated in foreign currency.

The second panel of Figure 6 shows monthly data for the share of household deposits that are denominated in foreign currency against the share of loans to firms denominated in foreign currency (both are stock variables). It is clear from the figure that domestic banks match deposits denominated in dollars to loans denominated in dollars. As discussed in Christiano et al. (2021), this pattern is partly driven by regulation requiring domestic banks to match the currency composition of their balance sheets. The main takeaway is that domestic households' demand for assets denominated in foreign currency is largely provided by domestic firms. In our analysis, this fact motivates modelling firms as borrowing directly from households.

Data sources. For Figure 1, we use data from the IMF Macroprudential Policies Database. We code a binary variable that takes value 1 if a country in a given year has tightened their LFC or LFX positions, and then sum across countries.

In Figure 2 we use data from the Argentinean, Chilean and Peruvian Central Banks. Regarding rates, our goal is to gather the rates on comparable instruments for which there are options both in local and foreign currency. Our decision over which instrument to pick is guided by the model and what we want to capture, the savings of representative households. In all cases we use expected depreciation of the domestic currency from survey data provided by the same central banks.

For Argentina, when calculating deposit dollarization we keep deposits made by natural persons from the private sector. When calculating loans we only keep loans taken by legal persons (firms) net of mortgages and car loans. For interest rates, we keep passive rates on time deposits of maturities higher than 60 days. We net out expected depreciation one year ahead the month we are considering. The Argentinean data for interest rates starts in 2004. The period between 2007 and 2015 included intervention of the statistics institute and stark control on capital outflows, making the observations for rates during this period not so useful for our purposes. We therefore drop these years.

For Chile, when calculating deposit dollarization and loans we keep deposits with maturity longer than a month (because for foreign currency deposits we do not have a finer disaggregation) and commercial loans. For passive interest rates from deposits we consider maturities from 1 to 3 years. We keep all years between 2001 and 2019.

For Peru, we keep deposits and loans for the private sector. For interest rates, we keep passive interest rate for 'savings'. We do not use data for longer maturities because they start only from 2010.