Heterogeneous Impact of the Global Financial Cycle*

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Abstract

I develop a heterogeneous-country model of the world economy to study the distributional impact of aggregate capital flight episodes. A global intermediary borrows from all countries and invests in their risky assets. Wealth heterogeneity between countries arises endogenously due to idiosyncratic shocks. A single global factor that combines the intermediary's wealth and risk-taking capacity determines capital inflows and risk premia in every country. A shock to the intermediary's risk-taking capacity generates global capital flight. Investors from rich countries use their external savings to replace foreign demand for domestic assets. These countries experience a "retrenchment" event: a sizable fall in outward flows. Their risky assets appreciate on impact. In poor countries, investors cannot replace foreign demand without a sharp rise in risk premia. Their asset markets adjust through prices rather than quantities, and prices fall. Estimating the model, I find that global financial shocks explain a quarter of the time-series variation in aggregate capital flows and a third of the variation in the relative performance of assets in advanced economies compared to emerging markets.

Key Words: capital flows, risk premium, global financial cycle, heterogeneity, retrenchment

JEL Classification Numbers: F32, F44, G12, G15

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1 Introduction

There is an aggregate cycle in international capital flows and asset prices. One factor explains 20% of the variation in gross capital flows across the world. This factor is strongly correlated with an equally powerful dominant factor in asset prices and various measures of global risk-taking capacity. In booms, when asset prices are high, investors tend to accumulate foreign holdings. In global downturns, they sell foreign assets and shift their portfolios toward domestic markets, which is what the literature calls "retrenchment".

Countries are not equally exposed to the global cycle. Emerging markets and developing economies are especially strongly affected by changes in the risk appetite of foreign investors. Asset prices are generally more volatile in these countries. At the same time, gross capital flows tend to be more strongly correlated with global aggregates in advanced economies. In downturns, investors from advanced economies sell more of their foreign assets than those from emerging markets, responding to negative shocks with more active retrenchment.

I construct a dynamic multiple-country model with a global financial cycle to interpret these patterns. The model jointly determines asset prices and gross capital flows in global equilibrium, generating international wealth distribution and a cross-section of country-specific risk premia. Motivated by a strong correlation between aggregate capital flows and measures of global risk appetite, I center the model around a global intermediary that trades assets in all countries. My main experiment focuses on capital flight episodes triggered by shocks to their risk-taking capacity. The objective of the model is to explain cross-country heterogeneity in the impact of these episodes. To this end, I propose a new mechanism that creates this heterogeneity endogenously through the behavior of local investors who interact with the intermediary.

This mechanism relies on tracking gross flows during capital flight events. The response of asset prices in these events depends on the wealth of local investors. In rich countries, they readily buy domestic assets using their large external holdings. This keeps local risk premia from rising, while the country's external assets and liabilities fall at the same time. In poor countries, local investors cannot absorb domestic assets as much, so risk premia rise to keep foreign investors from selling. Their asset markets adjust through prices rather than quantities. As a result, shocks to the risk-taking capacity of global intermediaries generate a rise in risk premia that is concentrated in poor countries, while retrenchment happens in rich ones. Capital flight events end up being driven by agents in rich countries selling their foreign assets to buy domestic ones.

The main equilibrium outcome is that risky assets in rich countries appreciate in these downturns, effectively acting as safe assets. The reason is that negative shocks to the risk-taking capacity of intermediaries lead to a fall in the global interest rate. Risk premia on assets in rich countries stay low due to retrenchment by local investors, so the fall in the interest rate raises their prices. In poor countries, the rise in risk premia dominates, and prices fall. This way, accounting for gross capital flows and the dynamics of short-term rates in the model leads to different signs of exposure to global shocks in fundamentally symmetric countries.

I use the following ingredients to illustrate this mechanism. There is a continuum of countries. In each one, there is an asset with idiosyncratic shocks to payoffs and a local agent who holds this asset and saves abroad, buying bonds issued by a global intermediary. The intermediary is based in a special country, interpreted as the US. It borrows from local agents and holds risky assets in all countries. Markets for risky assets are segmented: local agents can only invest in other countries through the intermediary. They clear country by country. The other two markets, for intermediary's bonds and for the single consumption good, clear globally.

The intermediary's risk-taking capacity is limited. Even though it has access to a continuum of assets with uncorrelated payoffs, it cannot take advantage of full diversification. The reason is that it is unsure about the right model for country-specific shocks and takes a cautious approach to investing, considering worst-case scenarios. This behavior reduces to a simple value-at-risk constraint, making the intermediary treat idiosyncratic risk as a real concern and giving it a separate risk-return trade-off for every country in its portfolio. As a result, its risk-taking capacity can be summarized by one preference parameter that varies over time, driving global dynamics.

Since the intermediary cannot fully absorb country-specific shocks, risk-sharing between countries is incomplete. This has two main implications. First, assets offer positive risk premia, and the intermediary makes profits. This affords the special country, where it is based, an "exorbitant privilege", allowing it to finance perpetual trade deficits even if it is in debt. Second, exposure to domestic idiosyncratic shocks creates a wealth distribution between countries. There is a cross-section of excess returns at any point in time, and countries continually move around the distribution due to good or bad dividend spells.

Due to market segmentation, asset prices in all countries depend on wealth accumulated by local agents. In rich countries, risk premia are low. They converge to zero in the limit of infinite wealth since any positive excess returns would create infinite demand from extremely rich domestic investors. Agents in these countries allocate a low share of their wealth to domestic risk, relying on safe external holdings. Despite that, they take over the home country's market due to their size. Foreign investors are not very important. In contrast, in poor countries, assets pay high risk premia, and local investors hold a small fraction of the market. These countries are more exposed to the intermediary's net worth and risk appetites.

When a negative shock hits the intermediary's risk-taking capacity, it seeks to sell risky assets in all countries simultaneously. In rich countries, local investors buy sizeable quantities without a large increase in excess returns. There are trades, with gross inflows and outflows offsetting each other. In poor countries, there is not enough wealth to do this, and excess returns have to rise to convince the intermediary to sell less. Prices adjust instead of quantities.

Despite a global increase in risk premia, the levels of asset prices in rich and poor countries move in opposite directions. The reason is that they also depend on the global interest rate in general equilibrium. The interest rate falls after the shock to clear the market for consumption goods: agents feel poorer due to portfolio losses and want to consume less, but aggregate output is fixed. In poor countries, the rise in risk premia outweighs the effect of the interest rate, and asset prices fall. In rich countries, risk premia do not increase so much, and prices go up. These responses have different signs even though fundamental dividend risk is the same everywhere. Retrenchment flows alone generate a full spectrum of exposure to aggregate shocks.

This result has a set of implications for international risk-sharing. First, the negative loadings of local returns on global shocks make rich countries richer in downturns. Second, other countries benefit from these negative loadings as well. This is because the intermediary makes capital gains on rich countries in bad times, which limits the fall in its net worth coming from losses it takes in poor ones. This, in turn, limits the contraction in foreign demand faced by everyone. Similar risk-sharing patterns were documented by Gourinchas and Rey (2022) and Dahlquist et al. (2022) for the US and the rest of the world: while the US provides insurance to other countries, its wealth share still increases in downturns. With my mechanism, a similar relationship emerges endogenously between rich and poor countries with symmetric fundamentals.

After showing the main mechanism, I take the model to data. The quantitative version has two additional features. First, local agents face a portfolio constraint. The risky share of their portfolios is bounded from above, which, in particular, prevents them from borrowing large amounts to increase leverage in downturns. Agents in poor countries now have even less capacity to buy assets from the intermediary in a capital flight episode, and their exposure to rick-off shocks rises.

Second, I introduce dividend shocks to the model. These shocks change aggregate output and can be called "real" as opposed to "financial" shocks to the intermediary's risk-taking capacity. In this more complete environment, I quantify the importance of these two types of shocks for the dynamics of financial flows, asset prices, and their heterogeneity across countries.

I estimate the model on aggregate capital flow and asset returns data. Estimation uses global averages of flows and prices, but the model successfully reproduces the differences between advanced economies and emerging markets, which correspond to rich and poor countries. It generates a larger volatility of outward flows from advanced economies, where outward flows are measured as a share of external assets, and a larger volatility of asset returns in emerging markets.

Another set of important patterns are correlations of outward flows and asset returns with global aggregates. Both in the model and in the data, outward flows in advanced economies fall by more when aggregate capital flows recede, indicating more active retrenchment. Conforming with the key mechanism, assets in advanced economies outperform those in emerging markets exactly in these times. Retrenchment insulates domestic asset prices from global shocks.

The model ascribes more than half of the variation in aggregate capital flows to real shocks. Financial shocks explain only about a quarter of it. Almost all variation in aggregate returns on risky assets is due to real shocks as well. At the same time, financial shocks are more important for the relative performance of assets in rich compared to poor countries. Real shocks move all asset prices in the same direction, while the loadings on financial shocks in rich and poor countries have opposite signs. Rich countries outperform poor ones in times of low risk-taking capacity. The model can explain half of the time-series variation in the relative performance of risky assets in advanced economies compared to emerging markets. Financial shocks alone can explain a third.

Finally, the fact that prices of risky assets in advanced economies rise in response to negative financial shocks decreases their cyclicality compared to emerging markets. Overall, asset prices are positively correlated with capital flows. In poor countries, the model delivers a correlation close to one: both real and financial shocks raise prices in times of high outward flows. In rich countries, the model-implied correlation is three times lower, since prices respond positively to output booms and negatively to booms in the intermediary's risk-taking capacity.

Related literature. A large literature explores global drivers of international capital flows and asset prices. Miranda-Agrippino and Rey (2022) provide a comprehensive review. The dominant global factor in a large panel of risky asset prices has been extracted by Miranda-Agrippino et al. (2020) and more recently updated by Miranda-Agrippino and Rey (2020). Habib and Venditti (2019) find a similar global component driving stock prices around the world. Jordà et al. (2017) document co-movement between risk-premia across the world.

Similarly strong co-movement has been documented for capital flows. Forbes and Warnock (2012) and Forbes and Warnock (2021) show co-movement between gross flows. Barrot and Serven (2018) identify common components in gross flows and show that these common components are strongly related to aggregate variables such as VIX, US dollar exchange rate, and interest rates. The main one is strongly correlated with the dominant factor in risky asset prices.

Part of this literature deals with heterogeneity between advanced economies and emerging markets. Avdjiev et al. (2022) show that synchronization between inflows and outflows is driven by banks, who are responsible for a larger share of flows in advanced economies as opposed to emerging markets. Barrot and Serven (2018) and Cerutti et al. (2019) show that flows in advanced economies are more responsive to common factors. I perform a simplified version of their factor extraction exercise to illustrate the differences in synchronization. I show that the dominant factor explains a larger share of variation in advanced economies and find that the cyclical component of flows, properly adjusted for size, in this group has a larger magnitude. This fact is at the heart of the model, which is built to generate more elastic asset markets in rich countries.

The literature studying distributions of returns and flows includes Chari et al. (2020), Gelos et al. (2022), and Eguren Martin et al. (2021). Kalemli-Özcan (2019) and Bräuning and Ivashina (2020) show that US monetary policy spillovers have a more pronounced effect on emerging markets. Chari et al. (2020) show the outsized effect of risk-off episodes on the worst realizations, the left tail. This is the response to a shock to risk-taking capacity in my model: the left tail of returns shifts significantly further, while the average stays very close to normal times.

Another strand of literature documents the special position of the US in the global financial system. Gourinchas et al. (2019) review evidence on various dimensions of its dominance. Gourinchas and Rey (2022) find that the US earns significant net returns on its net foreign asset position. Similarly to this gap between the US and the rest of the world, there is heterogeneity within the latter. Adler and Garcia-Macia (2018) show substantial differentials in returns on net foreign asset positions between advanced economies and emerging markets. My model assumes a special position of the US but generates the differences between other economies endogenously.

I contribute to the theoretical literature on the global financial cycle. The most closely related papers are Caballero and Simsek (2020a), Morelli et al. (2022), Bai et al. (2019), Davis and Van Wincoop (2022), Davis and Van Wincoop (2023), and Dahlquist et al. (2022).

Caballero and Simsek (2020a) show how retrenchment stabilizes domestic asset markets in a model where countries invest in each other's risky assets. Liquidity shocks trigger fire sales by foreign investors. Local investors then use their foreign holdings to pick up the unwanted asset and support its price. This mechanism is also present in Jeanne and Sandri (2023). I build a dynamic version of this model in the style of Brunnermeier and Sannikov (2014) with global intermediaries and endogenously arising differences in wealth, with the main focus on the distributional consequences of aggregate capital flight.

Morelli et al. (2022) model an economy built around a global intermediary that invests in many emerging markets. They find that shocks to the intermediary's net worth are an important determinant of borrowing costs around the world. Bai et al. (2019) use a similar model to measure the relative importance of global and local shocks in explaining the cross-section of sovereign spreads. I also study risk-off episodes driven by global intermediaries but focus on capital flows by local investors and their equilibrium implications.

Davis and Van Wincoop (2022) construct a multicountry model to generate gross flows after a shock to global risk aversion and show the importance of within-country heterogeneity. Davis and Van Wincoop (2023) additionally show that symmetric shocks to identical countries can generate positive co-movement between gross flows in and out as long as more risk-tolerant investors invest more abroad. I focus instead on the distributional consequences of global shocks in intermediated markets with endogenous heterogeneity between ex-ante identical countries.

Dahlquist et al. (2022) build a multicountry model with home-biased consumption and time-varying appetites for risk coming from deep habits. They show how an adverse output shock in a large country leads to an appreciation of its currency and, consequently, an increase in its wealth share, as its stock prices fall by less than foreign ones when adjusted for the exchange rate. I arrive at regressive redistribution in downturns through capital flows. In relative terms, rich countries become richer because they compensate for the falling demand from abroad. In absolute terms, they become richer because the shock to risk-taking capacity decreases the interest rate, while risk premia are held down by retrenchment, and asset prices rise.

Farboodi and Kondor (2022) study heterogeneous boom-buts dynamics with imperfect information about asset quality. In their model, shocks determine what investors learn about firms. In bad times, they flee from emerging markets to advanced economies, inducing a recession in the former and stabilizing output in the latter. Fu (2023) models joint determination of capital flows and exchange rates. He shows that currency betas are lower in countries where domestic investors have a higher propensity to retrench than foreign ones. The resulting link between retrenchment and cyclicality of returns is similar to that in my model. Zhou (2023) shows how exposure of assets to investor-specific shocks depends on the willingness of other market participants to absorb additional supply, focusing on the composition of the foreign investor base. A similar mechanism involving domestic investors is at the heart of my model.

The fact that country-specific returns in my model load on one global and one local factor is consistent with findings of Amiti et al. (2019), who find that local shocks are especially important for bank flows in crises. Aguiar and Gopinath (2007) argue that emerging markets exhibit frequent shocks to trend growth. Hassan et al. (2021) identify spikes in perceived country-specific riskiness and show that they are associated with falling asset prices and capital flight.

The role of intermediaries in the model is similar to that in Gourinchas et al. (2022), where they perform international arbitrage across otherwise disjoint markets. Other models with traders intermediating international asset markets include Jeanne and Rose (2002), Gabaix and Maggiori (2015), Greenwood et al. (2020), Fanelli and Straub (2021), and Itskhoki and Mukhin (2021).

Models with the US as a special country include, among others, Bruno and Shin (2015), Maggiori (2017), Farhi and Maggiori (2018), Jiang et al. (2020), Kekre and Lenel (2021), Sauzet (2023), and Devereux et al. (2023). In Jiang et al. (2020) and Kekre and Lenel (2021), the dollar carries a convenience yield. Kekre and Lenel (2021) study flight to safety caused by a shock to this convenience yield in a model with nominal frictions and investment.

Aversion to ambiguity in my model is built on the large theoretical literature dating back to Anderson et al. (2000), Hansen and Sargent (2001), and Chen and Epstein (2002). More recently, Ilut and Saijo (2021) use a similar specification in a model with a continuum of firms to generate empirically relevant co-movements and cyclical patterns in a business cycle model.

On the technical side, I use heterogeneous-agent tools that have mostly been used to model separate countries. I employ methods from Kaplan et al. (2018) to analyze non-linear solutions for aggregate one-time unanticipated shocks. Sequence-space methods of Auclert et al. (2021) and Auclert et al. (2020) and insights from Bhandari et al. (2023) adapted to continuous time allow me to speed up computations and linearize the model for estimation.

Layout. Section 2 shows a version of the model with the main mechanism. Section 3 presents the full model, Section 4 describes equilibrium, and Section 5 treats the shocks in detail. Section 6 explains estimation and results. Section 7 performs quantitative analysis with counterfactuals.

2 Simple Model

This section presents a simple model that delivers key insights with the necessary ingredients only. I set up an economy with heterogeneous countries and study an aggregate capital flight episode. Section 3 then adds other elements of the full model.

Time is continuous and runs forever. There is no aggregate uncertainty. The world is a unit measure of countries indexed by $i \in [0, 1]$ and a large special country populated by intermediaries. Each country has a Lucas tree in fixed unit supply. Output is homogeneous across countries and stochastic. Cumulative output of i's tree up to time t is denoted by y_{it}, and flow output is $dy_{it} = \nu dt + \sigma dZ_{it}$. Expected output ν and volatility σ are constant. The random increments dZ_{it} are standard Brownian. They are independent across countries.

Agents from these countries only invest in their domestic trees and bonds issued by global intermediaries. Bonds are riskless and short-term, paying interest $r_t dt$. The share price of i's tree is an endogenous stochastic process p_{it}. The instantaneous excess return on trees is

$$dR_{it} = (dy_{it} + dp_{it})/p_{it} - r_t dt$$

It includes dividends and capital gains. The wealth w_{it} of a representative agent from country i is her holdings of the tree and intermediary's bonds. The aggregate wealth of country i is $\underline{w}_{it}$, which equals w_{it} in equilibrium. The evolution of w_{it} is

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} - \frac{w_{it}}{\underline{w}_{it}} \cdot \lambda \underline{w}_{it} dt + \frac{w_{it}}{\underline{w}_{it}} \cdot \hat{\lambda} \underline{\hat{w}}_t dt \tag{1}$$

Here c_{it} is consumption. The second term represents returns on the tree, where θ_{it} is its portfolio share. The third and fourth terms reflect exogenous migration out of and into the country. They do not affect consumption and portfolio choice, and their only role is to induce stationarity. Appendix E shows another way to achieve that, using discount rates that depend on wealth.

The third term represents wealth emigration from country i to the special country. Agents in all regular countries die with intensity λ, and their wealth moves to intermediaries. The total outflow of wealth from i is $\lambda \underline{w}_{it} dt$. New agents are born instead. They start with zero wealth and instantly get transferred a portion of everyone's savings so that everyone in the country has the same net worth. This redistribution from continuing agents is in proportion to their w_{it}. Hence, survivors always make flow payments $w_{it}/\underline{w}_{it} \cdot \lambda \underline{w}_{it} dt = \lambda w_{it} dt$ to the newborns.

The last term represents the opposite process, immigration. Intermediaries die with intensity $\hat{\lambda}$, and their wealth is sent to one of the countries, where it is shared between local agents. The destination country is chosen uniformly, so each country i has an influx of wealth $\hat{\lambda} \underline{\hat{w}}_t dt$, where $\underline{\hat{w}}_t$ is the total wealth of intermediaries. Within i, each agent gets a share $w_{it}/\underline{w}_{it}$ of this transfer.

The sequence problem of the agent in the country i is

$$\max_{\{c_{is},\theta_{is}\}_{s\geq t}} \mathbb{E}_t\left[\rho\int_t^\infty e^{\rho(t-s)}\log(c_{is})ds\right]$$

subject to equation (1). Since everyone has the same w_{it}, in equilibrium $w_{it}=\underline{w}_{it}$, and

$$dw_{it}=(r_tw_{it}-c_{it})dt+\theta_{it}w_{it}dR_{it}+(\hat{\lambda}\hat{w}_t-\lambda w_{it})dt$$

Intermediaries and the special country. The special country is special for two reasons. First, its asset supply is different. Second, its local agents are global intermediaries. I explain these two properties below, using notation with hats to separate this country from regular ones.

In contrast to regular countries, the special country is large, with a finite measure $\hat{q}$ of trees that are pooled together in a fund. The random components of their yields wash out, so the total output over dt in the special country is $\hat{q}\nu dt$. These trees can only be traded as one, in a bundle with equal weights. I will refer to this fund as the special country's tree for convenience. Its price is $\hat{p}_t$, and the excess return is $d\hat{R}_t=(\nu dt+d\hat{p}_t)/\hat{p}_t-r_tdt$, idiosyncratic shocks washing out.

The wealth $\hat{w}_t$ of the local representative agent, who is also the global intermediary, is its holdings of trees less bonds outstanding. It evolves as

$$d\hat{w}_t=(r_t\hat{w}_t-\hat{c}_t)dt+\int[\hat{\theta}_{it}\hat{w}_tdR_{it}]di+\hat{\theta}_t\hat{w}_td\hat{R}_t+\frac{\hat{w}_t}{\underline{\hat{w}}_t}\cdot\lambda w_tdt-\frac{\hat{w}_t}{\underline{\hat{w}}_t}\cdot\hat{\lambda}\underline{\hat{w}}_tdt \tag{2}$$

Here $\hat{c}_t$ is consumption. The second term is excess returns on trees in all countries with portfolio weights $\{\hat{\theta}_{it}\}$. The third term is excess returns on the special country's tree, where $\hat{\theta}_t$ is the associated portfolio weight. The last two terms mirror migration terms in equation (1): there is an inflow λw_tdt, where $w_t=\int w_{it}di$ is the aggregate wealth of all countries, and an outflow $\hat{\lambda}\underline{\hat{w}}_tdt$. The aggregate wealth of intermediaries is $\underline{\hat{w}}_t$. Again, as in all countries, newborn intermediaries immediately receive transfers from survivors so that everyone's wealth is the same.

The intermediary's risk-taking capacity is limited. It faces a value-at-risk constraint:

$$\int\mathbb{V}[\hat{\theta}_{it}\hat{w}_tdR_{it}]di\leq\gamma_t\int\mathbb{E}[\hat{\theta}_{it}\hat{w}_tdR_{it}]di \tag{3}$$

This constraint aggregates idiosyncratic uncertainty over the intermediary's returns in all countries and bounds it by a multiple of expected profits on these assets. In Section 3, I derive it from ambiguity aversion and explain why it is reasonable to associate it with value at risk.

The parameter γ_t is key. It determines the intermediary's ability to hold country-specific risk. I call it risk-taking capacity. The intermediary could in principle take advantage of access to a continuum of uncorrelated assets and fully insure other agents, absorbing all idiosyncratic risk.

This case is nested as $\gamma_t = \infty$. With a finite γ_t, I can study non-trivial portfolios and trigger capital flight episodes by temporarily lowering it. The problem of the intermediary is

$$\max_{\{\hat{c}_s, \hat{\boldsymbol{\theta}}_{\boldsymbol{s}}\}_{s\geq t}} \mathbb{E}_t \left[\hat{\rho} \int_t^{\infty} e^{\hat{\rho}(s-t)} \log(\hat{c}_s) ds\right]$$

subject to equation (2) and equation (3). Since $\hat{w}_t = \underline{\hat{w}}_t$ in equilibrium, wealth evolves as

$$d\hat{w}_t = (r_t \hat{w}_t - \hat{c}_t)dt + \int [\hat{\theta}_{it} \hat{w}_t dR_{it}] di + \hat{\theta}_t \hat{w}_t d\hat{R}_t + (\lambda w_t - \hat{\lambda} \hat{w}_t)dt$$

Market clearing and equilibrium. Denote bond holdings of country i by b_{it} and their holdings of domestic trees by h_{it}. By construction, i's wealth is $w_{it} = b_{it} + p_{it}h_{it}$, and the risky share θ_{it} determines the split: $\theta_{it}w_{it} = p_{it}h_{it}$ and $(1-\theta_{it})w_{it} = b_{it}$.

To track the intermediary's holdings of trees and bond issuance, let $\hat{b}_t$ be the total bonds issued, let $\{\hat{h}_{it}\}$ be its holdings of trees in regular countries, and let $\hat{h}_t$ be its holdings of the special country's tree. By construction, $\hat{w}_t = \int p_{it}\hat{h}_{it}di + \hat{p}_t\hat{h}_t - \hat{b}_t$. Portfolio weights $\{\hat{\theta}_{it}\}$ satisfy $\hat{\theta}_{it}\hat{w}_t = p_{it}\hat{h}_{it}$ for all i, and $\hat{\theta}_t$ satisfies $\hat{\theta}_t\hat{w}_t = \hat{p}_t\hat{h}_t$. Bond issuance is $\hat{b}_t = (\int \hat{\theta}_{it}di + \hat{\theta}_t - 1)\hat{w}_t$.

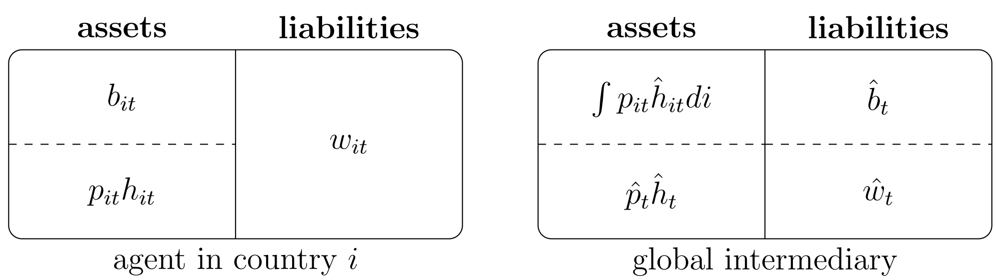


Figure 1: Schematic balance sheets of agents from a country i and the global intermediary.

With all holdings defined, I can define equilibrium.

Definition 1. *An equilibrium is a collection of price processes $\{r_t, \{p_{it}\}, \hat{p}_t\}_{t\geq 0}$, wealth processes $\{\{w_{it}\}, \{\underline{w}_{it}\}, \hat{w}_t, \underline{\hat{w}}_t\}_{t\geq 0}$, consumption processes $\{\{c_{it}\}, \hat{c}_t\}_{t\geq 0}$, and processes for asset holdings $\{\{h_{it}\}, \{\hat{h}_{it}\}, \{b_{it}\}, \hat{b}_t, \hat{h}_t\}_{t\geq 0}$ such that all agents optimize and*

- *aggregate wealth process agrees with individual wealth: $w_{it} = \underline{w}_{it}$ for all i and $\hat{w}_t = \underline{\hat{w}}_t$,*
- *markets for regular country trees clear: $h_{it} + \hat{h}_{it} = 1$ for all $i \in [0,1]$,*
- *market for the special country tree clears: $\hat{h}_t = \hat{q}$,*
- *bond market clears: $\int b_{it}di = \hat{b}_t$,*
- *market for consumption goods clears: $\int c_{it}di + \hat{c}_t = (1+\hat{q})\nu$.*

Markets for trees clear country by country, and the market for the intermediary's bonds clears globally. The market for consumption goods clears automatically as soon as other markets do.

I now characterize equilibrium. The main object of interest is asset prices p_{it}. I start with required returns dR_{it}, using market clearing conditions to characterize them, and then determine prices from them.

There is only idiosyncratic uncertainty in this economy. Only agents from country i and the intermediary have access to country i's tree, so excess returns dR_{it} only load on dZ_{it}:

$$dR_{it} = \mu_{it}^R dt + \sigma_{it}^R dZ_{it}$$

Drift and volatility $(\mu_{it}^R, \sigma_{it}^R)$ of excess returns are equilibrium objects: $\mu_{it}^R = (\nu + \mu_{it}^p)/p_{it} - r_t$ and $\sigma_{it}^R = (\sigma + \sigma_{it}^p)/p_{it}$. Here $(\mu_{it}^p, \sigma_{it}^p)$ are drift and volatility of asset prices: $dp_{it} = \mu_{it}^p dt + \sigma_{it}^p dZ_{it}$. As usual, volatility has an exogenous and an endogenous component.

The local agent's problem has a particularly simple solution due to log utility. Agents always consume a constant fraction of their wealth, $c_{it} = \rho w_{it}$, and choose a mean-variance portfolio:

$$\theta_{it} = \frac{\mu_{it}^R}{(\sigma_{it}^R)^2}$$

Unit elasticity of intertemporal substitution leads to a constant consumption-wealth ratio. Unit relative risk aversion in addition leads to portfolio choice with no hedging motive.

The intermediary's consumption and portfolio choice is similar: $\hat{c}_t = \hat{\rho}\hat{w}_t$ and

$$\hat{\theta}_{it} = \gamma_t \frac{\mu_{it}^R}{(\sigma_{it}^R)^2}$$

There is no real aggregate risk to its payoffs. Aversion to idiosyncratic uncertainty is due to the value-at-risk constraint. Its particular form obviates hedging motives, leading to a mean-variance portfolio with γ_t acting as time-varying risk tolerance.

Market clearing for each country's tree and the fact that $h_{it} = \theta_{it} w_{it}/p_{it}$ and $\hat{h}_{it} = \hat{\theta}_{it}\hat{w}_t/p_{it}$ lead to the following expression:

$$p_{it} = (w_{it} + \gamma_t \hat{w}_t) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2}$$

This expression offers one intuitive way to look at price determination. The first important factor is how much wealth is available for investing, $w_{it} + \gamma_t \hat{w}_t$, where wealth is weighted with effective risk tolerance coefficients for the two investors: 1 for the local agent and γ_t for the foreign one. The second factor is the risk-return profile of the asset, which determines how much of this wealth will be drawn in.

This is not a formula for prices in terms of exogenous objects because p_{it} enters μ_{it}^R and σ_{it}^R. Unpacking this dependence leads to the main equation of the model

$$\underbrace{\mu_{it}^p + \nu - r_t p_{it}}_{\text{risk premium}} = \underbrace{(\sigma_{it}^p + \sigma)^2}_{\text{quantity of risk}} \cdot \frac{1}{\gamma_t \hat{w}_t + w_{it}} \tag{4}$$

The risk premium offered by country i's tree is expected capital gains and dividends in excess of the global interest rate. The quantity of risk is the variance of dividends and capital gains. They both load on the country-specific shock dZ_{it}, so the volatilites simply add up. The price of risk is the inverse of the total available wealth, weighted by effective risk tolerance coefficients. The risk-taking capacity γ_t determines risk premia in all countries at the same time.

A useful benchmark is the case with $\gamma_t = \infty$. I describe it in Appendix G. In this case, nothing prevents the intermediary from taking advantage of a fully diversified portfolio in which country-specific risk washes out. Equation (4) shows that expected excess returns μ_{it}^R have to be zero in equilibirum. Otherwise, the intermediary would demand assets in unbounded quantities. Since $\mu_{it}^R = 0$, local agents are unwilling to hold any domestic assets because for them country-specific risks are a real concern. As a result, the intermediary holds the entire global supply of risky assets. Local agents are not exposed to risk and all have the same wealth in the long run.

When γ_t is finite, the intermediary is only willing to take idiosyncratic risk for a positive compensation. Local agents have to absorb the rest. Exposure to country-specific shocks makes history matter for their wealth, and there is a non-degenerate wealth distribution in equilibrium. It is convenient to illustrate heterogeneity in a steady state with a constant $\gamma_t = \gamma$.

Steady state. In the steady state, all aggregates are fixed. Excess returns on the special country's tree are zero, $d\hat{R} = 0$, so the tree is priced at fair value:

$$r\hat{p} = \nu$$

Regular countries continually move across the steady-state wealth distribution due to idiosyncratic shocks. All country-specific variables are functions of their current wealth w_{it} only and inherit its stochastic properties. Taking $\mu_w(w)$ and $\sigma_w(w)$ to be the drift and volatility of wealth in $dw = \mu_w(w)dt + \sigma_w(w)dZ$ and applying Itô's lemma to equation (4),

$$rp(w) = \nu - \underbrace{\frac{(\sigma_w(w)p'(w) + \sigma)^2}{w + \gamma\hat{w}}}_{\text{risk adjustment}} + \underbrace{\mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w)}_{\text{growth term}} \tag{5}$$

There are two additional terms relative to fair-value pricing. The first reflects the market premium for risk in dividends and capital gains. The growth term reflects the fact that prices are a function of wealth, so wealth dynamics introduce drift in prices.

The drift and volatility of wealth come from the budget constraint of local agents. After some algebra, they can be expressed as

$$\sigma_w(w) = \frac{\sigma w}{w(1 - p'(w)) + \gamma \hat{w}}$$
$$\mu_w(w) = (r - \rho - \lambda)w + \hat{\lambda}\hat{w} + \frac{\sigma_w(w)^2}{w}$$

Volatility originates in the dividend risk and is proportional to σ and wealth w. The denominator has two terms. The first is the total market wealth $w + \gamma\hat{w}$, which reflects risk sharing between the two participants. The second term is $-p'(w)w$, which accounts for endogenous risk amplification: dividend shocks hit wealth, which translates to prices and feeds back into wealth through them. The drift in wealth has a consumption-savings part $(r - \rho)w$, the migration part $\hat{\lambda}\hat{w} - \lambda w$, and the last term is the risk compensation.

I now describe three properties of the steady state with a non-explosive price function $p(w)$ that converges to a constant at $w \to \infty$. First, if this limit exists, it is the fair value:

$$\lim_{w \to \infty} rp(w) = \nu$$

This is evident from equation (5). The risk correction converges to zero because in these extremely rich countries, there is unlimited wealth available for investing, and any non-negligible risk premium would drive demand to infinity. The growth term goes to zero because $p(w)$ approaches a constant and becomes insensitive to wealth as a function. As a result, risky assets in rich countries are priced almost as the special country's tree.

Second, in rich countries, the domestic share of local investor's portfolios converges to zero. The equilibrium expression is

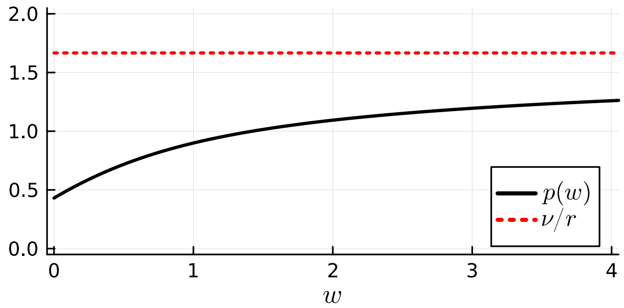
$$\theta(w) = \frac{p(w)}{w + \gamma\hat{w}}$$

Since $p(w)$ is bounded, $\theta(w)$ vanishes at infinity. Agents in rich countries mostly rely on safe foreign assets, and the volatility of their wealth does not scale with size: $\sigma_w(w) \to \sigma$ as $w \to \infty$.

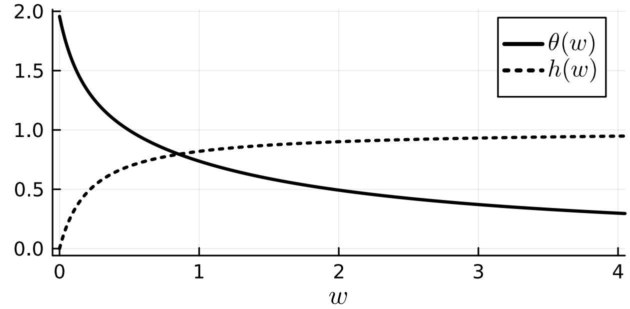
Third, local agents in rich countries take over the market. The equilibrium expression for their holdings $h(w)$ is

$$h(w) = \frac{w}{w + \gamma\hat{w}}$$

Even though their risky portfolio share vanishes at infinite wealth, they become so large that they dominate the domestic market. The opposite is true in poor countries: local investors hold smaller shares of trees, and at $w = 0$, the intermediary fully takes over.



(a) Asset prices and the fair value



(b) Holdings and risky shares of local agents

Figure 2: Panel (a): asset prices $p(w)$ and the fair value limit. Panel (b): tree holdings of local agents $h(w)$ and the risky portfolio shares $\theta(w)$.

Figure 2 illustrates the three properties by showing prices $p(w)$, domestic holdings $h(w)$, and the risky portfolio share $\theta(w)$ chosen by local agents. Appendix H.1 describes the calibration for this numerical example, and Appendix C provides derivations and additional details. In particular, I show that rich countries run trade and current account deficits, and poor countries run surpluses.

The steady-state wealth density $g(w)$ solves the following differential equation:

$$0 = -[\mu_w(w)g(w)]' + \frac{1}{2}[\sigma_w(w)^2 g(w)]''$$

The linear operator acting on $g(w)$ in this equation is the adjoint of that acting on $p(w)$ in the growth term in equation (5). These two equations are coupled and have to be solved jointly.

The intermediary takes positions in trees all around the wealth distribution. Since it has access to a continuum of uncorrelated returns, it makes sure profits, earning excess returns on its wealth with certainty. For the special country as a whole, these profits allow an "exorbitant privilege" of persistent trade deficits even if the country is in debt. Setting net migration flows to zero in the steady state, its trade deficit $\hat{c} - \hat{q}\nu$ can be written as

$$\hat{c} - \hat{q}\nu = r \cdot \underbrace{\left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right)}_{\text{net foreign assets}} + \gamma\hat{w} \int \left[\frac{\mu_R(w)}{\sigma_R(w)}\right]^2 dG(w)$$

Even if net foreign assets are negative, the special country can run persistent trade deficits financed by intermediation profits. These excess returns can be further traced to two sources: risk premium paid on trees and trading profits.

$$\hat{c} - \hat{q}\nu = r \cdot \left(\int p(w)\hat{h}(w)dG(w) - \hat{b}\right) + \underbrace{\int (\nu - rp(w))\hat{h}(w)dG(w)}_{\text{risk premium}} + \underbrace{\int \mu_p(w)\hat{h}(w)dG(w)}_{\text{trading profits}}$$

The risk premium is positive because trees are priced at less than their fair value. The intermediary takes advantage of that, borrowing at r and buying claims to a stream of dividends ν at a price $p(w) < \nu/r$. Dividend risk vanishes in its large portfolio.

Trading profits arise because countries move around the steady-state wealth distribution due to idiosyncratic shocks. This churn generates steady-state capital flows as the intermediary trades trees with local agents. Its trading strategy takes advantage of the drift in prices. Average drift is zero in the steady state, $\int \mu_p(w)dG(w) = 0$, but the intermediary takes positions $\hat{h}(w)$ that skew towards growing countries: $\int \mu_p(w)\hat{h}(w)dG(w) > 0$. As shocks reshuffle the wealth distribution, these countries become richer, their assets appreciate, and the intermediary sells them to buy cheaper assets from countries that arrive to the left tail.

I now illustrate the effects of an aggregate capital flight event that hits this heterogeneous country world. I trigger this episode by shocking the intermediary's risk-taking capacity.

Shock to risk-taking capacity. For illustration, suppose the economy is at the steady state at $t < 0$ and consider an unanticipated transitory shock to γ. For $t \geq 0$,

$$\gamma_t = \gamma - \Delta_\gamma e^{-\mu_\gamma t}$$

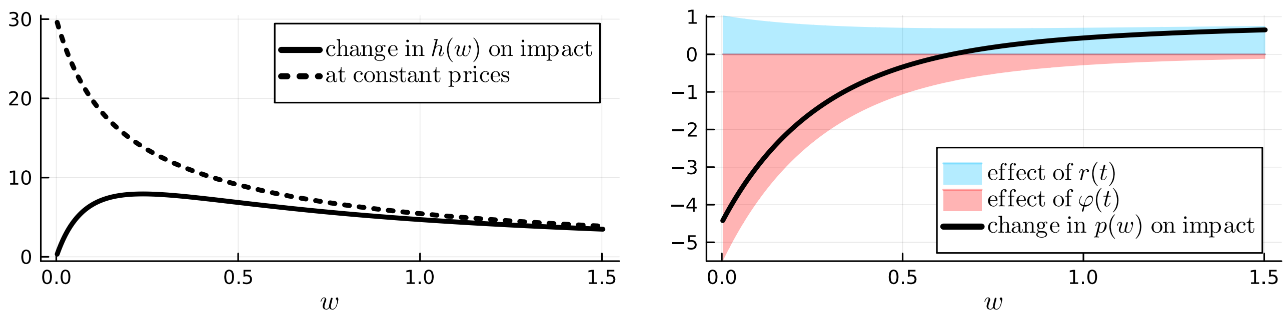
I choose $\Delta_\gamma > 0$ so that the shock to risk-taking capacity is negative, and demand falls.

The intermediary seeks to sell trees in all countries. Given returns, its desired portfolio share falls in the same proportion in every country. Since the intermediary holds a higher share of the market in poor countries, this means it tries to sell more assets in the left tail of the wealth distribution, and these countries are affected more.

Another difference between rich and poor countries is the reaction of local agents. They try to absorb the unwanted assets, and their ability to do it depends on wealth. In rich countries, there is enough wealth to replace foreign demand without a substantial rise in returns. Local agents need to shift a small share of their portfolio towards their home countries to buy trees from the intermediary. In poor countries, local agents cannot absorb what the intermediary wants to sell, and risk premia rise to convince it to sell less.

Panel (a) on Figure 3 shows the change in tree holdings by local agents on impact. The dotted line shows how much the intermediary wants to sell at constant prices, that is, if the trees continued to offer steady-state excess returns. This reflects the shift in its demand for risk. The solid line shows how much of that local agents actually buy in equilibrium. The difference between the two lines indicates how much excess returns have to adjust to stop the intermediary from selling. At large levels of wealth, there is almost no difference. Markets adjust to the shock through shifts in quantity, and there are trades. At low levels of wealth, adjustment is primarily through prices. When wealth approaches zero, local agents cannot absorb anything, so adjustment is through prices only. Excess returns have to rise to exactly match the fall in γ_t.

This shows why the reaction of excess returns is different across countries, but the ultimate object of interest is asset prices. Their response depends on what happens to the interest rate as well as risk premia. The interest rate falls on impact. This is because a fall in asset prices makes agents feel poorer, and they want to consume less. Output is fixed, however, so the interest rate falls to induce consumption.



(a) Change in domestic holdings

(b) Change in asset prices

Figure 3: Panel (a): change in tree holdings by local agents on impact (in percent of total supply). Panel (b): price change on impact (in percent of the steady-state value) with contributions of foreign demand $\varphi_t = \gamma_t \hat{w}_t$ and the interest rate r_t. Both responses as functions of wealth w right before the shock.

This fall in the interest rate revalues assets. How important is this for asset prices relative to the shifts in risk premia? Panel (b) in Figure 3 shows contributions of foreign demand $\varphi_t = \gamma_t \hat{w}_t$ and the interest rate r_t to the initial response of prices. In rich countries, the impact of the interest rate dominates, and prices go up. Their assets are risky but act like colloquial safe assets since they load negatively on the intermediary's risk-taking capacity. In poor countries, the contribution of foreign demand $\varphi_t = \gamma_t \hat{w}_t$ dominates, and prices fall.

This key insight survives in the quantitative model of Section 3. Local wealth determines the exposure of asset prices to global financial shocks. Retrenchment by local agents compresses risk premia, and risky assets in rich countries behave like safe assets in general equilibrium. Their betas on the global risk-taking capacity are endogenously negative. At the same time, it is this retrenchment that reflects capital flight in equilibrium. The fall in capital flows is driven by investors from rich countries repatriating their foreign assets. Appendix A shows empirical support for this mechanism, documenting that outward flows in advanced economies are more strongly correlated with global aggregates in the data.

Risk-sharing. Rich countries become relatively richer in downturns. At the same time, they indirectly provide insurance to the rest of the world. The intermediary makes capital gains on their trees, and everyone else is exposed to the intermediary's net worth. Without these capital gains, all countries would suffer an even larger shock to foreign demand, and poor countries would be hit particularly hard.

The intermediary's wealth increases on impact after a negative shock to γ_t. The reason is its exposure to the special country's tree. This tree appreciates on impact since the shock to risk-taking capacity does not apply to it, and its price only responds to the falling interest rate. The capital gains on this tree more than compensate for the losses on external position.

What happens to the intermediary's net worth is qualitatively and quantitatively significant in the model. To see why, notice that $\hat{w}_t$ enters all expressions for country-specific variables together with γ_t as a part of the product $\varphi_t = \gamma_t \hat{w}_t$. It is important for the behavior of $\hat{w}_t$ to conform with the data. Dahlquist et al. (2022) find that the US wealth share increases in downturns, even though it takes losses on its net foreign asset position. The reason is that its domestic market outperforms foreign markets more than enough to cover this international wealth transfer. This is exactly the role of the special country's tree here.

I next expand the model to make it better approximate data and do quantitative analysis to validate its predictions and run counterfactuals.

3 Full Model

I add three ingredients to the model to improve its quantitative performance. First, I introduce real shocks alongside financial shocks from Section 2. This amounts to making the dividend rate ν_t a time-varying exogenous process. Variation in output allows the model to capture swings in aggregate consumption, which is important for both capital flows and asset prices.

Second, I introduce a portfolio constraint on local agents. In Section 2, agents in poor countries borrow from global intermediaries, and this borrowing intensifies when γ_t falls. In global downturns, intermediaries lend to poor countries to enable them to buy back trees. Qualitatively, it does not change the fact that they are too small to insulate domestic risk premia from the shock. Quantitatively, their ability to borrow in downturns does make a difference for prices and other aggregates in equilibrium.

Finally, I work out the problem of the intermediary to incorporate ambiguity aversion. The intermediary is unsure about the right model for country-specific risk and takes a cautious approach to investing in risky assets. This feature prevents it from taking advantage of a continuum of uncorrelated assets and does it in exactly the same way as the value-at-risk constraint.

Portfolio constraint. Local savers solve the same problem as before with one more restriction:

$$\theta_{it} \leq \overline{\theta}$$

It caps the share of risky assets in their portfolio. The interpretation depends on the value of $\overline{\theta}$.

If $\overline{\theta} = 1$, the saver simply cannot borrow from the intermediary. If $\overline{\theta} > 1$, she can borrow from the intermediary up to a limit of $(\overline{\theta} - 1)w_{it}$. Since the multiple $\overline{\theta}$ of her wealth is the tree holdings, this means that borrowing is limited by $(\overline{\theta}-1)/\overline{\theta} < 1$ of their market value. Finally, if $\overline{\theta} < 1$, the saver must keep at least $1-\overline{\theta}$ of her wealth in riskless bonds. In this case, savers can be interpreted as including insurers and pension funds, who are mandated to hold some safe assets.

The reaction of constrained savers to foreign demand flight is the same in all three cases: they cannot buy what foreign investors wish to sell. Adjustment through prices rather than quantities takes an extreme form. However, the value of $\overline{\theta}$ matters for the level, as opposed to dynamics, of a constrained country's leverage. In my calibration, the relevant case is $\overline{\theta} < 1$, when all countries, including constrained ones, have at least some foreign savings like in the data.

The saver still consumes a constant share of wealth, $c_{it} = \rho w_{it}$, and chooses

$$\theta_{it} = \min\left\{\overline{\theta}, \frac{\mu^R_{it}}{(\sigma^R_{it})^2}\right\}$$

Intermediary with ambiguity aversion. I start by describing the problem of an intermediary who has access to a finite number of countries first. I then let the number of countries go to infinity to approximate the continuous limit. The evolution of the representative intermediary's wealth is

$$d\hat{w}_t = (r_t\hat{w}_t - \hat{c}_t)dt + \sum_i \hat{\theta}_{it}\hat{w}_t dR_{it} + \hat{\theta}_t\hat{w}_t d\hat{R}_t + \frac{\hat{w}_t}{\underline{\hat{w}}_t}\cdot\lambda w_t dt - \frac{\hat{w}_t}{\underline{\hat{w}}_t}\cdot\hat{\lambda}\underline{\hat{w}}_t dt$$

Here $\hat{\theta}_{it}$ is the portfolio share of a country i's tree and $\hat{\theta}_t$ is the share allocated to the special country's tree.

The intermediary is unsure about the right probability measure for dZ_{it}, the random component of dividends in every i. To account for possible misspecification, the intermediary considers other probability measures Q_i under which the process $\{\widetilde{Z}_{it}\}_{t\geq 0}$ given by $d\widetilde{Z}_{it} = dZ_{it} + \xi_{it}dt$ is a Brownian motion instead of $\{Z_{it}\}_{t\geq 0}$. The adapted sequences $\{\xi_{it}\}_{t\geq 0}$ index these alternative measures Q_i. The evolution of an individual intermediary's net worth can be rewritten as

$$d\hat{w}_t = (r_t\hat{w}_t - \hat{c}_t)dt + \sum\nolimits_i \hat{\theta}_{it}\hat{w}_t((\mu^R_{it} - \sigma^R_{it}\xi_{it})dt + \sigma^R_{it}d\widetilde{Z}_{it}) + \hat{\theta}_t\hat{w}_t d\hat{R}_t + \frac{\hat{w}_t}{\underline{\hat{w}}_t}(\lambda w_t - \hat{\lambda}\underline{\hat{w}}_t)dt \tag{6}$$

Each of the alternative measures Q_i is associated with a likelihood ratio M_{it} relative to the original measure. These likelihood ratios are equal to 1 at $t = 0$ and evolve as

$$dM_{it} = -\xi_{it}M_{it}dZ_{it} \tag{7}$$

The intermediary would like to consider scenarios with heavy losses, finding corrections ξ_{it} that minimize its payoff. Discipline is provided by a penalty that keeps the intermediary from deviating too much from the baseline measure. The penalty function per unit of time for every i is proportional to the expected value of $\eta(w_{it})dm_{it}$, where dm_{it} is the increment of the log-likelihood ratio $m_{it} = \log(M_{it})$, and $\eta(\cdot)$ is a non-decreasing (perhaps, constant) function of country i's wealth. This allows for different levels of caution when assessing rich and poor countries. I add this to better approximate the distribution of external assets and liabilities.

The problem of an individual intermediary is

$$\max_{\{\hat{c}_s,\hat{\theta}_s,\hat{\boldsymbol{\theta}}_s\}_{s\geq t}} \inf_{Q\in\mathcal{Q}} \mathbb{E}_t^Q \left[\hat{\rho}\int_t^\infty e^{\hat{\rho}(s-t)}\log(\hat{c}_s)ds + \frac{1}{2}\sum_i \int_t^\infty e^{\hat{\rho}(s-t)}\hat{\gamma}_s\eta(w_{is})dm_{is}\right]$$

subject to equation (6). Here $\mathcal{Q}$ is the set of alternative measures that can be represented by equation (7) and $\{\hat{\gamma}_t\}_{t\geq 0}$ is an adapted sequence of cost parameters.

The intermediary's behavior can be described as looking at value-at-risk measures since it considers adverse scenarios. The difference relative to imposing a value-at-risk constraint directly is that cautious behavior here derives from preferences rather than regulatory constraints. Declines in risk-taking capacity can be ascribed to shifts in attitude to risk rather than tightening regulations.

The cost parameter $\hat{\gamma}_t$ captures appetites for risk since it determines how far the intermediary is willing to go with alternative adverse scenarios. A low $\hat{\gamma}_t$ means low costs of considering models with large potential losses, making risky assets less attractive. Unlimited capacity is nested as $\hat{\gamma}_t = \infty$, which leads the intermediary to set $dm_{it} = 0$ and stick to the baseline probability measure. In this case, the intermediary takes advantage of a perfectly diversified international portfolio and absorbs all idiosyncratic risk globally.

The intermediary solves the minimization problem first, choosing an alternative measure for every country i and their product measure Q to account for losses suggested by models in the admissible set $\mathcal{Q}$. It then solves the usual maximization problem for given model adjustments, choosing consumption and portfolio. In general, $\hat{\gamma}_t$ could be varying over time, generating temporary episodes of low risk-taking capacity.

The increment of the log-likelihood ratio dm_{it} can be rewritten as

$$dm_{it} = -\xi_{it}dZ_{it} - \frac{1}{2}\xi_{it}^2 dt$$

Since $dZ_{it} = d\widetilde{Z}_{it} - \xi_{it}dt$, where $d\widetilde{Z}_{it}$ is a standard Brownian increment under Q_i, the expectation of this under Q_i is $\mathbb{E}_t^Q[dm_{it}] = \xi_{it}^2 dt/2$. The problem transforms into

$$\max_{\{\hat{c}_s,\hat{\theta}_s,\hat{\boldsymbol{\theta}}_s\}_{s\geq t}} \inf_{\{\boldsymbol{\xi}_s\}_{s\geq t}} \mathbb{E}_t^{Q(\boldsymbol{\xi}_t)} \left[\int_t^\infty e^{\hat{\rho}(s-t)}\left(\hat{\rho}\log(\hat{c}_s) + \frac{\hat{\gamma}_s}{2}\sum_i \eta(w_{is})\xi_{is}^2\right)ds\right]$$

subject to equation (6). This formulation acknowledges that the choice of measure $Q \in \mathcal{Q}$ is equivalent to selecting drift corrections $\boldsymbol{\xi}_t = \{\xi_{it}\}$. Importantly, alternative models are chosen separately for all countries, and the intermediary thinks that $d\tilde{Z}_{it}$ are independent across i.

Fix the number of regular countries at n. Consumption of the intermediary is $\hat{c}_t = \hat{\rho}\hat{w}_t$, and portfolio weights and drift correction for each country i are

$$\hat{\theta}_{it}^{(n)} = \frac{\hat{\gamma}_t \eta(w_{it})}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2}$$
$$\xi_{it}^{(n)} = \frac{1}{\hat{\gamma}_t \eta(w_{it})} \cdot \theta_{it}^{(n)} \sigma_{it}^R = \frac{1}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{it}^R}$$

The intermediary's portfolio weight of each risky asset is proportional to the mean-variance ratio. It also increases in the risk-taking capacity $\hat{\gamma}_t$ and the country's weight $\eta(w_{it})$. Whenever $\hat{\gamma}_t$ is finite, ambiguity aversion attenuates portfolio weights, as the multiplier before the mean-variance ratio is between 0 and 1. The drift correction $\xi_{it}^{(n)}$ is proportional to the Sharpe ratio, meaning that the intermediary takes a more cautious view of high-yielding countries.

To take the continuous limit, I let the number of countries n go to infinity. Payoffs in the limit have to include integrals over $i \in [0, 1]$, and portfolio weights $\hat{\theta}_{it}$ in the limit become a density:

$$\hat{\theta}_{it} = \lim_{n\to\infty} n\hat{\theta}_{it}^{(n)}$$

To ensure that this limit exists, I let the risk-taking capacity $\hat{\gamma}_t$ decrease as n diverges: $\hat{\gamma}_t = \gamma_t/n$. This double limit of portfolio weights is

$$\hat{\theta}_{it} = \gamma_t \eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{8}$$

Why does risk-taking capacity have to go to zero as the number of countries grows? If the intermediary's aversion to uncertainty does not rise as it gets access to a continuum of uncorrelated returns, it will take infinite positions as the law of large numbers wipes out all the risk. The limiting density in equation (8) does not exist unless $\hat{\gamma}_t$ decreases at least as fast as γ_t/n.

Another way to look at it is to account for total payoffs. Increasing n indefinitely gives the intermediary access to more and more uncorrelated assets, so without a commensurate rise in aversion to uncertainty the intermediary's portfolio would blow up. It would borrow and invest without an upper bound, and its wealth would not have well-defined dynamics in the limit.

The drift corrections approach the Sharpe ratio:

$$\xi_{it} = \lim_{n\to\infty} \xi_{it}^{(n)} = \frac{\mu_{it}^R}{\sigma_{it}^R}$$

Finally, the wealth of the special country in the limit follows

$$d\hat{w}_t = (r_t \hat{w}_t - \hat{c}_t)dt + \int_0^1 \hat{\theta}_{it} \hat{w}_t dR_{it} di + \hat{\theta}_t \hat{w}_t d\hat{R}_t + (\lambda w_t - \hat{\lambda} \hat{w}_t)dt$$

Borovička et al. (2023) similarly take a double limit to study aversion to ambiguity with recursive preferences. In their case, uncertainty vanishes at the same speed as the aversion to ambiguity increases, delivering a tractable limit.

Appendix F.1 reverses the order and poses the problem with a continuum of assets instead of solving a finite-country one and taking the limits of the solutions. The results are the same. Solving the finite-country problem first has a technical advantage: alternative measures chosen by the intermediary are absolutely continuous with respect to the true measure. With a continuum of countries, this does not hold, as all aggregates become deterministic and measures degenerate.

Ilut and Saijo (2021) use the same continuous limit in a model with a large portfolio of firms to generate endogenous movements in confidence that respond to aggregates. They explain the effect of idiosyncratic uncertainty on an ambiguity-averse decision-maker and relate it to work on laws of large numbers by Marinacci (1999) and Epstein and Schneider (2003).

Discussion. Two elements of the model are technical: migration between countries and intermediary's weights $\eta(w_{it})$ on country-specific ambiguity. Migration enforces stationarity and does not affect consumption or portfolio decisions. I could use variable discount rates instead:

$$\rho(w_{it}, \hat{w}_t) = \rho + \lambda - \hat{\lambda}\hat{w}_t/w_{it}$$
$$\hat{\rho}(\hat{w}_t, w_t) = \hat{\rho} + \hat{\lambda} - \lambda w_t/\hat{w}_t$$

This would change consumption but not portfolio choice or wealth dynamics. Importantly, aggregate consumption given the distribution of wealth would not change either.

The weights $\eta(w_{it})$ make the steady-state distribution of external assets and liabilities better approximate the data. If $\eta(\cdot)$ is increasing, the intermediary is less concerned about ambiguity when it comes to richer countries, so the penalty for choosing alternative measures is higher for high w_{it}. Allowing for country-specific ambiguity captures some variation in investors' perceptions of countries that go beyond observable returns. Through $\eta(w_{it})$, fluctuations in w_{it} change the intermediary's attitude to i's assets given its risk-return profile. Hassan et al. (2021) provide evidence for dynamically changing perception of country-specific risk that affects investor preferences.

4 Equilibrium

This section describes equilibrium, the steady state, and calibration. The definition of equilibrium is exactly like in Section 2. Four types of markets must clear: for shares of regular country trees, shares of the special country tree, bonds, and consumption goods. Individual wealth of representative savers must also agree with aggregate wealth in their respective countries.

I drop the subscripts and refer to countries using their current wealth w. This step could not be done before solving the intermediary's problem because countries with different identities provide independent returns and have to be accounted for separately, even if their local dynamics are the same. But now, with portfolio choice given by equation (8), I can work directly with prices $p(w,t)$ and portfolio shares $\theta(w,t)$ and $\hat{\theta}(w,t)$ that map into holdings $h(w,t)$ and $\hat{h}(w,t)$.

Rewriting the market clearing conditions for each regular country,

$$\mu_R(w,t) = \sigma_R(w,t)^2 \cdot \max\left\{\frac{p(w,t)}{\varphi(t)\eta(w)+w}, \frac{p(w,t)-\overline{\theta}w}{\varphi(t)\eta(w)}\right\} \tag{9}$$

This equation shows how returns are determined by available cash in the market given by local wealth w and intermediary's demand $\varphi(t)\eta(w)$, where the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$ is common to all markets, as in Section 2.

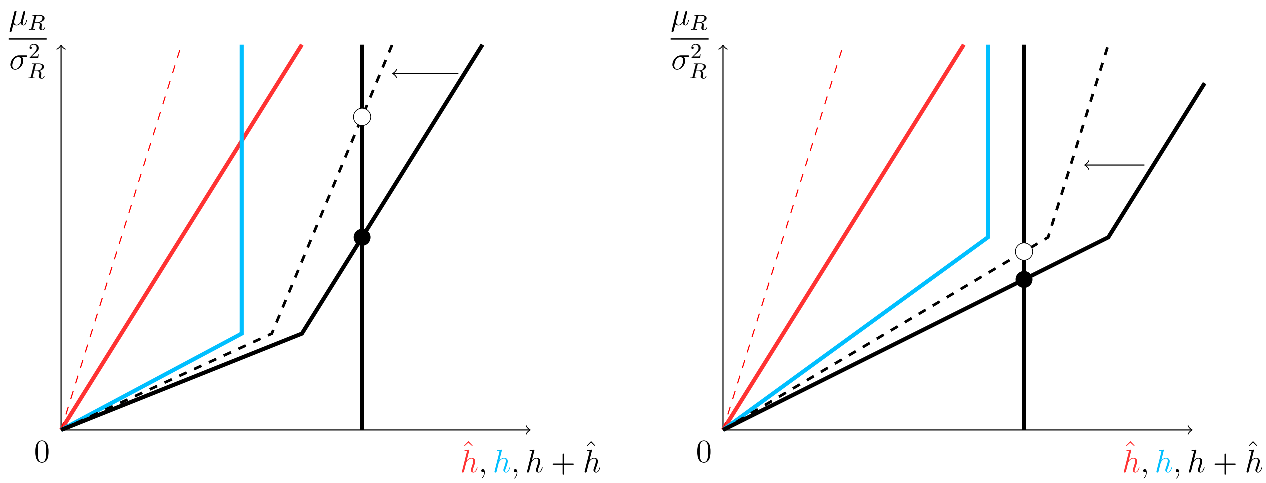
The max operator here shows that the local savers can be constrained or unconstrained in their portfolio choice. When the constraint is slack, the price of risk is given by the ratio of the total supply of assets $p(w,t)\cdot 1$ divided by the total demand $\varphi(t)\eta(w)+w$. Both local and foreign investors are marginal. The elasticity of excess returns to the global factor $\varphi(t)$ depends on the country's wealth. In countries with a large w, this elasticity approaches zero. Local savers can absorb all fluctuations in foreign demand without drastic movements in required returns. In poor countries, local savers can do it less effectively, and the elasticity approaches -1 as $w \to 0$. This case is well described by the simple model of Section 2.

When the constraint binds for local savers, only the intermediary is marginal, so the price of risk is the residual supply $p(w,t)-\overline{\theta}w$ divided by its demand $\varphi(t)\eta(w)$. This case is new relative to Section 2. The intermediary is the only marginal investor in these countries, and the elasticity of excess returns to $\varphi(t)$ is -1. Domestic investors cannot absorb any shifts in foreign demand, so they pass through to required returns one for one.

A constrained country is illustrated on panel (a) of Figure 4. The red line depicts demand from the intermediary, local demand is in blue, and their horizontal sum in black is the overall demand. This line has a kink where the mean-variance ratio becomes so high that local savers hit their portfolio constraint. The supply line is vertical, and the supply-demand intersection is in the region where domestic demand is fully inelastic.

If a negative shock to $\varphi(t)$ leads to withdrawal of foreign demand, total demand shifts left. The new supply-demand intersection is at a higher mean-variance ratio. The blue line is vertical at this point, so local investors cannot increase their holdings. Since there is nobody to sell to, the intermediary has to accept the old quantities, and excess returns must rise. Adjustment fully happens through prices, which in Section 2 only happened in the limit $w \to 0$.

A more elastic case is illustrated on panel (a) of Figure 4. The fall in foreign demand still leads to an increase in required excess returns, but this time local savers are unconstrained and can compensate for capital flight. There is trade. They increase their holdings in response to wider spreads, and excess returns do not move as much.



(a) Constrained savers

(b) Unconstrained savers

Figure 4: Supply and demand for country i's trees as a function of the mean-variance ratio μ_R/σ_R^2 for fixed w and p. Supply is vertical. Demand $\hat{h}$ from the intermediary in red, h from the local savers in blue. The total demand is a horizontal sum of the two. Local savers are constrained on panel (a) and unconstrained on panel (b). Dashed lines show shifted curves after foreign demand is partially withdrawn. Equilibria move from the black to the white dots.

The extent to which trades can insulate prices from the shock to $\varphi(t)$ depends on w, which in the picture maps into the slope of the blue line left of the kink. Large w means a low slope, with a horizontal line in the limit or $w \to \infty$. Excess returns do not move at all in this case. Small w makes the blue line closer to vertical. Poor unconstrained countries behave like constrained ones.

General equilibrium. Figure 4 shows partial equilibrium shifts. To get the full general equilibrium picture, I have to account for the following things. First, higher promised returns come from future price growth, which requires asset prices $p(w,t)$ to fall in equilibrium. This revalues portfolios and impacts wealth w and $\hat{w}$. Second, consumption responds to changes in wealth. This leads to a change in the interest rate that further revalues assets, including the safe asset issued in the special country that is not affected by $\varphi(t)$ in the first place.

Capital flows offer another general equilibrium perspective. When local investors buy domestic assets from the intermediary, they run down their foreign holdings. Since all unconstrained countries do this simultaneously, a fall in $\varphi(t)$ leads to a wave of global retrenchment, which is reflected in the shrinking balance sheets of global intermediaries, who now raise less funding to finance their investments. This deleveraging affects short-term rates, changing prices and revaluing wealth.

The main insights on heterogeneity from Figure 4 survive, though. Constrained markets show very limited trading activity, and adjustment happens through prices. Unconstrained markets in sufficiently rich countries see large trading volumes and, because of that, limited price movements. Closing the model pins down levels of responses in addition to differences between countries that are visible on Figure 4 already.

To recover prices, define equilibrium drift and volatility functions $(\mu_w, \mu_p, \sigma_w, \sigma_p)$ as

$$
\begin{aligned}
dw &= \mu_w(w,t)dt + \sigma_w(w,t)dZ \\
dp &= \mu_p(w,t)dt + \sigma_p(w,t)dZ
\end{aligned}
$$

Here dZ are increments of the country-specific Brownian motions. Using the definition of returns,

$$
\begin{aligned}
\mu_R(w,t) &= (\mu_p(w,t) + \nu(t))/p(w,t) - r(t) \\
\sigma_R(w,t) &= (\sigma_p(w,t) + \sigma)/p(w,t)
\end{aligned}
$$

Plugging this into market clearing and using Itô's lemma leads to a partial differential equation for $p(w,t)$. A related partial differential equation determines the evolution of the regular country wealth distribution $G(w,t)$ with the associated density $g(w,t)$.

PROPOSITION 1. *The prices $p(w,t)$ and density $g(w,t)$ solve the following system:*

$$
\begin{aligned}
r(t)p(w,t) - \partial_t p(w,t) &= y(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{1}{2}\sigma_w(w,t)^2 \partial_{ww} p(w,t) \\
\partial_t g(w,t) &= -\partial_w[\mu_w(w,t)g(w,t)] + \frac{1}{2}\partial_{ww}[\sigma_w(w,t)^2 p(w,t)]
\end{aligned}
$$

subject to suitable boundary conditions. The risk-adjusted payoff function $y(w,t)$ is given by

$$
y(w,t) = \nu(t) - \left(\frac{\sigma}{1-\epsilon(w,t)\theta(w,t)}\right)^2 \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1 - \frac{\overline{\theta} w}{p(w,t)}\right)\right\}
$$

where $\epsilon(w,t) = w/p(w,t) \cdot \partial_w p(w,t)$ is the wealth elasticity of price.

The price $p(w,t)$ solves a standard Kolmogorov backward equation akin to those describing value functions. The discount rate is $r(t)$, and the reward function $y(w,t)$ is the tree yield adjusted for risk. The wealth distribution solves a standard forward equation, and these two equations are coupled. The feedback loop between them goes through $r(t)$, which clears the global bond market and therefore depends on the distribution of wealth, and the global factor $\varphi(t)$, which includes $\hat{w}(t)$, the special country's net worth that depends on the distribution of prices.

A slight complication in solving the coupled system is that the payoff function depends on $p(w,t)$ and its derivative in a non-linear way. On top of that, one needs to know $p(w,t)$ to compute the drift and volatility of wealth. This turns the problem posed by Proposition 1 into finding a fixed point. I describe my solution algorithm in Appendix I.

Steady state. Suppose $\gamma(t)$ and $\nu(t)$ are constant and the economy is in the steady state with a steady-state wealth distribution and price, return, and portfolio weight functions. Notation for these steady-state versions omits the time argument.

Table 1: Steady-state calibration.

	model	target	source
aggregates:			
US wealth share	31.5%	32.3%	Credit Suisse (2022)
US output share	23.7%	22.8%	World Bank
average risk premium	2.62pp	2.5pp	Gourinchas and Rey (2022)
emerging market premium	2.22pp	2.3pp	Adler and Garcia-Macia (2018)
external assets to external liabilities:			
mean	1.071	1.075	IFS (IMF)
standard deviation	0.686	0.685	IFS (IMF)
q25	0.614	0.621	IFS (IMF)
q50	0.849	0.877	IFS (IMF)
q75	1.285	1.249	IFS (IMF)

Relative to Section 2, there is a new source of heterogeneity between rich and poor countries. Namely, there is a threshold $\tilde{w}$ such that the local portfolio constraint $\theta(w) \leq \overline{\theta}$ binds in countries with $w < \tilde{w}$. In poor countries, prices are depressed by low wealth in the market. This renders expected returns high, so domestic investors want to allocate a high share of their portfolio to the tree and hit the limit $\overline{\theta}$. As a country grows rich, it outgrows the constraint since the price increases with wealth, lowering expected returns and making the tree less attractive. As in Section 2, the risky portfolio share of local investors $\theta(w,t)$ falls in w.

Portfolio constraint makes the elasticity of excess returns to foreign demand $\varphi(t)$ equal to -1 in a positive measure of countries at once. In the world without the constraint, as described in Section 2, this extreme elasticity is only present in the limit $w \to 0$. Introducing $\overline{\theta}$ spreads this behavior to all countries with $w < \tilde{w}$. In unconstrained countries, the elasticity of excess returns to $\varphi(t)$ still increases in wealth and approaches zero when $w \to \infty$, as in Section 2.

Calibration. Table 1 presents the steady-state targets and model fit. I choose the model parameters to approximate the following moments. First, I designate the US as the special country and target US wealth share of 32.3% from Credit Suisse (2022) and US share of GDP of 25.4% from the World Bank data. Second, Gourinchas and Rey (2022) estimate that the US gets an annual return of 2-3 percent on its external position. Third, Adler and Garcia-Macia (2018) estimate that emerging markets earn a 2.3pp lower real return on NFA compared to advanced economies. Finally, I make the model reproduce some moments of the empirical distribution of external assets relative to external liabilities. See Appendix H for details.

I further normalize average output ν and set $\hat{\lambda}$ to a value that induces zero net migration flows in the steady state. The parameters are $(\rho, \hat{\rho}, \lambda, \hat{\lambda}, \nu, \overline{\theta}, \sigma, \gamma, \hat{q}, \zeta)$, where ζ sets the intermediary's weights: $\eta(w) = \zeta + (1-\zeta)w$.

Table 2: Model parameters.

parameter	value	meaning
regular countries		
ρ	0.0793	discount rate
λ	0.0177	emigration rate
ν	0.0600	output rate
σ	0.0647	output volatility
$\overline{\theta}$	0.7059	upper limit on risky asset share
special country		
$\hat{\rho}$	0.0844	discount rate
$\hat{\lambda}$	0.0384	emigration rate
$\hat{q}$	0.3096	asset stock
ζ	0.3824	country weight intercept
γ	0.6698	risk-taking capacity

This affine specification has two properties. First, at $w \to 0$ it pans out to a constant, so the total demand for assets is bounded away from zero even in countries with vanishing wealth. These countries are taken over by the intermediary, and the prices of their trees are bounded away from zero. Second, at $w \to \infty$ the intermediary's demand is proportional to that of the local investors, so the trees are not completely taken over by domestic agents even as they become infinitely rich. This prevents the rich countries from having to hold the entire supply of their risky assets, in which case fluctuations in foreign demand would mechanically have very little effect.

The portfolio constraint parameter $\overline{\theta} = 0.71$ means that not only are regular countries unable to borrow, but they also must hold some riskless debt. This can be associated with regulatory mandates on investment vehicles. The share of unconstrained countries in the steady state is about 12%, which is close to the number of advanced economies in the world.

5 Financial and Real Shocks

This section illustrates a shock to risk-taking capacity γ in the full model and discusses its general equilibrium implications in detail. I then compare this financial shock to a real shock that exogenously hits output and explain what features of aggregate data these two shocks map to.

Shock to risk-taking capacity. For illustration, suppose the economy is at the steady state during $t < 0$. At $t = 0$, there is an unanticipated shock to future path of γ:

$$\gamma(t) = \gamma - \Delta_\gamma e^{-\mu_\gamma t} \mathbb{1}\{t \geq 0\}$$

I set the persistence parameter $\mu_\gamma = 0.24$ and the size of the shock ϵ_γ at 7.9% of the steady-state

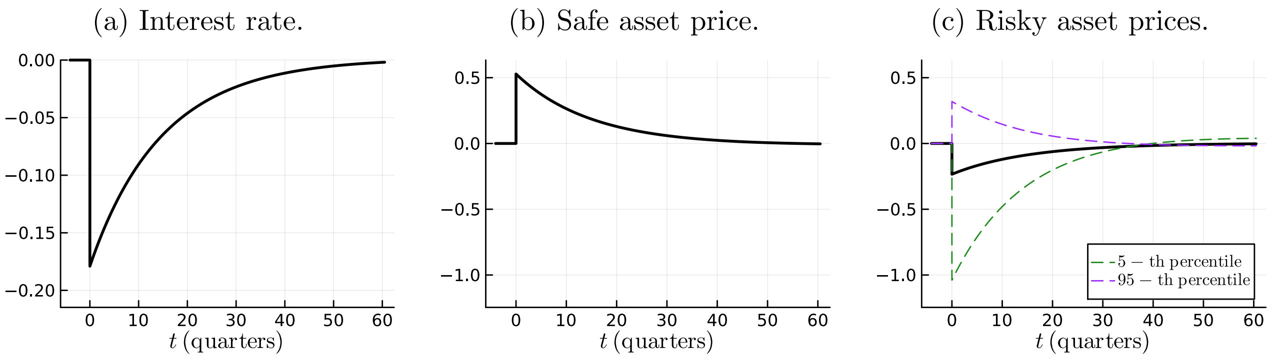


Figure 5: Impulse responses of prices. Panel (a): global interest rate $r(t)$, percentage points. Panel (b): safe asset price $\hat{p}(t)$, percent of the steady-state value. Panel (c), solid line: average risky asset price $p(w,t)$, percent of the steady-state average. Dashed lines: responses of the prices at the 5-th percentile of the wealth distribution (in green) and at the 95-th percentile (in purple).

value of γ in accordance with my estimation results from Section 6.

The shock to $\gamma(t)$ has an immediate effect on the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$. The intermediary loses its appetite for risky assets and withdraws demand from all markets in regular countries. Expected excess returns rise. If the interest rate $r(t)$ was fixed, these expected returns would come from the initial fall in prices $p(w,t)$ and a positive drift as they gradually recover.

However, as prices fall, investors lose money on risky assets and want to consume less. With output fixed, $r(t)$ has to fall to convince them to consume the old amount on aggregate. The fall in the intermediary's risk-taking capacity thus generates a fall in the global interest rate. This revalues assets, so the fall in $r(t)$ limits the overall fall in prices.

Figure 5 shows the responses of $r(t)$, $\hat{p}(t)$, and the average risky asset price $p(w,t)$. In addition to the average, panel (c) has the responses of prices at the 5-th and 95−th percentiles of the wealth distribution. They are different and even have the opposite signs. At the 95-th percentile, the tree gains value on impact. As explained in Section 2, this fall in the interest rate more than compensates the rise in risk premium in rich countries since retrenchment fully neutralizes the fall in foreign demand. This leads to instant appreciation. In poor countries, the rise in risk premiums dominates, and assets depreciate on impact.

The safe asset, which is the special country's tree, appreciates as well. In fact, the increase in its price $\hat{p}(t)$ is enough to increase the special country's wealth $\hat{w}(t)$ on impact. This is consistent with the results in Dahlquist et al. (2022) and Jiang et al. (2022). The special country loses money on its foreign holdings but ends up increasing its wealth share.

What would happen without the constraint $\overline{\theta}$? Agents in the left tail of the wealth distribution would borrow from the intermediary to buy assets, so lending to poorer countries would rise during global downturns. These countries would be effectively issuing riskless debt to the intermediary to

Figure 6: Gains and losses of the intermediary and local savers on impact. Measured in percent of global GDP, weighted by density at $t = +0$

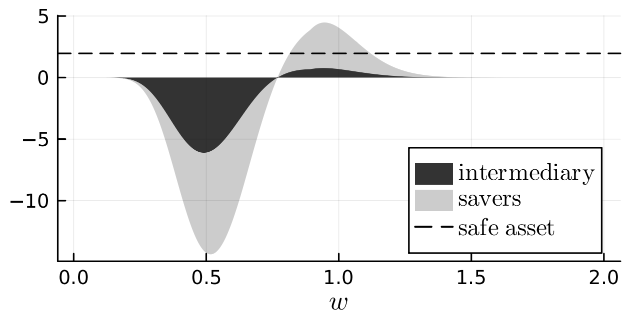


Table 3: Distribution of gains and losses on impact in percent of global GDP. Low w countries are those constrained in steady state, $\theta(w) = \overline{\theta}$

intermediary (safe asset)	1.96%
intermediary (risky assets, low w)	−1.06%
intermediary (risky assets, high w)	0.06%
savers in low w countries	−1.34%
savers in high w countries	0.31%

fund purchases of their risky assets, against empirical evidence. In my calibration, $\overline{\theta} = 0.71$ means that regular countries cannot borrow at all, and there is no spike in lending during global busts.

Distributional impact. What do risky assets issued by different countries contribute to gains and losses of the global intermediary's portfolio? And how are gains and losses distributed among investors in regular countries? Figure 6 shows them in cross-section, measured in percent of global GDP and weighted by the steady-state density so that they can be integrated directly to compute the totals. Table 3 aggregates gains and losses into those made by the intermediary on the safe assets and risky assets from countries that are initially constrained and unconstrained. Constrained countries have low wealth, so they map into emerging markets. Table 3 also aggregates the impact of the shock on the wealth of local investors.

Two things stand out. First, total wealth revaluation is close to zero. Because the elasticity of intertemporal substitution is equal to one, it always holds that

$$\int \rho w dG(w,t) + \hat{\rho}\hat{w}(t) = \nu(1 + \hat{q})$$

The shock to risk-taking capacity $\gamma(t)$ does not affect output. This means that the interest rate in general equilibrium will adjust to revalue assets enough to keep the weighted sum of global wealth on the left-hand side of this equation constant. An implication is that the intermediary's wealth and that of the rest of the world cannot go down at the same time, and there has to be redistribution with an approximately zero sum. If $\rho = \hat{\rho}$, it is exactly zero-sum.

Second, there is substantial heterogeneity. Losses are concentrated in poorer countries. In contrast, risky assets issued in rich countries endogenously behave as almost safe due to retrenchment. At the same time, the intermediary shares in the losses in poor countries more than in the gains that rich countries make. This is because the intermediary's portfolio is skewed towards high-yielding emerging markets.

Losses on risky assets in regular countries translate into a fall in the special country's net foreign asset position. This country makes a wealth transfer to the rest of the world by absorbing part of their losses, consistent with observations in Gourinchas and Rey (2022). On the other hand, the intermediary's exposure to the special country's tree and trees in rich countries limits its losses and increases its wealth share on impact. Rich countries contribute to stabilizing the intermediary's net worth $\hat{w}(t)$ and, through that, help arrest the fall in $\varphi(t) = \gamma(t)\hat{w}(t)$. The upshot is that rich countries partly insure the global intermediary, which in turn partly insures the poor countries.

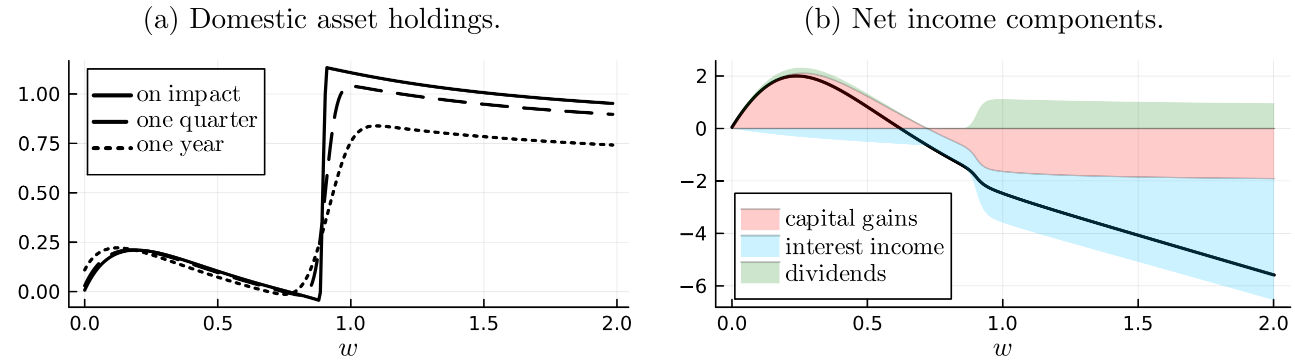


Figure 7: Panel (a): holdings of domestic assets by local investors in percent of total supply (difference relative to the steady state). Solid line for the change on impact, dashed for the expected change one quarter out, dotted for the expected change one year out. Panel (b): changes in components of expected wealth accumulation over the first quarter. Changes in prices in red, changes in interest income in blue, changes in dividends in green. Horizontal axis corresponds to wealth at $t = 0$, right before the shock hits.

Adjustment in regular countries. How do regular countries adjust? Panel (a) on Figure 7 shows the expected changes in holdings of domestic assets relative to the steady state. Expectations here are taken with respect to idiosyncratic shocks, and the horizontal axis corresponds to wealth at $t = 0$. See details in Appendix J.

Investors in rich countries retrench on impact and buy about 1% of the total supply of domestic assets. They then gradually sell them back in transition. These transactions, of course, are the other side of the intermediary's asset purchases.

Panel (b) decomposes expected wealth accumulation in regular countries over the first quarter. Changes in asset prices are positive for poor countries and negative for rich ones as the world reverts back to normal. Interest income declines for everyone, but disproportionately so for rich investors. Dividend income increases for rich investors since they retrench and have more risky assets in portfolios right after the shock. To finance consumption, investors from rich countries have to sell assets since their interest income declines more than their dividend income rises.

The world becomes more unequal in terms of wealth and in terms of spreads after the shock. Realized returns right after the shock are negative in poorer countries and close to zero in richer countries. This is consistent with the results in Chari et al. (2020), who show that the left tail of the return distribution moves more than the right tail during risk-off episodes. The distribution of required returns in the model also becomes more dispersed and skewed.

Shocks to output. I now describe another type of shock: a persistent decline in output. I hit the economy separately with two unanticipated shocks, one to output in regular countries and one to the special country's output. They both have the following shape:

$$\nu(t) = \nu - \Delta_\nu e^{-\mu_\nu t} \mathbb{1}\{t \geq 0\}$$

I set the size of the shock Δ_ν to 2.2% of the steady-state value ν and persistence parameter to $\mu_\nu = 0.78$ in accordance with my estimation results from Section 6.

The aggregate effects of these two shocks are qualitatively similar. Output falls on impact, so the interest rate has to rise to make agents consume less. Asset prices fall because of lower future dividends, and the rising interest rate depresses them even further. Magnitudes differ by about three times: the output share of the special country is approximately one-quarter of the total.

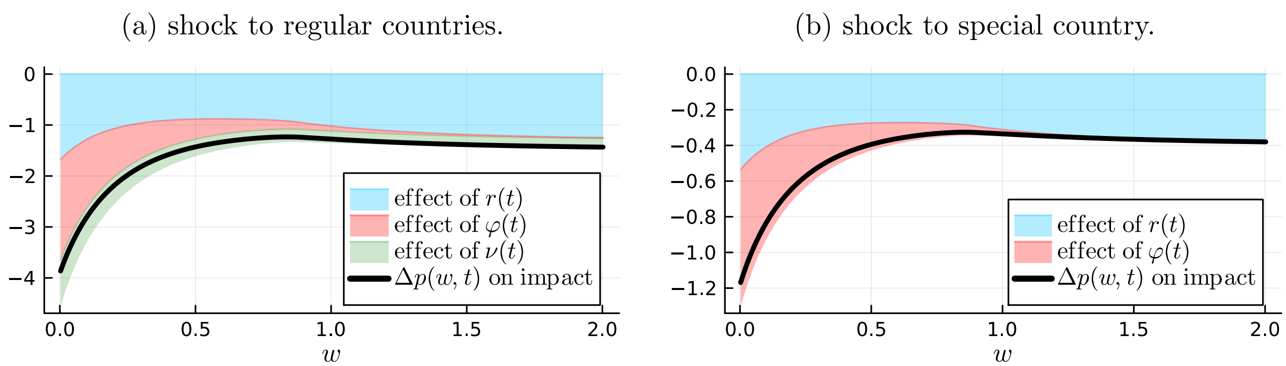


Figure 8: Changes in risky asset prices on impact: percentage change in $p(w,t)$ relative to the steady state decomposed into the effect of interest rate $r(t)$, foreign demand $\varphi(t)$, and dividends $\nu(t)$. Shock to ν in regular countries on panel (a) and the special country on panel (b).

The falling asset prices revalue everyone's wealth. In particular, the intermediary's net worth $\hat{w}(t)$ takes a hit, which affects the global factor $\varphi(t) = \gamma(t)\hat{w}(t)$. From the perspective of the equilibrium condition on excess returns in equation (9), it looks like a fall in the intermediary's risk-taking capacity. Importantly, this happens in both cases. It is natural to expect losses to be distributed between everyone when the shock hits regular countries: both local agents and global intermediaries hold their assets. The shock to the special country, on the other hand, hits dividends on the safe asset, which is only held by the intermediary. The fact that it spreads to regular countries through $\varphi(t)$ and $r(t)$ reflects strong contagion forces.

Figure 8 shows the cross-section of impact changes in asset prices. Unlike with a shock to risk-taking capacity $\gamma(t)$, prices fall everywhere because the interest rate rises. Panel (a) shows a rough decomposition of the impact effect into parts that come from a jump in $r(t)$, a fall in $\varphi(t)$, and a fall in $\nu(t)$. The interest rates drive asset prices in the right tail of the wealth distribution.

In poor countries, prices respond to foreign demand $\varphi(t)$ and discounted future cash flows more. The decomposition is not exactly additive since the price function is highly non-linear.

Empirically, US shocks are an important driver of global dynamics. Boehm and Kroner (2023) show that news about the US economy has strong effects on risky asset prices globally. Miranda-Agrippino and Rey (2020) provide evidence for the global impact of contractionary monetary policy in the US. Kalemli-Özcan (2019) documents spillovers of US monetary policy that are particularly large for emerging markets. My model does not have a nominal dimension and monetary policy, but the real projection of a richer nominal model could generate spillovers from the special country to the rest of the world through the same mechanism.

Differences between shocks to γ and ν. Prices react to financial and real shocks in different ways. After a negative shock to output, all prices fall, showing strong procyclicality with respect to ν. After a negative shock to risk-taking capacity, prices in poor countries fall, and those in rich countries rise. The former are procyclical with respect to γ, and the latter countercyclical. Special country's tree shows the same pattern as those in rich countries.

Another contrast between these shocks to ν and γ is capital flows on impact. Output shocks generate flows of similar magnitudes across countries. The reason is that they do not raise disagreement on the risk properties of assets. The absolute risk aversion of the intermediary is $\gamma(t)\hat{w}(t)\eta(w)$, and that of the local investor is simply w. Since γ is constant, they only change relative to each other to the extent that w and $\hat{w}(t)$ change by different amounts. Hence, trades are driven by differences in wealth revaluation between local agents and intermediaries. These differences are similar across countries.

With a shock to γ, this is not the case: the fall in γ dominates movements in wealth, so the shock opens a large gap between absolute risk aversion coefficients, generating trades. Capital flows are strongly procyclical in rich countries and less so in poor ones.

I next estimate the model using data on asset returns and capital flows. Output shocks are essential since data show large swings in aggregate returns. Since the effects of output shocks are similar when they originate in the special country and in the rest of the world, I shock ν in all countries at the same time.

6 Estimation

To estimate the model, I work with its first-order approximation. Shocks to γ and ν drive the dynamics. I use the following processes for their deviations from the steady state:

$$d\tilde{\gamma}(t) = -\mu_\gamma \tilde{\gamma}(t)dt + \sigma_\gamma \cdot dW(t)$$
$$d\tilde{\nu}(t) = -\mu_\nu \tilde{\nu}(t)dt + \sigma_\nu \cdot dW(t)$$

Here $\tilde{\gamma}(t) = \gamma(t) - \gamma$ and $\tilde{\nu}(t) = \nu(t) - \nu$ with (γ, ν) being the steady-state values. The shock increments $dW(t)$ are two-dimensional standard Brownian. Parameters μ_γ and μ_ν govern persistence. Loadings σ_γ and σ_ν are two-dimensional vectors. The first-order approximation of the model is around $\sigma_\gamma = \sigma_\nu = (0, 0)$.

I work in the sequence space and compute sequence-space Jacobians. Take two sequences of deviaitons $\{\tilde{\gamma}(t)\}_{t\geq 0}$ and $\{\tilde{\nu}(t)\}_{t\geq 0}$ that become known at $t = 0$. Take also a sequence $\{z(w,t)\}_{t\geq 0}$. For any $t \geq 0$, the first-order deviation of $z(w,t)$ is

$$\tilde{z}(w,t) = \int_0^\infty J_{z,\gamma}(w,t,\tau)\tilde{\gamma}(\tau)d\tau + \int_0^\infty J_{z,\nu}(w,t,\tau)\tilde{\nu}(\tau)d\tau$$

Here $J_{z,\gamma}$ and $J_{z,\nu}$ are sequence-space Jacobians of z. They are defined for $t \geq 0$, $\tau \geq 0$, and all w. The fact the deviation sequences $\{\tilde{\gamma}(t), \tilde{\nu}(t)\}_{t\geq 0}$ become known at $t = 0$ is important. Shocks dW that will happen in the future cannot affect the economy now through the deviations they have not yet created. The time-varying function of wealth $z(w,t)$ can represent, for example, asset prices $p(w,t)$ or the probability density of wealth $g(w,t)$.

I find these sequence-space Jacobians by solving the linearized version of the coupled system of equations from Proposition 1. Appendix L shows the linearized system and explains the algorithm.

First-order deviations of aggregate sequences $\{x(t)\}_{t\geq 0}$ can be written as

$$\tilde{x}(t) = \int_0^\infty J_{x,\gamma}(t,\tau)\tilde{\gamma}(\tau)d\tau + \int_0^\infty J_{x,\nu}(t,\tau)\tilde{\nu}(\tau)d\tau$$

with $J_{x,\gamma}$ and $J_{x,\nu}$ being its sequence-space Jacobians.

Since these deviations are first-order, $\tilde{x}(t)$ additively subsumes the first-order reaction of $x(t)$ to all deviations in γ and ν that became known prior to t. At any $s \leq t$, the shock $dW(s)$ reveals a new devition in γ and ν that has the following time profile: $\tilde{\gamma}(s+\tau) = e^{-\mu_\gamma \tau}\sigma_\gamma \cdot dW(s)$ and $\tilde{\nu}(s+\tau) = e^{-\mu_\nu \tau}\sigma_\nu \cdot dW(s)$ for all $\tau \geq 0$. The deviation $\tilde{x}(t)$ adds up responses to all these happening before t:

$$\tilde{x}(t) = \int_{-\infty}^t \left(\int_0^\infty J_{x,\gamma}(t-s,\tau)e^{-\mu_\gamma\tau}d\tau\right)\sigma_\gamma \cdot dW(s) + \int_{-\infty}^t \left(\int_0^\infty J_{x,\nu}(t-s,\tau)e^{-\mu_\nu\tau}d\tau\right)\sigma_\nu \cdot dW(s)$$

This can be rewritten as

$$\tilde{x}(t) = \int_{-\infty}^t (\hat{J}_{x,\gamma}(t-s;\mu_\gamma)\sigma_\gamma + \hat{J}_{x,\nu}(t-s;\mu_\nu)\sigma_\nu) \cdot dW(s)$$

Here the reduced Jacobians $\hat{J}_{x,\gamma}$ and $\hat{J}_{x,\nu}$ are

$$\hat{J}_{x,\gamma}(t;\mu_\gamma) = \int_0^\infty J_{x,\gamma}(t,\tau)e^{-\mu_\gamma \tau}d\tau$$
$$\hat{J}_{x,\nu}(t;\mu_\nu) = \int_0^\infty J_{x,\nu}(t,\tau)e^{-\mu_\nu \tau}d\tau$$

For estimation, a sample path of $\tilde{x}(t)$ can be integrated from a simulated sequence of shocks dW given parameters. Jacobians $J_{x,\gamma}$ and $J_{x,\nu}$ only require knowing the steady state. Parameters that determine the steady state come from the calibration procedure explained in Section 4. To estimate the parameters of the processes for γ and ν, I use the method of simulated moments.

Data. I use two aggregates $x(t)$: total outflows $m(t)$ and average risky asset prices $p(t)$ given by

$$m(t) = \int (1-\theta(w,t))wdG(w,t)$$
$$p(t) = \int p(w,t)dG(w,t)$$

I compute their first-order deviations in the model and normalize them by the steady-state values. These quantities are denoted by $\tilde{m}(t)$ and $\tilde{p}(t)$.

To construct the data analog for outward flows, I take the Balance of Payments and International Investment Positions data from the IMF. For a country i in quarter t, the quantity f_{it}^{raw} denotes net purchases of foreign assets. Following Forbes and Warnock (2012) and Forbes and Warnock (2021), I take a smooth version:

$$f_{it} = \sum_{t-3}^{t} f_{it}^{\text{raw}} - \sum_{t-7}^{t-4} f_{it}^{\text{raw}}$$

I restrict attention to portfolio debt, portfolio equity, and "other" assets (banking flows). The analog of $\tilde{m}(t)$ is

$$M(t) = \frac{\sum_i f_{it}}{\sum_i A_{i,t-1}}$$

where $A_{i,t-1}$ is the stock of these assets one quarter before in country i. See Appendix B for details of data construction and sample statistics.

I construct the analog of $\tilde{p}(t)$ from the MCSI asset price index that excludes the US. Denote the quarterly version of this index by Q_t. The analog of $\tilde{p}(t)$ is quarterly returns smoothed over the four-quarter window: $P(t) = \sum_{t-3}^{t} Q_s/Q_{s-1}$.

Figure 9 shows the data. The sample is 85 quarters long, starting in Q4 of 2001, which is the first point at which I have more than 10 emerging markets in the sample for capital flows. I

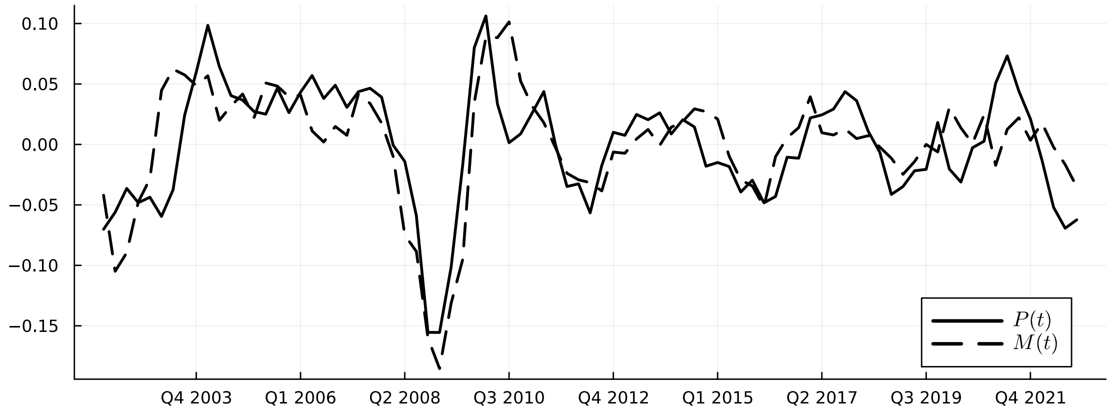


Figure 9: Constructed data series $P(t)$ and $M(t)$.

compute moments of these series and compare them to moments of simulated sequences, looking for a combination of parameters that minimizes a quadratic distance. Table 4 shows the targets.

Table 4: Targeted moments.

	std($\tilde{p}_t$)	std($\tilde{m}_t$)	corr($\tilde{p}_t, \tilde{m}_t$)	corr($\tilde{p}_t, \tilde{p}_{t-1}$)	corr($\tilde{m}_t, \tilde{m}_{t-1}$)
data	0.048	0.049	0.738	0.785	0.828
model	0.048	0.049	0.740	0.779	0.839

I estimate five parameters in total. The first two are persistence levels (μ_γ, μ_ν). Both loading vectors are two-dimensional: $\sigma_\gamma = (\sigma_{\gamma 1}, \sigma_{\gamma 2})$ and $\sigma_\nu = (\sigma_{\nu 1}, \sigma_{\nu 2})$. I normalize $\sigma_{\nu 1} = 0$. The first dimension of the shock, $dW_1(t)$, only affects $\tilde{\gamma}(t)$, while $dW_2(t)$ hits both $\tilde{\gamma}(t)$ and $\tilde{\nu}(t)$. Increments $d\tilde{\gamma}(t)$ and $d\tilde{\nu}(t)$ are potentially correlated. Table 5 shows estimated parameters. I estimate standard errors using parametric bootstrap. See Appendix H.2 for details.

Table 5: Estimation results.

μ_γ	μ_ν	$\sigma_{\gamma 1}$	$\sigma_{\gamma 2}$	$\sigma_{\nu 2}$
0.244	0.776	0.126	0.084	0.0039
(0.093)	(0.045)	(0.031)	(0.010)	(0.0001)

Figure 10 shows the first-order responses of $\tilde{m}(t)$ and $\tilde{p}(t)$ to the standard paths of $\tilde{\gamma}_t$ and $\tilde{\nu}_t$:

$$\tilde{\gamma}(t) = |\sigma_\gamma| e^{-\mu_\gamma t} \cdot \mathbb{1}\{t \geq 0\}$$
$$\tilde{\nu}(t) = |\sigma_\nu| e^{-\mu_\nu t} \cdot \mathbb{1}\{t \geq 0\}$$

The shock to γ affects outward flows more strongly than prices since prices in rich and poor countries move in opposite directions. The shock to ν affects prices more strongly, shifting all of them in the same direction. In aggregate data, the volatility of $P(t)$ and $M(t)$ is similar, so the

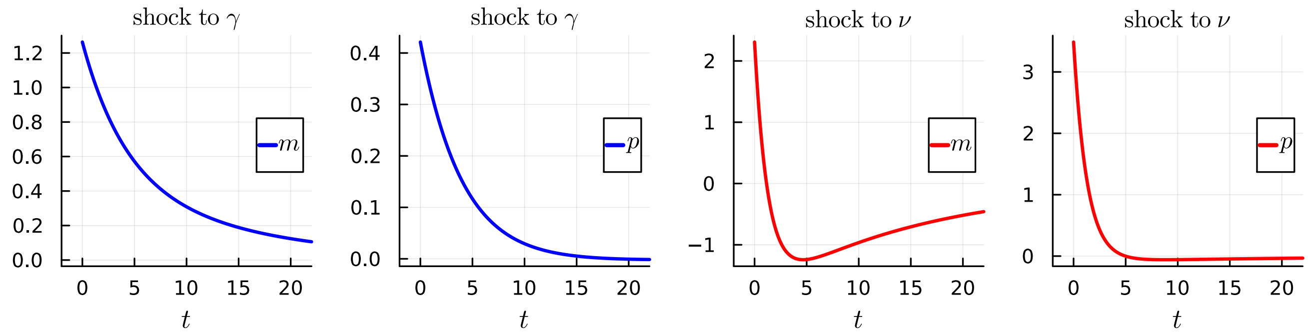


Figure 10: First-order responses of $\tilde{m}(t)$ and $\tilde{p}(t)$ to standard-sized shocks to γ and ν, measured in percent of the steady-state value.

model needs both shocks. In the next section, I analyze their contributions to the cyclicality of key quantities separately.

7 Quantitative Analysis

This section dissects the effect of financial and real shocks on global aggregates and on rich and poor countries separately. I start by showing that the model captures key differences between rich and poor countries along the cycle, even though I only use global sequences to estimate parameters of aggregate shocks.

Untargeted moments. I map rich countries in the model to advanced economies in the data. A natural threshold in the model is the level of wealth at which the portfolio constraint starts to bind in the steady state. I designate countries richer than this cutoff as advanced economies and those below as emerging markets.

Table 6: untargeted volatilities.

	outward flows		**asset prices**	
	std($\tilde{m}_t^{AE}$)	std($\tilde{m}_t^{EM}$)	std($\tilde{p}_t^{AE}$)	std($\tilde{p}_t^{EM}$)
data	0.045	0.035	0.042	0.059
model	0.074	0.027	0.030	0.048

Table 6 shows untargeted volatilities of outward flows and asset prices. The model captures two aspects of heterogeneity from the data. First, the volatility of outward flows as a share of assets is larger in advanced economies. In the model, this property derives naturally from the portfolio constraint that stops investors from poorer economies from reacting to global shocks. Second, asset prices are more volatile in emerging markets. The model generates the same pattern by allowing investors from rich economies to retrench and compensate for the shortfall of foreign demand without much higher risk premia.

The model overestimates the difference in the volatility of flows. This is because there is an artificially sharp change in behavior around the portfolio constraint. Redefining advanced economies and emerging markets so that some advanced economies are also constrained in the steady state partly alleviates this. The same applies to asset prices. The model definition of advanced economies overstates the difference in volatility, and designating some constrained countries as rich would make this less pronounced.

Table 7: untargeted correlations.

	$\text{corr}(\tilde{m}_t^{AE} - \tilde{m}_t^{EM}, \tilde{m}_t)$	$\text{corr}(\tilde{p}_t^{AE} - \tilde{p}_t^{EM}, \tilde{m}_t)$	$\text{corr}(\tilde{p}_t^{AE} - \tilde{p}_t^{EM}, \tilde{m}_t^{AE} - \tilde{m}_t^{EM})$
data	0.67	-0.16	-0.17
model	0.13	-0.55	-0.59

Table 7 shows correlations between the relative performance of rich and poor economies and aggregates. The correlation between $\tilde{m}_t^{AE} - \tilde{m}_t^{EM}$, which is how much more outward investment rich countries do, and global outward lows $\tilde{m}_t$ is positive in the data. This means, for example, that investors from rich countries more actively retrench in downturns. Adding this fact to that in Table 6 means that not only are outward flows more volatile in advanced economies, but they are also more closely timed to global shocks. The model reproduces this pattern, although it does not hit the correlation quantitatively.

The correlation of relative performance of assets $\tilde{p}_t^{AE} - \tilde{p}_t^{EM}$ with aggregate flows $\tilde{m}_t$ is negative. Advanced economies outperform emerging markets more in global downturns, which is when capital flows recede. In the model, a strong negative correlation is generated by more active retrenchment in rich countries insulating asset prices from foreign demand shocks. The last column confirms this, showing that advanced economies outperform emerging markets more when there is a deeper fall in their outward flows.

The converse of this is, of course, the better relative performance of emerging markets in global booms, which is when aggregate outward flows are on the rise. Advanced economies grow their external investment by more, and this reduction in domestic demand partly compensates for large inflows generated by intermediaries. In poor countries, domestic investors are constrained, so the reduction in the desired portfolio share of the domestic tree does not cause a reduction in the actual share. A rising foreign demand meets inelastic residual supply.

Financial and real shocks. Shocks to the intermediary's risk-taking capacity and output shocks have the same implications for poor countries in the model: asset prices fall after a decrease in either of the two. In rich countries, asset prices load positively on ν but negatively on γ. The same happens in the special country. The interest rate responds positively to output and negatively to risk-taking capacity. Figure 11 shows these patterns using impulse responses obtained with sequence-space Jacobians.

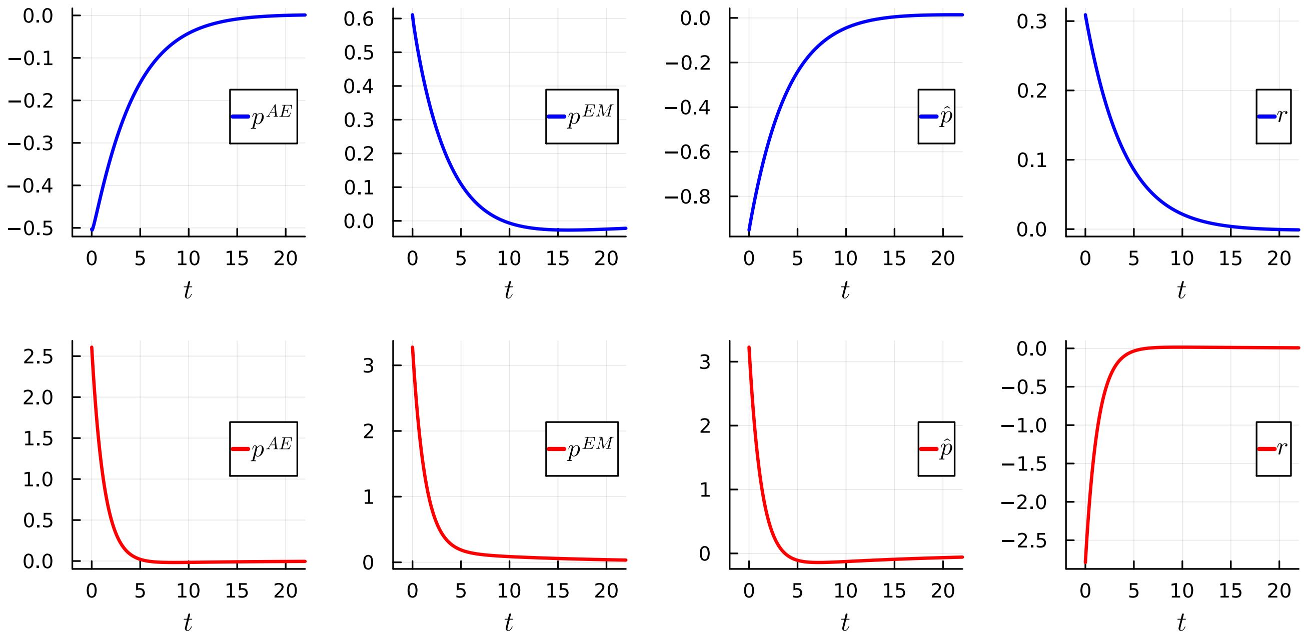


Figure 11: First-order impulse responses of asset prices averaged across poor countries p^{EM}, asset prices averaged across rich countries p^{AE}, the special country's price $\hat{p}$, and the interest rate r. Top row: shock to γ. Bottom row: shock to ν.

Table 8 shows variance decomposition for first-order deviations of total outward flows m_t, the global average of asset prices p_t, and separately averaged asset prices in advanced economies and emerging markets p^{AE} and p^{EM}. The last row shows the decomposition for p_t^{AE} - p_t^{EM}, which measures relative performance of assets in advanced economies compared to emerging markets.

	data	full model	only γ	only ν
m_t	0.049	0.049	0.024	0.044
p_t	0.048	0.048	0.007	0.044
p_t^{AE}	0.042	0.030	0.009	0.033
p_t^{EM}	0.059	0.048	0.010	0.042
p_t^{AE} - p_t^{EM}	0.035	0.026	0.019	0.010

Table 8: standard deviations of first-order responses. Responses measured in units of the respective steady-state values, except for the last row.

One observation is that output shocks contribute more to variance of both aggregate flows m_t and average asset prices p_t. The model attributes a large share of variation in the data to ν, so the picture would be incomplete without including it. Moreover, I estimate the shocks to have a positive correlation of 0.56, so output is often falling in times of low risk-taking capacity.

Another observation is that the relative contribution of shocks to risk-taking capacity to flows m_t is larger than that to prices p_t. This is consistent with the fact that shocks to γ are redistributive and do not destroy aggregate wealth. Financial shocks explain 24% of the variance in aggregate capital flows.

The model can explain around 55% of the variance in the $p_t^{AE} - p_t^{EM}$, the relative performance of assets in advanced economies compared to emerging markets. Financial shocks alone explain about a third of the total variance, and real shocks in isolation about 10%.

The relative performance of advanced economies is one quantity to which output shocks contribute less than the shock to risk-taking capacity. The reason is that advanced economies outperform emerging markets in downturns due to more active retrenchment, which is tightly connected to risk-taking capacity shocks. Output shocks do not generate differential retrenchment patterns, since flows in times of low output are mainly generated by domestic agents losing less wealth than intermediaries and becoming relatively richer, which leads to them taking over the tree. This happens in all countries, with little effect of size or the portfolio constraint.

Cyclicality. Since the two shocks move asset prices in advanced economies in oposite directions, it is natural to expect the cyclicality of prices to be different from that in emerging markets. To assess cyclicality, I use their correlation with aggregate outward flows $\tilde{m}_t$. The response of $\tilde{m}_t$ to both shocks is positive, and $\tilde{m}_t$ is a natural barometer of the global financial cycle. I compute the correlations in the full model and then shut down the shocks to γ and ν separately to gauge their respective contributions.

	full model	only γ	only ν
$\hat{p}_t$	0.43	-0.96	0.66
p_t^{AE}	0.52	-0.97	0.58
p_t^{EM}	0.69	0.93	0.48
r_t	-0.62	0.97	-0.57
$p_t^{AE} - p_t^{EM}$	-0.55	-0.95	-0.18

Table 9: correlations of first-order responses with total outflows $\tilde{m}_t$

Table 9 shows the correlations. All prices are procyclical in the full model, and the interest rate is countercyclical: it rises in downturns. In emerging markets, correlation with $\tilde{m}_t$ is the highest, since both shocks to γ and ν contribute positively. In advanced economies and in the special country, shocks to risk-taking capacity contribute negatively, which partly offsets positive comovement coming from shocks to output, and the overall correlation is lower. As a result, relative performance of assets in advanced economies compared to emerging markets is countercyclical.

Table 10 shows what this implies for the cyclicality of wealth. The correlation of wealth in advanced economies and the special country with $\tilde{m}_t$ is three times lower than that in emerging markets. This difference is coming from the shocks to risk-taking capacity that redistribute to rich countries in downturns, suppressing the volatility of their wealth.

	full model	only γ	only ν
$\hat{w}_t$	0.30	-0.95	0.11
w_t^{AE}	0.32	-0.89	0.97
w_t^{EM}	0.94	0.97	0.99

Table 10: correlations of first-order responses with total outflows $\tilde{m}_t$

The shock to risk-taking capacity turns out to have a stabilizing impact in rich countries, especially since it is positively correlated with the output shock. In times of low output, the risk-taking capacity of global intermediaries is often low as well, which benefits rich countries since their propensity to retrench supports their asset prices in downturns.

Limitations of the model. The model is an exchange economy without differentiated goods, nominal rigidities, investment, policymakers, or within-country heterogeneity.

Adding differentiated goods and nominal rigidities can help incorporate the feedback between aggregate demand and output. I estimate a positive correlation between shocks to risk-taking capacity and output shocks. This suggests that a model with endogenous output could fit the data just as well if the output is occasionally demand-determined and risk-taking capacity shocks affect aggregate demand through asset prices, as described by Caballero and Simsek (2020b).

Nominal rigidities are important for monetary policy. Kalemli-Özcan (2019) shows that advanced economies and emerging markets are exposed to the US monetary policy in different ways. Their own policy responses are different too, as shown by De Leo et al. (2022) and Das et al. (2022). This heterogeneity could have aggregate implications.

The fact that in reality, all countries issue assets denominated in dollars as well as their local currency generates another contagion mechanism, as pointed out by Jiang et al. (2020). Tracking the distribution of countries and gross capital flows is important for studying these questions. However, the picture is incomplete without fluctuating exchange rates since exchange rates absorb non-trivial fractions of movements in risk premia and asset prices.

Investment and, more generally, endogenous supply of assets is necessary to model more realistic macro adjustment to short-run shocks and long-run growth. When the supply of assets is not fixed, gross flows matter for borrowing costs and have an important connection to the real economy. Adding these elements to the model can open the door to studying capital controls, industrial policy, and their external spillovers.

The data I use do not include reserves, which in reality play an important role in external adjustment. Reserves are deployed by social planners and do not only respond to risks and returns. I also do not consider capital control or financial repression policies. Local planners may turn out to create international externalities, posing a non-trivial problem for the global planner. Considering the local planner's problem could be a useful next step, as well as thinking about a global planner that addresses possible coordination issues between local planners around the world.

8 Conclusion

I develop a model of the world economy with two tiers of heterogeneity between countries. The special country occupies the place of a global intermediary and issues safe assets. Regular countries issue risky assets and endogenously differ in wealth. The differences in wealth lead to different responses to global shocks that drive risk premia and capital flows.

In particular, when the intermediary's risk-taking capacity decreases and it sells risky assets around the world, investors in rich countries retrench and support the prices of domestic assets. Poor countries do not have enough wealth and hit the constraint that prevents them from massively borrowing to replicate the response of rich countries with leverage. In their markets, prices adjust instead of quantities and drop sharply. The falling interest rate revalues assets, appreciating those issued by rich countries, which effectively makes them good substitutes for safe assets. The distributional impact is regressive, although rich countries insure the poor ones.

I find that real shocks are as important as financial shocks for the model to capture aggregate fluctuations in capital flows and asset prices. However, financial shocks are the main driver of the relative performance of assets in rich countries compared to those in poor ones. Asset prices in rich countries load negatively on the intermediary's risk-taking capacity, and this makes their wealth less volatile and less cyclical.

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A Two facts about capital flows

In this section, I establish the fact that outward flows from advanced economies are more strongly correlated with global aggregates, and their cyclical components are more volatile.

"Outflows" are net purchases of foreign assets by domestic investors. "Inflows" are net purchases of local assets by foreign investors. Both are "gross" flows, even though they can be negative. They do not have to add up to zero across countries, in contrast to net flows. I zoom in on outflows to focus on the role of domestic investors in shaping international heterogeneity.

The main point this section makes is that private outflows from advanced economies are more synchronized with the global financial cycle, both in terms of correlation and in terms of magnitude. Investors from advanced economies time their retrenchment to downturns more precisely and retrench more actively than those in emerging markets.

The data come from the Balance of Payments and International Investment Positions statistics provided by the IMF. I supplement these with GDP data from the World Bank. See Appendix A for details. Following the detrending procedure from Forbes and Warnock (2012) and Forbes and Warnock (2021), I construct the following variables:

$$f_{it} = \sum_{s=t-3}^{s=t} FA_{i,s} - \sum_{s=t-7}^{s=t-4} FA_{i,s} \tag{A.1}$$

Here t is a quarter, i is a country, and $FA_{i,t}$ records net purchases of foreign assets by i. Forbes and Warnock (2012) and Forbes and Warnock (2021) use similar measures to detect extreme capital flow episodes such as stops, surges, flight, and retrenchment. The position-adjusted version is

$$\overline{f}_{it} = \sum_{s=t-3}^{s=t} \frac{FA_{i,s}}{A_{i,s-1}} - \sum_{s=t-7}^{s=t-4} \frac{FA_{i,s}}{A_{i,s-1}} \tag{A.2}$$

Here A_{it} is the total stock of i's external assets in quarter t. Both FA_{it} and A_{it} are measured in dollars and exclude reserves and FDI. Of course, private outflows do not fully describe adjustment to shocks, since countries regularly deploy reserves, often at a large scale. These interventions and other operations not driven by profit maximization are beyond the scope of my paper.

Note that $FA_{i,t}$ is different from the total change in position $A_{i,t} - A_{i,t-1}$ because the latter includes valuation effects. The variables f_{it} and $\overline{f}_{it}$ are constructed to account for trades, not price changes. Equation (A.1) calculates a de-trended and de-seasoned version of changes in assets due to trades and equation (A.2) calculates percentage changes due to trades.

To track the global cycle, I extract the principal component ϕ_t, which I call the outflow factor, from a balanced subpanel of f_{it} covering the last 20 years. I rescale all series in the panel to have the same volatility so that ϕ_t is not mechanically driven by countries with the largest flows.

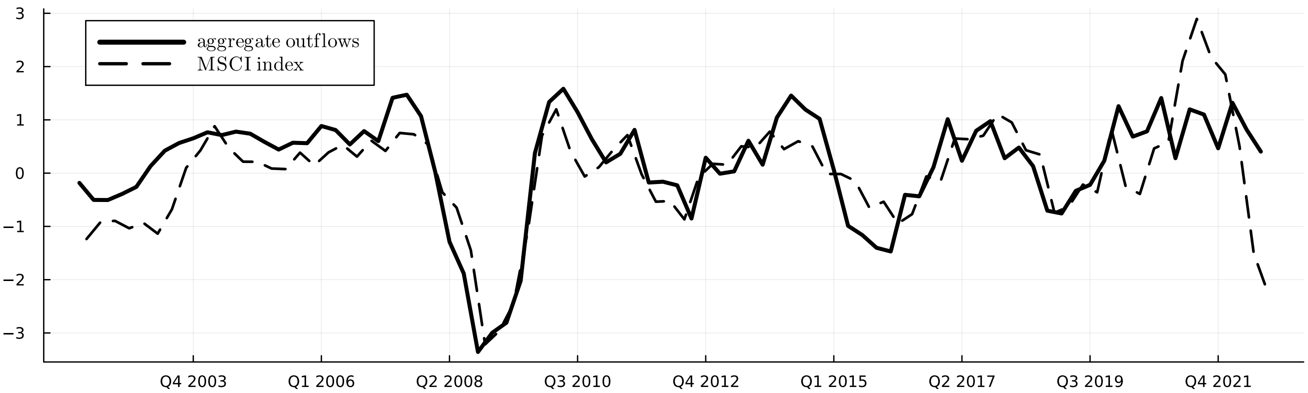

Figure A.1: Principal component ϕ_t of outflows f_{it} and quarterly returns on MSCI ACWI smoothed over the four-quarter windows, both normalized to have zero mean and unit standard deviation.

Figure A.1 shows ϕ_t with returns on MSCI ACWI, a global index of asset prices with a broad international coverage. A large literature (see Miranda-Agrippino and Rey (2022) for a review) documents co-movement between capital flows and measures of global risk appetite. Outflows from countries that load more on ϕ_t should be more aligned with these measures and more closely follow the global cycle. These more exposed countries turn out to be advanced economies.

Fact 1: outflows from advanced economies are more correlated with global factors. The principal component ϕ_t of capital outflows explains more variation in outflows f_{it} from advanced economies than from emerging markets. I measure this by computing R^2 of

$$f_{it} = \alpha_i + \beta_i \phi_t + \varepsilon_{it}$$

Figure A.2 shows results for individual countries in the sample. The average over advanced economies is 26% and that over emerging markets is 10%. Outflows from advanced economies are more tightly connected to global factors than those from emerging markets.

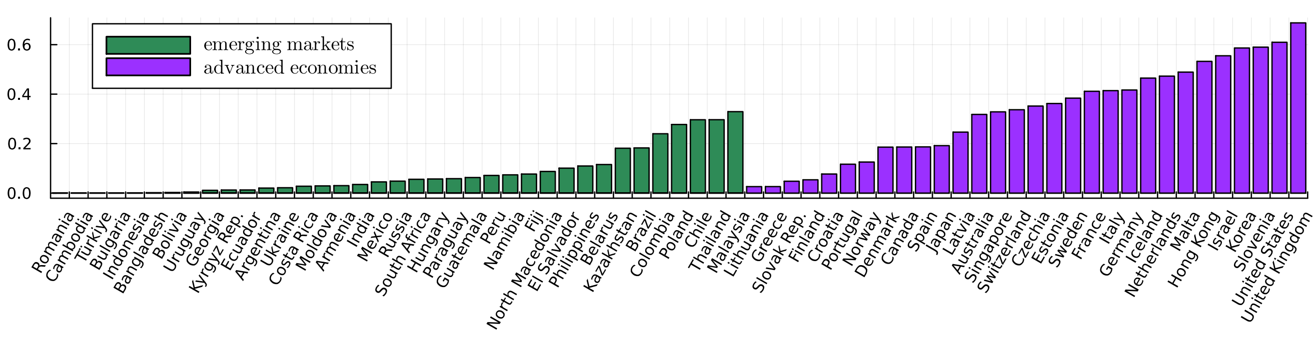

Figure A.2: Share of time-series variation in f_{it} explained by ϕ_t for a given country i.

This means that outflows from advanced economies co-move with measures or risk appetites more closely since they are more tightly connected to ϕ_t. Table 11 confirms this by showing correlations between within-group averages and time series associated with the global financial cycle.

Table 11: Correlation between aggregate series and averages $\{f_t^{AE}, \overline{f}_t^{AE}, f_t^{EM}, \overline{f}_t^{EM}\}$

	f_t^{AE}	f_t^{EM}	$\overline{f}_t^{AE}$	$\overline{f}_t^{EM}$
outflow factor ϕ_t	0.95	0.24	**0.86**	0.29
VIX (negative)	0.36	0.20	**0.38**	0.15
asset price factor, Miranda-Agrippino and Rey (2020)	0.42	0.46	**0.32**	0.04
intermediary factor, He et al. (2017)	0.24	0.01	**0.21**	-0.16
treasury basis, Jiang et al. (2021)	0.27	0.03	**0.27**	0.00

Here the variables f_t^{AE} and $\overline{f}_t^{AE}$ average f_{it} and $\overline{f}_{it}$ over advanced economies, while f_t^{EM} and $\overline{f}_t^{EM}$ average outflows from emerging markets. The asset price factor from Miranda-Agrippino and Rey (2020) is a dominant component that they extract from 858 series of risky asset prices around the world. The intermediary factor from He et al. (2017) traces the capital ratios of financial intermediaries, is highly cyclical, and explains variation in expected returns on large classes of assets. The treasury basis from Jiang et al. (2021) is a measure of convenience yield on short-term US treasury bonds that rises with demand for these safe assets.

The differences in correlations between advanced economies and emerging markets are noticeable in most cases and particularly salient when flows are measured relative to outstanding positions (the highlighted column). Outflows from advanced economies are more strongly synchronized with measures of global risk appetites. I next show that they are larger in magnitude.

Fact 2: cyclical component of outflows relative to outstanding positions is more volatile in advanced economies. Measuring magnitudes requires an adjustment for size. Flows relative to outstanding positions $\overline{f}_{it}$ is a natural candidate. They show how much of their aggregate portfolio investors move during the cycle. To extract the cyclical component, I measure the group-specific loadings of $\overline{f}_{it}$ on the dominant component in outward flows ϕ_t:

$$\overline{f}_{it} = \alpha_i + \gamma\phi_t + \beta \mathbb{1}\{i \in \text{AE}\}\phi_t + \varepsilon_{it}$$

The coefficient β measures the difference between the loadings of position-adjusted flows on the dominant factor between advanced economies and emerging markets. Table 12 shows that the loadings are larger in advanced economies. The quantitative interpretation is that investors from advanced economies repatriate 2.5% more of their external assets when the dominant component ϕ_t is one standard deviation down.

Fact 1 shows that investors in advanced economies time their purchases of foreign assets to the global financial cycle. Fact 2 now additionally shows that they rebalance portfolios more actively, as measured by flows relative to outstanding positions. The same applies to busts: advanced

Table 12: the dependent variable $\overline{f}_{it}$ is expressed as percentage.

ϕ_t	1.290
	(0.433)
$\mathbb{1}\{i \in \text{AE}\}\phi_t$	**2.523**
	(0.483)
$R^2 = 0.04$, $N = 5965$	

economies retrench in crises and do it more actively than emerging markets.

Figure A.3 summarizes the patterns of synchronization in the two groups. Solid lines show the average components $\overline{f}_t^{AE}$ and $\overline{f}_t^{EM}$. This cross-sectional dispersion for every t is shown by the shaded area. Position-adjusted outflows in advanced economies follow the outflow factor more closely on average and are less dispersed around it. Outflows are overall more volatile in emerging markets, but this is because idiosyncratic variation dominates. The average component in advanced economies is more volatile than that in emerging markets. I show this with a variance decomposition in Appendix B.

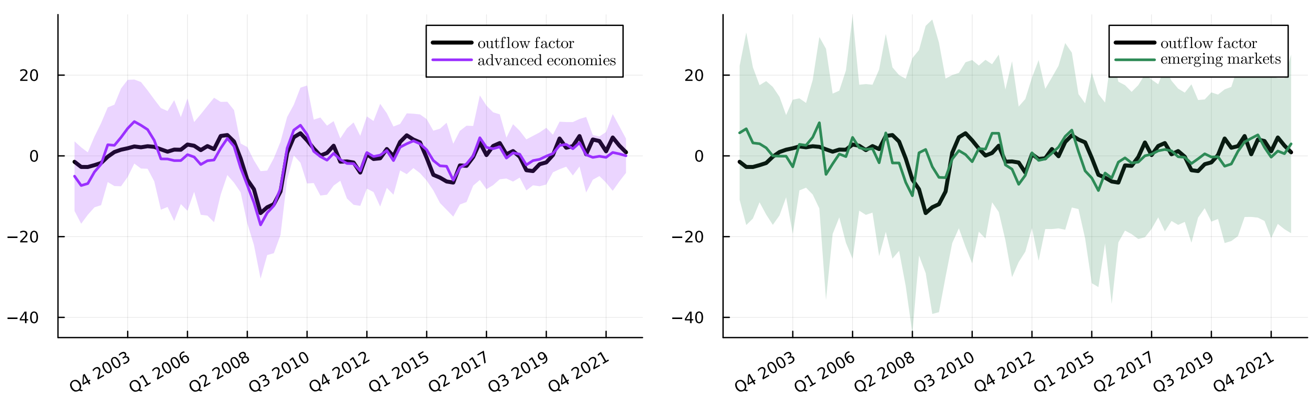

Figure A.3: Aggregate outflows from advanced economies and emerging markets relative to external assets. Black line: outflow factor ϕ_t, rescaled for illustration. Solid line and shaded areas show cross-sectional averages and standard deviations of $\overline{f}_{it}$ for each quarter t.

Appendix B also shows that these results are stronger for GDP-adjusted flows. The reason is that advanced economies have more external assets relative to GDP, so flows as a share of GDP are more volatile in both aggregate and idiosyncratic components. Flows relative to positions are more informative because they quantify investor activity relative to their wealth. The fact that wealth is not equally comparable with output in all economies speaks to the economics of long-run asset accumulation. The fact that investors in advanced economies, on aggregate, move larger shares of their portfolio with the global financial cycle, is instead more informative about their demand for risky assets and the distribution of short-run demand elasticities. These observations motivate the model.

B Additional details for empirical facts

I use IMF data on assets and liabilities from the International Investment Position dataset and financial accounts from the Balance of Payments dataset. I combine “portfolio investment” (both debt and equity) with “other investment”, which contains bank loans. The GDP data come from the World Bank.

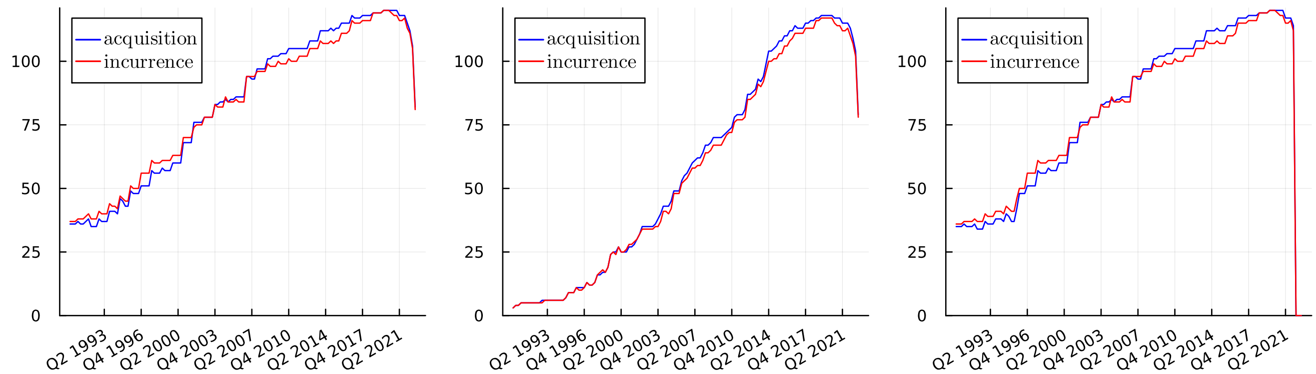

Figure A.4: Number of countries in the sample. Left: acquisition of external assets and incurrence of external liabilities. Center: acquisition of assets and incurrence of liabilities relative to assets and liabilities outstanding. Right: acquisition of assets and incurrence of liabilities to GDP.

The panels are highly unbalanced. Figure A.4 shows the time-varying size of the cross-section. To extract the outflow factor, I choose the time period from 2001 Q3 to 2022 Q3. The country groups for this exercise are:

- Advanced economies: Australia, Canada, Croatia, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong, Iceland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Netherlands, Norway, Portugal, Singapore, Slovak Rep., Slovenia, Spain, Sweden, Switzerland, United Kingdom

- Emerging markets: Argentina, Armenia, Bangladesh, Belarus, Bolivia, Brazil, Bulgaria, Cambodia, Chile, Colombia, Costa Rica, Ecuador, El Salvador, Georgia, Guatemala, Hungary, India, Indonesia, Kazakhstan, Kyrgyz Rep., Mexico, Moldova, Namibia, North Macedonia, Pakistan, Peru, Philippines, Romania, South Africa, Thailand, Türkiye, Ukraine

I designate the United States, Cyprus, and Malta, who are also available, as offshores.

In addition to the measures of outflows f_{it} and $\overline{f}_{it}$ defined in the main text, I define a GDP-adjusted measure $\underline{f}_{it}$:

$$\underline{f}_{it} = \sum_{s=t-3}^{s=t} \frac{FA_{i,s}}{GDP_{i,s-1}} - \sum_{s=t-7}^{s=t-4} \frac{FA_{i,s}}{GDP_{i,s-1}} \tag{A.3}$$

I interpolate GDP from annual data linearly. Table 13 supplements Table 11 with the same correlations computed for $\underline{f}_t^{AE}$ and $\underline{f}_t^{EM}$.

Table 13: Correlation between aggregate series and averages $\{f_t^{AE}, \overline{f}_t^{AE}, \underline{f}_t^{AE}, f_t^{EM}, \overline{f}_t^{EM}, \underline{f}_t^{EM}\}$

	f_t^{AE}	f_t^{EM}	$\overline{f}_t^{AE}$	$\overline{f}_t^{EM}$	$\underline{f}_t^{AE}$	$\underline{f}_t^{EM}$
outflow factor ϕ_t	0.95	0.24	**0.86**	0.29	0.90	0.62
VIX (negative)	0.36	0.20	**0.38**	0.15	0.40	0.30
asset price factor, Miranda-Agrippino and Rey (2020)	0.42	0.46	**0.32**	0.04	0.49	0.29
intermediary factor, He et al. (2017)	0.24	0.01	**0.21**	-0.16	0.27	0.24
treasury basis, Jiang et al. (2021)	0.27	0.03	**0.27**	0.00	0.28	0.16

The ordering of correlations for this measure is similar to those for f_{it} and $\overline{f}_{it}$.

B.1 Extracting factors

The balanced panel includes $N = 70$ countries and contains $T = 79$ quarters. The model is

$$\boldsymbol{f} = F\Lambda + \boldsymbol{\varepsilon} \tag{A.4}$$

Here $\boldsymbol{f}$ is a $T \times N$ matrix that collects cross-sections as rows, $\boldsymbol{f}_{ti} = f_{it}$. The matrix F is $T \times r$, where r is the number of factors. The matrix Λ is $r \times N$ and contains factor loadings. Error terms are in the $T \times N$ matrix $\boldsymbol{\epsilon}$.

I extract the principal component following Doz et al. (2012). The estimate S of the variance-covariance matrix is

$$S = \frac{1}{T}\boldsymbol{f}'\boldsymbol{f} \tag{A.5}$$

This matrix is $N \times N$. I denote by $\mathbb{D}$ the diagonal $r \times r$ matrix collecting r of its largest eigenvalues, and by $\mathbb{W}$ the $N \times r$ matrix collecting the corresponding eigenvectors as columns. The first r estimated components (the first denoted by ϕ_t) comprise the columns of the $T \times r$ matrix $\hat{F}$

$$\hat{F} = \boldsymbol{f}\mathbb{W}\mathbb{D}^{-1/2} \tag{A.6}$$

Table 11 shows correlations between in-group averages $\{f_t^{AE}, f_t^{EM}, \overline{f}_t^{AE}, \overline{f}_t^{EM}\}$ and time series measuring the global risk-taking capacity. Table 13 adds correlations for $\{\underline{f}_t^{AE}, \underline{f}_t^{EM}\}$. I download the quarterly VIX from the FRED website. It is available until Q4 2022. The asset price factor from Miranda-Agrippino et al. (2020) is available until Q4 2018, the intermediary factor from He et al. (2017) until Q4 2022, and the treasury basis from Jiang et al. (2021) until Q2 2017. The starting date for all these series, except ϕ_t, is Q1 1990. My particular vintage of ϕ_t starts in Q3 2001, giving the sample about 20 years.

This sample does not have to be balanced and uses more countries. The groups are

- Advanced economies: Australia, Austria, Belgium, Canada, Croatia, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hong Kong, Iceland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Netherlands, New Zealand, Norway, Portugal, Singapore, Slovak Rep., Slovenia, Spain, Sweden, Switzerland, United Kingdom

- Emerging markets: Afghanistan, Albania, Angola, Argentina, Armenia, Aruba, Azerbaijan, Bahamas, Bangladesh, Belarus, Belize, Bhutan, Bolivia, Bosnia and Herzegovina, Brazil, Brunei, Bulgaria, Cabo Verde, Cambodia, Cameroon, Chile, China, Colombia, Congo, Costa Rica, Curaçao and Sint Maarten, Djibouti, Dominican Rep., Ecuador, Egypt, El Salvador, Ethiopia, Fiji, Georgia, Guatemala, Guinea, Guyana, Haiti, Honduras, Hungary, India, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Kiribati, Kosovo, Kuwait, Kyrgyz Rep., Laos, Lebanon, Lesotho, Madagascar, Malaysia, Mauritania, Mauritius, Mexico, Moldova, Mongolia, Montenegro, Morocco, Mozambique, Myanmar, Namibia, Nepal, Nicaragua, Nigeria, North Macedonia, Pakistan, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Qatar, Romania, Russia, Rwanda, Samoa, Saudi Arabia, Serbia, Sint Maarten, Solomon Islands, South Africa, Sri Lanka, Sudan, Suriname, São Tomé and Príncipe, Tajikistan, Tanzania, Thailand, Tonga, Trinidad and Tobago, Türkiye, Uganda, Ukraine, Uruguay, Uzbekistan, Vanuatu, Venezuela, Vietnam, West Bank and Gaza, Yemen, Zambia, Zimbabwe

- Offshores: Bahrain, Bermuda, Cyprus, Ireland, Luxembourg, Malta, Panama, Seychelles, Timor-Leste, United States

Having extracted the factor, I run the following specification in the main text:

$$\overline{f}_{it} = \alpha_i + \gamma\phi_t + \beta\mathbb{1}\{i \in \text{AE}\}\phi_t + \varepsilon_{it} \tag{A.7}$$

An alternative, less flexible specification shuts down country-specific intercepts:

$$\overline{f}_{it} = \alpha + \gamma\phi_t + \beta\mathbb{1}\{i \in \text{AE}\}\phi_t + \varepsilon_{it} \tag{A.8}$$

This removes country fixed effects. Alternatively, I can make the specification more flexible by replacing $\gamma\phi_t$ with quarter-specific intercepts:

$$\overline{f}_{it} = \alpha + \gamma_t + \beta\mathbb{1}\{i \in \text{AE}\}\phi_t + \varepsilon_{it} \tag{A.9}$$

This adds time fixed effects. Table 14 shows that the results are very similar across specifications. The central column corresponds to the baseline from the main text.

Table 14: the dependent variable $\overline{f}_{it}$ is expressed as percentage.

ϕ_t	1.301	1.290	
	(0.448)	(0.433)	
$\mathbb{1}\{i \in \text{AE}\}\phi_t$	2.510	**2.523**	2.423
	(0.492)	(0.483)	(0.508)
R^2	0.02	0.04	0.05
country FE	NO	YES	YES
time FE	NO	NO	YES
$N = 5965$ across all specifications			

B.2 Variance decomposition

The variance of these size-adjusted flows can be decomposed into time-series dispersion of the averages $\overline{f}_t$ or $\underline{f}_t$ and cross-sectional dispersion around these averages. Taking $\overline{f}_{it}$,

$$\mathbb{V}[\overline{f}_{it}] = \underbrace{\mathbb{V}[\mathbb{E}[\overline{f}_{is}|s=t]]}_{\text{aggregate}} + \underbrace{\mathbb{E}[\mathbb{V}[\overline{f}_{is}|s=t]]}_{\text{idiosyncratic}} \tag{A.10}$$

Table 15: Decomposition of total variance of $\overline{f}_{it}$ within advanced economies and emerging markets. The sample spans Q1 2003 through Q4 2022 and contains 28 AE and 47 EM on average.

	standard deviation of $\overline{f}_{it}$	total variance	aggregate	idiosyncratic
advanced economies	0.10	0.0107	**0.0020**	0.0087
emerging markets	0.21	0.0456	**0.0012**	0.0444

Table 15 shows this decomposition conditional on advanced economies and emerging markets. Importantly, the aggregate component is larger in advanced economies. Not only are the average outflows from advanced economies more tightly connected to aggregates, but they are also more volatile.

Table 16: Decomposition of variance in $\underline{f}_{it}$ between advanced economies and emerging markets.

	standard deviation of $\overline{f}_{it}$	total variance	aggregate	idiosyncratic
advanced economies	0.17	0.0297	**0.0060**	0.0237
emerging markets	0.05	0.0026	**0.0001**	0.0025

Table 16 is the analog of Table 15 for flows adjusted by GDP. The differences between advanced economies and emerging markets are so much more pronounced because advanced economies have more external assets relative to GDP. Figure A.5 shows the ratio between average assets over GDP in advanced economies and emerging markets, $\underline{f}_t^{AE}/\underline{f}_t^{EM}$. It also shows the same ratio for liabilities over GDP. Advanced economies accumulate more assets relative to the size of their economies, so it is not particularly surprising that purchases and sales of foreign assets adjusted for output are larger in magnitude in these countries.

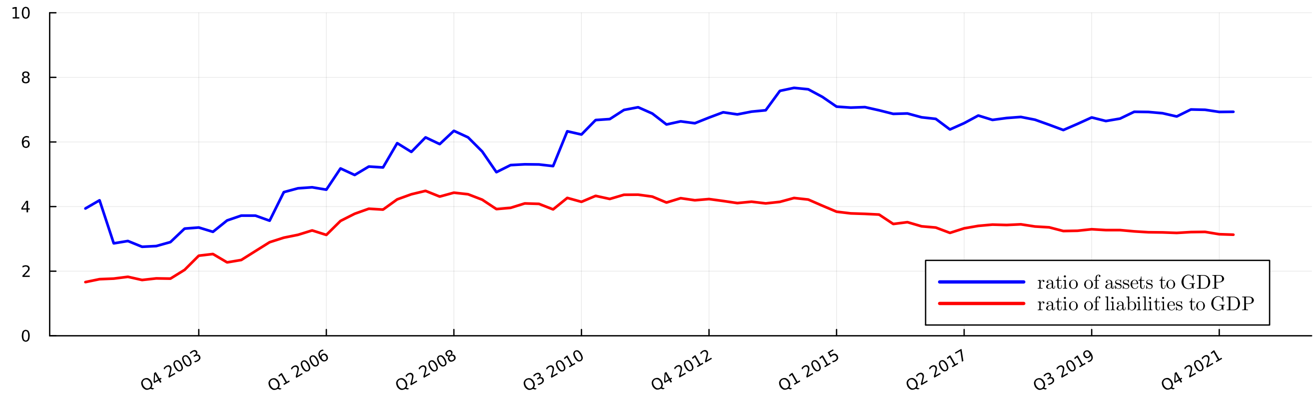

Figure A.5: Ratio of average assets-to-GDP and liabilities-to-GDP in advanced economies and emerging markets.

The synchronization of GDP-adjusted measures of flow with the global flows is also more pronounced in advanced economies. Figure A.6 shows the analog of Figure A.3 for $\underline{f}_t^{AE}$ and $\underline{f}_t^{EM}$. Emerging markets look even less synchronized with the global capital flows than on Figure A.3 since ϕ_t on these pictures is normalized to have a standard deviation that is the average of that of $\underline{f}_t^{AE}$ and $\underline{f}_t^{EM}$, and these two are substantially different. Using position-adjusted flows, as I do in Appendix A, is more informative because they are commensurate in size across advanced and emerging economies and because they capture the intensity of trading relative to wealth, which has a very different relationship to output in different countries.

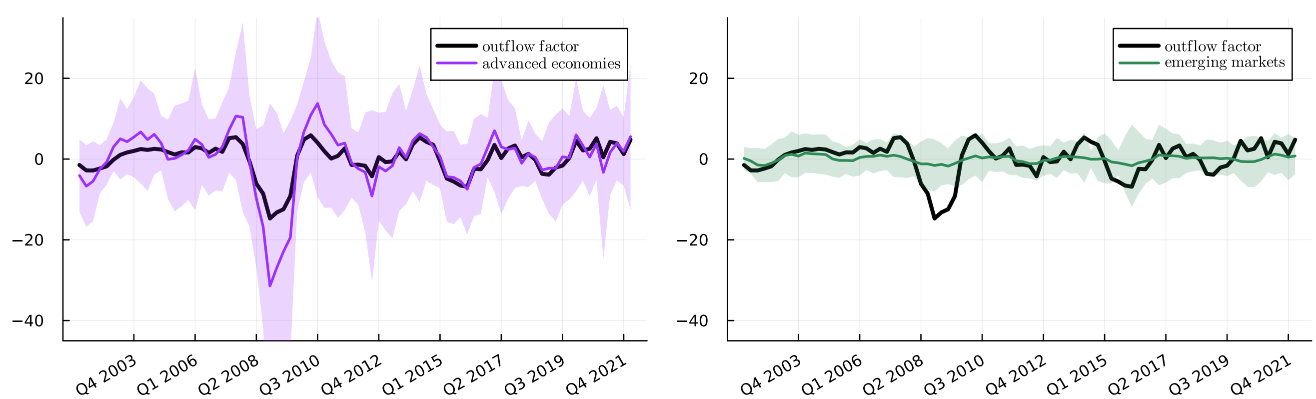

Figure A.6: Aggregate outflows from advanced economies and emerging markets as a percentage of GDP. Black line: outflow factor ϕ_t normalized to have the average standard deviation between $\underline{f}_{it}^{AE}$ and $\underline{f}_{it}^{EM}$. Error bands show cross-sectional standard deviations of $\overline{f}_{it}$ in the two groups for each quarter t.

Figure A.7 presents the time series of cross-sectional size for advanced economies and emerging markets. Advanced economies dominate the sample of position-adjusted outflows before 2008, but the groups are of comparable size. After 2008, emerging markets overtake advanced economies, outnumbering them by slightly more than two times at the peak. The idiosyncratic component of the total variance in the emerging market panel is five times larger. In the sample of outflows normalized by GDP, they always outnumber advanced economies by 2-2.5 times, and the idiosyncratic component of the total variance is 9 times smaller.

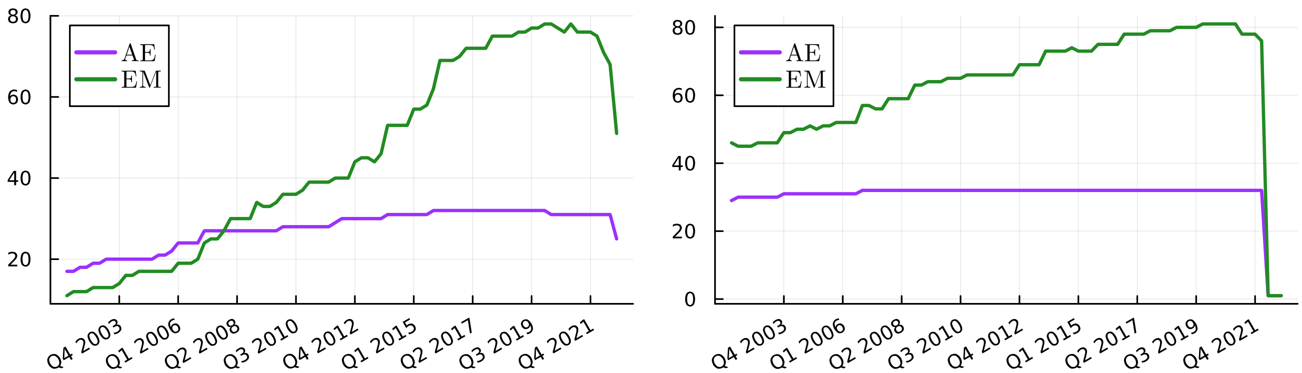

Figure A.7: Number of countries in the sample for the two groups: advanced economies and emerging markets. Left: the panel of outflows normalized by positions. Right: the panel of outflows normalized by GDP.

C Details of the simple model

In this section, I explain the properties of the steady state in the simple model that appear in the text of Section 2.

Holdings. I start by deriving the equilibrium expressions for $h(w)$ and $\theta(w)$. First, from the optimal choice of agents,

$$\theta(w) = \frac{\mu_R(w)}{\sigma_R(w)^2} \tag{A.11}$$

$$\hat{\theta}(w) = \gamma \frac{\mu_R(w)}{\sigma_R(w)^2} \tag{A.12}$$

Using the fact that $p(w)h(w) = \theta(w)w$ and $p(w)\hat{h}(w) = \hat{\theta}(w)\hat{w}$, the market clearing condition $h(w) + \hat{h}(w) = 1$ can be transformed into

$$p(w) = \theta(w)w + \hat{\theta}(w)\hat{w} \tag{A.13}$$

Using the fact that $\hat{\theta}(w) = \gamma\theta(w)$,

$$\theta(w) = \frac{p(w)}{w + \gamma\hat{w}} \tag{A.14}$$

From this,

$$h(w) = \frac{w}{w + \gamma\hat{w}} \tag{A.15}$$

Wealth dynamics. I next derive the drift and volatility of regular country wealth. The definitions of $\mu_w(w)$, $\sigma_w(w)$, $\mu_p(w)$, and $\sigma_p(w)$ are

$$dw = \mu_w(w)dt + \sigma_w(w)dZdp \qquad = \mu_p(w)dt + \sigma_p(w)dZ \tag{A.16}$$

Here dZ is the increment of the country-specific dividend shock. The budget constraint of a regular country is

$$dw = (r - \lambda)wdt + \hat{\lambda}\hat{w}dt - c(w)dt + \theta(w)w[\mu_R(w)dt + \sigma_R(w)dZ] \tag{A.17}$$

From this,

$$\sigma_w(w) = \theta(w)w\sigma_R(w) = \theta(w)w\frac{\sigma_p(w) + \sigma}{p(w)} = \frac{w}{w + \gamma\hat{w}}(\sigma_p(w) + \sigma) \tag{A.18}$$

Here I used the fact that $\sigma_R(w) = (\sigma_p(w) + \sigma)/p(w)$ and equation (A.14) for $\theta(w)$.

By Itô's lemma, $\sigma_p(w) = p'(w)\sigma_w(w)$, so

$$\sigma_w(w) = \frac{w}{w + \gamma\hat{w}}(p'(w)\sigma_w(w) + \sigma) = \frac{\sigma w}{w + \gamma\hat{w} - p'(w)w} \tag{A.19}$$

The volatility of wealth originates in the dividend volatility and is amplified through prices: when a dividend shock hits wealth, it transmits to prices in equilibrium and feeds back into wealth. To see this intuitively, rewrite the above expression as

$$\sigma_w(w) = \sigma h(w) \cdot \frac{w + \gamma\hat{w}}{w + \gamma\hat{w} - p'(w)w} \tag{A.20}$$

Now turning to $\mu_w(w)$,

$$\begin{aligned}
\mu_w(w) &= (r - \rho - \lambda)w + \hat{\lambda}\hat{w} + w\theta(w)\mu_R(w) \\
&= (r - \rho - \lambda)w + \hat{\lambda}\hat{w} + w\theta(w)^2\sigma_R(w)^2 \\
&= (r - \rho - \lambda)w + \hat{\lambda}\hat{w} + \frac{\sigma_w(w)^2}{w}
\end{aligned} \tag{A.21}$$

Here I used the fact that consumption is $c(w) = \rho w$ and in optimum.
Properties. Taking the market clearing condition again,

$$\mu_R(w) = \frac{p(w)\sigma_R(w)^2}{w + \gamma\hat{w}} \tag{A.22}$$

Plugging $\mu_R(w) = (\mu_p(w) + \nu)/p(w) - r$ and $\sigma_R(w) = (\sigma_p(w) + \sigma)/p(w)$,

$$\mu_p(w) + \nu - rp(w) = \frac{(\sigma_p(w) + \sigma)^2}{w + \gamma\hat{w}} \tag{A.23}$$

Using Itô's lemma,

$$\mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w) + \nu - rp(w) = \frac{(\sigma_w(w)p'(w) + \sigma)^2}{w + \gamma\hat{w}} \tag{A.24}$$

Plugging the formula for $\sigma_w(w)$ into the right-hand side and rearranging,

$$rp(w) = \nu - \frac{\sigma^2(w + \gamma\hat{w})}{(w + \gamma\hat{w} - p'(w)w)^2} + \mu_w(w)p'(w) + \frac{\sigma_w(w)^2}{2}p''(w) \tag{A.25}$$

This is the differential equation that $p(w)$ has to satisfy. I work with equilibria in which prices are non-explosive, meaning $p(w)$ has a constant limit as $w \to \infty$. In such equilibria, $p'(w) \to 0$ as $w \to \infty$, and $\mu_w(w)$ and $\sigma_w(w)$ have the following properties.

First, $\sigma_w(w) \to \sigma$ as $w \to \infty$. The volatility of wealth does not scale with its size as a country grows rich. Large portfolios have relatively little risk. Second, from this, it follows that $\mu_w(w)/w \to r - \rho - \lambda$. Drift in wealth does scale with its size. If $r < \rho$, large portfolios are simply consumed away, with λ adding some wealth drain through emigration. Plugging this into equation (A.25), taking the limit of both sides as $w \to \infty$, and using the fact that $p'(w)$ and $p''(w)$ both converge to zero,

$$r \lim_{w\to\infty} p(w) = \nu + (r - \rho - \lambda) \lim_{w\to\infty} p'(w)w \tag{A.26}$$

Since the limit of $p(w)$ exists by assumption, that of $p'(w)w$ exists too. But if it exists, it must be zero. Otherwise, $p'(w)w$ is bounded away from zero for all $w > \bar{w}$ for some $\bar{w}$, which means $|p'(w)|$ can be bounded from below by δ/w for some $\delta > 0$, and hence $|p(w)| > \delta(\log(w) - \log(\bar{w})) + |p(\bar{w})|$, which contradicts $p(w)$ having a limit as $w \to \infty$.

The upshot is that

$$\lim_{w\to\infty} p(w) = \frac{\nu}{r} \tag{A.27}$$

Asset prices converge to fair value as wealth goes to infinity, and the risk premium vanishes. Risky assets in rich countries are priced almost as the special country's tree.

Balance of payments. There is trade in goods and assets in this economy. Components of the payment balance have the following form:

$$\text{trade balance} = \nu dt + \sigma dZ - \rho w dt \tag{A.28}$$

$$\text{factor payments} = rb(w)dt - \nu\hat{h}(w)dt \tag{A.29}$$

$$\text{transfers} = \hat{\lambda}\hat{w}dt - \lambda w dt \tag{A.30}$$

$$\text{current account balance} = \text{trade balance} + \text{factor payments} + \text{transfers} \tag{A.31}$$

Trade balance is equal to net exports, which is everything a country produces in excess of its consumption. Factor payment balance is receipts (interest payments on intermediary's debt) less dividends sent abroad. Transfers account for the migration of wealth. Taken together, these three variables constitute the current account.

It is clear that rich countries run trade deficits and poor countries run trade surpluses. The difference is due to consumption. The factor payment balance can be rewritten as

$$\text{factor payments} = rwdt - \nu dt - \sigma dZ + h(w)(\nu - rp(w))dt \tag{A.32}$$

In rich countries, interest payments from abroad dominate as a source of factor income. They scale with wealth, while the rest converges to a constant. In poor countries, factor payments converge to $-(\nu dt + \sigma dZ)$, the dividends sent abroad. This article balances their positive net exports.

Taking all three articles together, the current account balance can be rewritten as

$$\text{current account balance} = (r - \rho - \lambda)wdt + \hat{\lambda}\hat{w}dt + h(w)(\nu dt + \sigma dZ - rp(w)dt) \tag{A.33}$$

Rich countries run current account deficits that scale with wealth approximately as $(r-\rho-\lambda)wdt$. Poor countries run surpluses, which in the limit of $w \to 0$ converge to $\hat{\lambda}\hat{w}dt$, the wealth immigration.

The net foreign asset position of a country is

$$N(w) = b(w) - p(w)\hat{h}(w) = w - p(w) \tag{A.34}$$

The dynamic evolution in $N(w)$ is

$$\begin{aligned} dN(w) = dw - dp(w) &= (r - \rho - \lambda)wdt + \hat{\lambda}\hat{w}dt + (dp(w) + \nu dt + \sigma dZ)h(w) - dp(w) \\ &= \text{current account balance} - \hat{h}(w)dp(w) \end{aligned} \tag{A.35}$$

Net foreign assets rise as a result of current account surplus or due to valuation adjustments. Since this integrates to zero in the steady state, the current account of all regular countries as a whole against the special country is

$$\text{total current account balance} = \left[\int \hat{h}(w)\mu_p(w)dG(w)\right] dt \tag{A.36}$$

In my calibration, this is positive. The drift in prices integrates to zero,

$$\int \mu_p(w)dG(w) = 0 \tag{A.37}$$

It is not monotone as a function, but the sign only changes once: $\mu_p(w)$ is positive for small w and negative for large w. Since $\hat{h}(w)$ is a decreasing positive function,

$$\int \hat{h}(w)\mu_p(w)dG(w) > 0 \tag{A.38}$$

The special country thus runs a current account deficit against regular countries. Since the current account is equal to the negative of the financial account, it equivalently runs a financial account surplus. What allows the special country to run a current account deficit is its trading profits: it chooses positions $\hat{h}(w)$ that skew toward growing countries with positive drift in prices. As countries churn in the wealth distribution, the intermediary makes capital gains on average and realizes them through trading.

D Details of portfolio choice

In this section, I derive the portfolio choice of the agents and characterize their value functions. This setup features wealth migration between countries. Appendix E shows a specification with discount rates that vary with wealth instead.

Regular country savers. I start with the savers from regular countries. The evolution of an individual saver's w_{it} given her country's aggregate wealth $\underline{w}_{it}$ is

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} + \frac{w_{it}}{\underline{w}_{it}} \cdot \hat{\lambda}\hat{w}_t dt - \frac{w_{it}}{\underline{w}_{it}} \cdot \lambda \underline{w}_{it} dt \tag{A.39}$$

Here c_{it} is consumption, and the first term in equation (A.39) represents the consumption-savings trade-off. The second term represents returns on the domestic tree, where θ_{it} is its portfolio share that the saver chooses. She still operates under a portfolio constraint $\theta_{it} \leq \theta$.

The last two terms represent exogenous migration in and out of regular countries. The third term in equation (A.39) represents wealth immigration from the special country. Savers in that country die with intensity $\hat{\lambda}$, and their money is sent to one of the regular countries, where it is shared between the local savers in proportion to net worth. The destination country is chosen uniformly, so each country i has an influx of wealth $\hat{\lambda}\hat{w}_t dt$, where $\hat{w}_t$ is the special country's wealth. In i, each saver gets a share $w_{it}/\underline{w}_{it}$ of this transfer.

Finally, the fourth term represents emigration to the special country. In regular countries, savers die with an intensity λ, and their wealth is sent to the special country. The total outflow of money from i is $\lambda \underline{w}_{it} dt$. New savers are born instead. They start with zero wealth and instantly get transferred a portion of everyone's savings so that everyone within the country has the same net worth. This redistribution from continuing savers is in proportion to their w_{it}. Hence, conditional on continuing, they always make flow payments $\lambda w_{it} dt$ to the newborns.

The sequence problem of the saver in the country i is

$$V_{it} = \max_{(c_{is},\theta_{is})_{s\geq t}} \mathbb{E}_t \left[\rho \int_t^{\infty} e^{\rho(t-s)} \log(c_{is}) ds \right] \tag{A.40}$$

subject to equation (A.39) and $\theta_{it} \leq \theta$. Savers choose consumption rate and the portfolio share allocated to risky assets. They take the interest rate r_t, tree price p_{it}, and aggregate net worth $\underline{w}_{it}$ as given. The discount rate ρ is constant.

Since everyone has the same w_{it}, in equilibrium $w_{it} = \underline{w}_{it}$, and the evolution of the total wealth in country i is

$$dw_{it} = (r_t w_{it} - c_{it})dt + \theta_{it} w_{it} dR_{it} + (\hat{\lambda}\hat{w}_t - \lambda w_{it})dt$$

The proposition below characterizes the solution to their problem in equation (A.40).

Proposition 2. *Given the time paths of the global interest rate r_t, the special country's wealth $\hat{w}_t$, and the drift and volatility of the excess return process $(\mu_{it}^R, \sigma_{it}^R)$,*

$$V_{it} = \log(\rho w_{it}) + \kappa(\underline{w}_{it}, t) \tag{A.41}$$

where $\kappa(\underline{w}_{it}, t)$ satisfies a partial differential equation. Consumption and portfolio choice are

$$c_{it} = \rho w_{it} \tag{A.42}$$

$$\theta_{it} = \min\left\{\overline{\theta}, \frac{\mu_{it}^R}{(\sigma_{it}^R)^2}\right\} \tag{A.43}$$

Proof of Proposition 2. Since there is no aggregate uncertainty, state variables for a saver in country i are her own wealth $\underline{w}_{it}$, aggregate wealth of her country w_{it}, and time t. Dropping the subscript i, define the drift and volatility of w_{it} and $\underline{w}_{it}$:

$$d\underline{w} = \mu_{\underline{w}}(\underline{w}, t)dt + \sigma_{\underline{w}}(\underline{w}, t)dZ \tag{A.44}$$

$$dw = \mu_w(\underline{w}, w, t; c, \theta)dt + \sigma_w(\underline{w}, w, t; \theta)dZ \tag{A.45}$$

The saver correctly assesses the functions $\mu_{\underline{w}}(\underline{w}, t)$ and $\sigma_{\underline{w}}(\underline{w}, t)$ but does not internalize the effect of her choices on w. The drift and volatility of individual wealth depend on consumption and portfolio choice (c, θ):

$$\mu_w(\underline{w}, w, t; c, \theta) = r(t)w - c + \theta\mu_R(\underline{w}, t)w + w\left(\hat{\lambda}\frac{\hat{w}(t)}{\underline{w}} - \lambda\right) \tag{A.46}$$

$$\sigma_w(\underline{w}, w, t; \theta) = \theta\sigma_R(\underline{w}, t)w \tag{A.47}$$

Here the dependence on time comes from the drift and volatility of returns $\mu_R(\underline{w}, t)$ and $\sigma_R(\underline{w}, t)$ as well as the global interest rate $r(t)$ and the net worth of the special country $\hat{w}(t)$. The HJB equation for the saver's value $V(\underline{w}, w, t)$ is, suppressing the arguments,

$$\begin{aligned}\rho V - \partial_t V = \max_{c, \theta \leq \overline{\theta}}\ &\rho\log(c) + \mu_w(\underline{w}, w, t; c, \theta)\partial_w V + \frac{\sigma_w(\underline{w}, w, t; \theta)^2}{2}\partial_{ww}V \\ &+ \mu_{\underline{w}}(\underline{w}, t)\partial_{\underline{w}}V + \frac{\sigma_{\underline{w}}(\underline{w}, t)^2}{2}\partial_{\underline{w}\underline{w}}V + \sigma_{\underline{w}}(\underline{w}, t)\sigma_w(\underline{w}, w, t; \theta)\partial_{\underline{w}w}V\end{aligned} \tag{A.48}$$

Now guess that the value function $V(\underline{w}, w, t)$ has the following form:

$$V(\underline{w}, w, t) = \log(\rho w) + \kappa(\underline{w}, t) \tag{A.49}$$

Plugging this into equation (A.48),

$$\rho \log(\rho w) + \rho\kappa(\underline{w}, t) - \partial_t \kappa(\underline{w}, t) = \max_{c,\theta \leq \overline{\theta}} \ \rho \log(c) + \frac{\mu_w(\underline{w}, w, t; c, \theta)}{w} - \frac{\sigma_w(\underline{w}, w, t; \theta)^2}{2w^2}$$
$$+ \mu_{\underline{w}}(\underline{w}, t)\partial_{\underline{w}}\kappa(\underline{w}, t) + \frac{\sigma_{\underline{w}}(\underline{w}, t)^2}{2}\partial_{\underline{w}\underline{w}}\kappa(\underline{w}, t) \tag{A.50}$$

Notice that the cross-derivative term drops out. Now using the functional forms for $\mu_w(\underline{w}, w, t; c, \theta)$ and $\sigma_w(\underline{w}, w, t; \theta)$ from equation (A.46) and equation (A.47), the optimal choices are

$$c^* = \rho w \tag{A.51}$$
$$\theta^* = \min\left\{\frac{\mu_R(\underline{w}, t)}{\sigma_R(\underline{w}, t)^2}, \overline{\theta}\right\} \tag{A.52}$$

This shows that savers consume a constant fraction of their wealth and choose a mean-variance portfolio whenever they can.

To get the partial differential equation that describes $\kappa(\underline{w}, t)$, use the consistency requirement $\underline{w} = w$, which also implies $\mu_{\underline{w}}(\underline{w}, t) = \mu_w(w, w, t; c^*, \theta^*)$ and $\sigma_{\underline{w}}(\underline{w}, t) = \sigma_w(w, w, t; \theta^*)$. Plugging this into equation (A.51) and equation (A.50),

$$\rho\kappa(\underline{w}, t) - \partial_t\kappa(w, t) = \frac{\mu_w(w, t)}{w} - \frac{\sigma_w(w, t)^2}{2w^2} + \mu_w(w, t)\partial_w\kappa(w, t) + \frac{\sigma_w(w, t)^2}{2}\partial_{ww}\kappa(w, t) \tag{A.53}$$

Boundary conditions for this equation in general depend on the properties of loadings $\mu_R(\underline{w}, t)$ and $\sigma_R(\underline{w}, t)$. Plugging the optimal choice of controls in equation (A.51) and equation (A.52),

$$\mu_{\underline{w}}(\underline{w}, t) = (r(t) - \rho - \lambda)w + \hat{\lambda}\hat{w}(t) + \min\left\{\frac{\mu_R(\underline{w}, t)}{\sigma_R(\underline{w}, t)^2}, \overline{\theta}\right\}\mu_R(\underline{w}, t)w \tag{A.54}$$
$$\sigma_{\underline{w}}(\underline{w}, t) = \min\left\{\frac{\mu_R(\underline{w}, t)}{\sigma_R(\underline{w}, t)^2}, \overline{\theta}\right\}\sigma_R(\underline{w}, t)w \tag{A.55}$$

At $\underline{w} = 0$, the drift of wealth is not equal to zero. This property helps avoid $\underline{w} = 0$ being an absorbing state. However, $\kappa(\underline{w}, t)$ might diverge around small $\underline{w}$. Assuming that $\mu_R(\underline{w}, t)$ is bounded, the limiting behavior of $\kappa(\underline{w}, t)$ around $\underline{w} = 0$ is

$$\lim_{x\to 0}\frac{\kappa(x, t)}{\log(x)} = -1 \tag{A.56}$$

Assuming that $\mu_R(w, t)/\sigma_R(w, t)$ approaches zero as $\underline{w} \to \infty$,

$$\lim_{x\to\infty}\rho\kappa(x, t) - \partial_t\kappa(x, t) = r(t) - \rho - \lambda \tag{A.57}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\kappa(\underline{w}, t)$ as the limiting terminal condition at infinity. Assuming that $\mu_R(w,t)$ is bounded and $\mu_R(\underline{w}, t)/\sigma_R(\underline{w}, t) \to 0$ as $\underline{w} \to \infty$, this completes the characterization of $\kappa(\underline{w}, t)$ given the general equilibrium objects $r(t)$, $\hat{w}(t)$, $\mu_R(\underline{w}, t)$, and $\sigma_R(\underline{w}, t)$. $\square$

Intermediaries. I next describe the special country. The individual net worth $\hat{w}_t$ of an intermediary, who is also a saver in the special country, evolves as

$$d\hat{w}_t = (r_t\hat{w}_t - \hat{c}_t)dt + \sum_i \hat{\theta}_{it}\hat{w}_t dR_{it} + \hat{\theta}_t\hat{w}_t d\hat{R}_t + \frac{\hat{w}_t}{\underline{\hat{w}}_t} \cdot \lambda w_t dt - \frac{\hat{w}_t}{\underline{\hat{w}}_t} \cdot \hat{\lambda}\underline{\hat{w}}_t dt \tag{A.58}$$

Consumption is $\hat{c}_t$ and $\underline{\hat{w}}_t$ is the special country's aggregate wealth. The second and third terms are excess returns on trees in regular countries and at home. Positions $(\hat{\theta}_{it}, \hat{\theta}_t)$ are akin to θ_{it} in the regular country's problem. Portfolios with a continuum of idiosyncratic returns require special care, so I write the total return as a sum without specifying its type. Below, I work out this problem for a finite number of countries and then let the number of countries grow to infinity.

The last two terms are the mirror image of wealth migration terms in equation (1): there is an inflow of $\lambda w_t dt$, where $w_t = \int_0^1 w_{it} di$ is the aggregate wealth of the regular countries, and an outflow of $\hat{\lambda}\underline{\hat{w}}_t dt$. Again, as in regular countries, newborn savers immediately receive transfers from everyone else so that everyone's wealth is the same.

The individual wealth $\hat{w}_t$ of the special country savers aggregates into $\underline{\hat{w}}_t$ evolving as

$$d\hat{w}_t = (r_t\hat{w}_t - \hat{c}_t)dt + \sum\nolimits_i \hat{\theta}_{it}\hat{w}_t dR_{it} + \hat{\theta}_t\hat{w}_t d\hat{R}_t + (\lambda w_t - \hat{\lambda}\hat{w}_t)dt \tag{A.59}$$

Taking into account drift corrections, the evolution of an individual intermediary's net worth can be rewritten as

$$d\hat{w} = (r_t\hat{w}_t - \hat{c}_t)dt + \sum\nolimits_i \hat{\theta}_{it}\hat{w}_t((\mu^R_{it} - \sigma^R_{it}\xi_{it})dt + \sigma^R_{it}d\widetilde{Z}_{it}) + \hat{\theta}_t\hat{w}_t d\hat{R}_t + \frac{\hat{w}_t}{\underline{\hat{w}}_t}(\lambda w_t - \hat{\lambda}\underline{\hat{w}}_t)dt \tag{A.60}$$

The problem of an individual intermediary is

$$\hat{V}_t = \max_{\{\hat{c}_s, \hat{\theta}_s, \boldsymbol{f}_s\}_{s\geq t}} \inf_{Q\in\mathcal{Q}} \mathbb{E}^Q_t\left[\hat{\rho}\int_t^\infty e^{\hat{\rho}(s-t)}\log(\hat{c}_s)ds + \frac{1}{2}\int_t^\infty e^{\hat{\rho}(s-t)}\hat{\gamma}_s\sum_i \eta(w_{is})dm_{is}\right] \tag{A.61}$$

subject to equation (A.60). Note a constant discount rate $\hat{\rho}$.

Proposition 3. *Fix the number of regular countries at n. Given the path of the global interest rate r_t and the vector x_t of aggregate wealth in every country including $\underline{\hat{w}}_t$, the value function of an individual special country saver is*

$$\hat{V}^{(n)}_t = \log(\hat{\rho}\hat{w}_t) + \hat{\kappa}^{(n)}(x_t, t) \tag{A.62}$$

The function $\hat{\kappa}_t$ solves a first-order ordinary differential equation. The choice of portfolio weights and drift correction for each country i is

$$\hat{f}_{it}^{(n)} = \frac{\hat{\gamma}_t \eta(w_{it})}{1+\hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.63}$$

$$\xi_{it}^{(n)} = \frac{1}{\hat{\gamma}_t \eta(w_{it})} \cdot f_{it}^{(n)} \sigma_{it}^R = \frac{1}{1+\hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.64}$$

Proof of Proposition 3. Fix the number of regular countries n. The state variables of the global bank are its wealth $\hat{w}$, a vector x that combines aggregate special country wealth $\underline{\hat{w}}$ and all other (w_i), and time t that summarizes all other variables. The evolution of individual wealth $\hat{w}$ is

$$d\hat{w} = (r_t \hat{w}_t - \hat{c}_t)dt + \sum_i \hat{\theta}_{it} \hat{w}_t((\mu_{it}^R - \sigma_{it}^R \xi_{it})dt + \sigma_{it}^R d\widetilde{Z}_{it}) + \hat{\theta}_t \hat{w}_t d\hat{R}_t + \frac{\hat{w}_t}{\underline{\hat{w}}_t}(\lambda w_t dt - \hat{\lambda} \underline{\hat{w}}_t)dt \tag{A.65}$$

Combine all $(d\tilde{Z}_{it})$ into a vector $d\tilde{Z}_t$. Aggregate vector x evolves as

$$dx = \mu_x(x,t)dt + \sigma_x(x,t)d\tilde{Z}_t \tag{A.66}$$

The HJB equation for $\hat{V}(\hat{w}, x, t)$ is, suppressing arguments,

$$\hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c},\hat{\theta},(\hat{\theta}_i)} \min_{(\xi_i)} \ \hat{\rho}\log(\hat{c}) + \frac{\hat{\gamma}(t)}{2}\sum_i \eta(w_{it})\xi_i^2 + \mu_{\hat{w}}(\hat{w}, x, t; \hat{c}, \hat{\theta}, (\hat{\theta}_i, \xi_i))\partial_{\hat{w}}\hat{V} + \mu_x(x,t)'\partial_x \hat{V}$$
$$+ \frac{\sigma_{\hat{w}}(\hat{w}, x, t; (\hat{\theta}_i))^2}{2}\partial_{\hat{w}\hat{w}}V + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\hat{w}x}(\hat{w}, x, t; (\hat{\theta}_i))'\partial_{\hat{w}x}V \tag{A.67}$$

Here the drift and variance of $\hat{w}$ conditional on controls c, $\hat{\theta}$, $(\hat{\theta}_i)$, and (ξ_i) are

$$\mu_{\hat{w}}(\hat{w}, x, t; \hat{c}, \hat{\theta}, (\hat{\theta}_i, \xi_i)) = (r(t) - \hat{\lambda} + \mathbb{E}_t[\hat{R}_t])\hat{w} - c + \sum_i \hat{w}\hat{\theta}_i(\mu_{it}^R - \xi_i \sigma_{it}^R) + \frac{w(t)}{\hat{w}(t)}\lambda\hat{w} \tag{A.68}$$

$$\sigma_{\hat{w}}(\hat{w}, x, t; (\hat{\theta}_i))^2 = \hat{w}^2 \sum_i \hat{\theta}_i^2 (\sigma_{it}^R)^2 \tag{A.69}$$

$$\sigma_{\hat{w}x}(\hat{w}, x, t; (\hat{\theta}_i)) = \hat{w}\sigma_x(x,t)v(x,t,(\hat{\theta}_i)) \tag{A.70}$$

The vector $v(x, t, (\hat{\theta}_i))$ in equation (A.70) collects the products $(\hat{\theta}_i \sigma_{it}^R)$.

The solution for the weight on the special country's tree $\hat{\theta}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I assume that this is the case. The optimal weight $\hat{\theta}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu_{\hat{w}}(\hat{w}, x, t; \hat{c}, \hat{\theta}^*, (\hat{\theta}_i, \xi_i^*))$ below.

The expressions for variance of $d\hat{w}$ and co-variance of $d\hat{w}$ and dx per unit of time in equation (A.69) and equation (A.70) use independence between $d\tilde{Z}_{it}$.

Solving the minimization problem over (ξ_i),

$$\xi_i^* = \frac{1}{\hat{\gamma}(t)\eta(w_{it})} \cdot \hat{\theta}_i \sigma_{it}^R \cdot \hat{w}\partial_{\hat{w}}\hat{V} \tag{A.71}$$

Plugging this into equation (A.68),

$$\mu_{\hat{w}}(\hat{w}, x, t; \hat{c}, (\hat{\theta}_i, \xi_i^*)) = (r(t) - \hat{\lambda})\hat{w} - \hat{c} + \sum_i \left(\hat{w}\hat{\theta}_i \mu_{it}^R - \partial_{\hat{w}}\hat{V} \frac{(\hat{w}\sigma_{it}^R)^2}{\hat{\gamma}(t)\eta(w_{it})} \hat{\theta}_i^2 \right) + \frac{w(t)}{\hat{w}(t)}\lambda\hat{w} \tag{A.72}$$

The problem in equation (A.67) is now

$$\begin{aligned} \hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c}, (\hat{\theta}_i)} \ & \hat{\rho}\log(\hat{c}) + \frac{(\partial_{\hat{w}}\hat{V})^2}{2\hat{\gamma}(t)} \sum_i \frac{(\hat{w}\sigma_{it}^R)^2}{\eta(w_{it})} \hat{\theta}_i^2 + \mu_{\hat{w}}(\hat{w}, x, t; \hat{c}, (\hat{\theta}_i, \xi_i^*))\partial_{\hat{w}}\hat{V} + \mu_x(x,t)'\partial_x \hat{V} \\ & + \frac{\sigma_{\hat{w}}(\hat{w}, x, t; (\hat{\theta}_i))^2}{2} \partial_{\hat{w}\hat{w}}\hat{V} + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\hat{w}x}(\hat{w}, x, t; (\hat{\theta}_i))'\partial_{\hat{w}x}\hat{V} \end{aligned} \tag{A.73}$$

Plugging equation (A.72) and equation (A.69) into this,

$$\begin{aligned} \hat{\rho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c}, (\hat{\theta}_i)} \ & \hat{\rho}\log(\hat{c}) - \hat{c}\partial_{\hat{w}}\hat{V} + \partial_{\hat{w}}\hat{V} \sum_i \hat{w}\mu_{it}^R \hat{\theta}_i - \frac{(\partial_{\hat{w}}\hat{V})^2}{2\hat{\gamma}(t)} \sum_i \frac{(\hat{w}\sigma_{it}^R)^2}{\eta(w_{it})} \hat{\theta}_i^2 \\ & + \left(r(t) - \hat{\lambda} + \lambda \frac{w(t)}{\hat{w}(t)} \right) \hat{w}\partial_{\hat{w}}\hat{V} + \hat{w}^2 \sum_i \frac{\hat{\theta}_i^2 (\sigma_{it}^R)^2}{2} \partial_{\hat{w}\hat{w}} V \\ & + \mu_x(x,t)'\partial_x \hat{V} + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\hat{w}x}(\hat{w}, x, t; (\hat{\theta}_i))'\partial_{\hat{w}x}V \end{aligned} \tag{A.74}$$

Guess that the value function $\hat{V}(\hat{w}, t)$ has the following form

$$\hat{V}(\hat{w}, x, t) = \log(\hat{\rho}\hat{w}) + \hat{\kappa}(x, t) \tag{A.75}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\rho}\hat{w} \tag{A.76}$$

Replacing this in equation (A.74),

$$\begin{aligned} \hat{\rho}\hat{\kappa}(x,t) - \partial_t \hat{\kappa}(x,t) = \max_{(\hat{\theta}_i)} \ & \sum_i \left(\mu_{it}^R \hat{\theta}_i - \frac{(\sigma_{it}^R)^2}{2\hat{\gamma}(t)\eta(w_{it})} \hat{\theta}_i^2 - \frac{(\sigma_{it}^R)^2}{2} \hat{\theta}_i^2 \right) + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \\ & + \mu_x(x,t)'\partial_x \hat{\kappa}(x,t) + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}\hat{\kappa}(x,t)\sigma_x(x,t)) \end{aligned} \tag{A.77}$$

The optimal choice of portfolio weights ($\hat{\theta}_i$) is

$$\hat{\theta}_i^* = \frac{\hat{\gamma}(t)\eta(w_{it})}{1 + \hat{\gamma}(t)\eta(w_{it})} \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.78}$$

This leads to the following expression for the optimal drift correction:

$$\xi_i^* = \frac{1}{1 + \hat{\gamma}(t)\eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.79}$$

The differential equation for $\hat{\kappa}(t)$ becomes

$$\begin{aligned}\hat{\rho}\hat{\kappa}(x,t) - \partial_t \hat{\kappa}(\hat{w},t) = \frac{1}{2}\sum_i \frac{\hat{\gamma}(t)\eta(w_{it})}{1 + \hat{\gamma}(t)\eta(w_{it})} \left(\frac{\mu_{it}^R}{\sigma_{it}^R}\right)^2 + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \\ + \mu_x(x,t)'\partial_x \hat{\kappa}(x,t) + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}\hat{\kappa}(x,t)\sigma_x(x,t))\end{aligned} \tag{A.80}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\hat{\kappa}$ as the terminal limit at $t \to \infty$. $\square$

E Portfolio choice with time-varying discounting

In this section, I derive the portfolio choice of the agents and characterize their value functions under a specification without wealth migration and with subjective discount rates that vary with wealth. The main takeaway from this exercise is that portfolio choice and aggregate consumption are the same as in the baseline.

Regular country savers. I start with the savers from regular countries. The proposition below characterizes the solution to their problem in equation (A.40).

Proposition 4. *Given the time paths of the global interest rate r_t, the special country's wealth $\hat{w}_t$, and the drift and volatility of the excess return process $(\mu^R_{it}, \sigma^R_{it})$,*

$$V_{it} = \log(\varrho_{it} w_{it}) + \kappa(\underline{w}_{it}, t) \tag{A.81}$$

where $\kappa(\underline{w}_{it}, t)$ satisfies a partial differential equation. Consumption and portfolio choice are

$$c_{it} = \varrho_{it} w_{it} \tag{A.82}$$

$$\theta_{it} = \min\left\{\overline{\theta}, \frac{\mu^R_{it}}{(\sigma^R_{it})^2}\right\} \tag{A.83}$$

Proof of Proposition 4. Since there is no aggregate uncertainty, state variables for a saver in country i are her own wealth w_{it}, aggregate wealth of her country $\underline{w}_{it}$, and time t. Dropping the subscript i, define the drift and volatility of w_{it} and $\underline{w}_{it}$:

$$dw = \mu_w(\underline{w}, w, t; c, \theta)dt + \sigma_w(\underline{w}, w, t; \theta)dZ \tag{A.84}$$

$$d\underline{w} = \mu_{\underline{w}}(\underline{w}, t)dt + \sigma_{\underline{w}}(\underline{w}, t)dZ \tag{A.85}$$

The saver correctly assesses the functions $\mu_{\underline{w}}(\underline{w}, t)$ and $\sigma_{\underline{w}}(\underline{w}, t)$ but does not internalize the effect of her choices on $\underline{w}$. The drift and volatility of individual wealth depend on consumption and portfolio choice (c, θ):

$$\mu_w(\underline{w}, w, t; c, \theta) = r(t)w - c + \theta\mu_R(\underline{w}, t)w \tag{A.86}$$

$$\sigma_w(\underline{w}, w, t; \theta) = \theta\sigma_R(\underline{w}, t)w \tag{A.87}$$

Here the dependence on time comes from the global interest rate $r(t)$, the net worth of the special country $\hat{w}(t)$, and time-dependent terms $\mu^R(\underline{w}, t)$ and $\sigma^R(\underline{w}, t)$. The HJB equation for the saver's value $V(\underline{w}, w, t)$ is, suppressing the arguments,

$$\varrho V - \partial_t V = \max_{c,\theta \le \overline{\theta}} \varrho \log(c) + \mu_w(\underline{w}, w, t; c, \theta)\partial_w V + \frac{\sigma_w(\underline{w}, w, t; \theta)^2}{2}\partial_{ww} V$$
$$+ \mu_{\underline{w}}(\underline{w}, t)\partial_{\underline{w}} V + \frac{\sigma_{\underline{w}}(\underline{w}, t)^2}{2}\partial_{\underline{w}\underline{w}} V + \sigma_{\underline{w}}(\underline{w}, t)\sigma_w(\underline{w}, w, t; \theta)\partial_{\underline{w}w} V \tag{A.88}$$

Now guess that the value function $V(\underline{w}, w, t)$ has the following form:

$$V(\underline{w}, w, t) = \log(w) + \log(\varrho(\underline{w}, t)) + \kappa(\underline{w}, t) \tag{A.89}$$

Plugging this into equation (A.88) and suppressing arguments of $\varrho(\underline{w}, t)$ again,

$$\varrho \log(\varrho \underline{w}) + \varrho\kappa(\underline{w}, t) - \partial_t \kappa(\underline{w}, t) = \max_{c,\theta \le \overline{\theta}} \varrho \log(c) + \frac{\mu_w(\underline{w}, w, t; c, \theta)}{w} - \frac{\sigma_w(\underline{w}, w, t; \theta)^2}{2w^2}$$
$$+ \mu_{\underline{w}}(\underline{w}, t)\partial_{\underline{w}}\kappa(\underline{w}, t) + \frac{\sigma_{\underline{w}}(\underline{w}, t)^2}{2}\partial_{\underline{w}\underline{w}}\kappa(\underline{w}, t) \tag{A.90}$$

Notice that the cross-derivative term drops out. Now using the functional forms for $\mu_w(\underline{w}, w, t; c, \theta)$ and $\sigma_w(\underline{w}, w, t; \theta)$ from equation (A.86) and equation (A.87), the optimal choices are

$$c^* = \varrho(\underline{w}, t) w \tag{A.91}$$
$$\theta^* = \min\left\{\frac{\mu_R(\underline{w}, t)}{\sigma_R(\underline{w}, t)^2}, \overline{\theta}\right\} \tag{A.92}$$

This shows that savers consume a constant fraction of their wealth and choose a mean-variance portfolio whenever they can.

To get the partial differential equation that describes $\kappa(\underline{w}, t)$, use the consistency requirement $\underline{w} = w$, which also implies $\mu_{\underline{w}}(\underline{w}, t) = \mu_w(w, w, t; c^*, \theta^*)$ and $\sigma_{\underline{w}}(\underline{w}, t) = \sigma_w(\underline{w}, w, t; \theta^*)$. Plugging this into equation (A.91) and equation (A.90),

$$\varrho\kappa(\underline{w}, t) - \partial_t\kappa(\underline{w}, t) = \frac{\mu_{\underline{w}}(\underline{w}, t)}{\underline{w}} - \frac{\sigma_{\underline{w}}(\underline{w}, t)^2}{2\underline{w}^2} + \mu_{\underline{w}}(\underline{w}, t)\partial_{\underline{w}}\kappa(\underline{w}, t) + \frac{\sigma_{\underline{w}}(\underline{w}, t)^2}{2}\partial_{\underline{w}\underline{w}}\kappa(\underline{w}, t) \tag{A.93}$$

Boundary conditions for this equation in general depend on the properties of loadings $\mu_R(\underline{w}, t)$ and $\sigma_R(\underline{w}, t)$. Plugging the optimal choice of controls and the functional form of $\varrho(\underline{w}, t)$ in equation (A.91) and equation (A.92),

$$\mu_{\underline{w}}(\underline{w}, t) = (r(t) - \rho - \lambda)\underline{w} + \hat{\lambda}\hat{w}(t) + \min\left\{\frac{\mu_R(\underline{w}, t)}{\sigma_R(\underline{w}, t)^2}, \overline{\theta}\right\}\mu_R(\underline{w}, t)\underline{w} \tag{A.94}$$
$$\sigma_{\underline{w}}(\underline{w}, t) = \min\left\{\frac{\mu_R(\underline{w}, t)}{\sigma_R(\underline{w}, t)^2}, \overline{\theta}\right\}\sigma_R(\underline{w}, t)\underline{w} \tag{A.95}$$

At $\underline{w} = 0$, the drift of wealth is not equal to zero. This property helps avoid $\underline{w} = 0$ being an absorbing state. However, $\kappa(\underline{w}, t)$ might diverge around small $\underline{w}$. Assuming that $\mu_R(\underline{w}, t)$ is bounded, the limiting behavior of $\kappa(\underline{w}, t)$ around $\underline{w} = 0$ is

$$\lim_{x \to 0} \frac{\kappa(x,t)}{\log(x)} = -1 \tag{A.96}$$

Assuming that $\mu_R(\underline{w}, t)/\sigma_R(\underline{w}, t)$ approaches zero as $\underline{w} \to \infty$,

$$\lim_{x \to \infty} \varrho(x,t)\kappa(x,t) - \partial_t \kappa(x,t) = r(t) - \rho - \lambda \tag{A.97}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\kappa(\underline{w}, t)$ as the limiting terminal condition at infinity. Assuming that $\mu_R(\underline{w}, t)$ is bounded and $\mu_R(\underline{w}, t)/\sigma_R(\underline{w}, t) \to 0$ as $\underline{w} \to \infty$, this completes the characterization of $\kappa(\underline{w}, t)$ given the general equilibrium objects $r(t)$, $\hat{w}(t)$, $\mu_R(\underline{w}, t)$, and $\sigma_R(\underline{w}, t)$. $\square$

Problem of the intermediary. The next proposition deals with the problem of the intermediary, which is also the special country's saver.

PROPOSITION 5. *Fix the number of regular countries at n. Given the path of the global interest rate r_t and the vector x_t of aggregate wealth in every country including $\underline{\hat{w}}_t$, the value function of an individual special country saver is*

$$\hat{V}_t^{(n)} = \log(\hat{\varrho}_t \hat{w}_t) + \hat{\kappa}^{(n)}(x_t, t) \tag{A.98}$$

The function $\hat{\kappa}_t^{(n)}$ solves a first-order ordinary differential equation. The choice of portfolio weights and drift correction for each country i is

$$\hat{\theta}_{it}^{(n)} = \frac{\hat{\gamma}_t \eta(w_{it})}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.99}$$

$$\xi_{it}^{(n)} = \frac{1}{\hat{\gamma}_t \eta(w_{it})} \cdot f_{it}^{(n)} \sigma_{it}^R = \frac{1}{1 + \hat{\gamma}_t \eta(w_{it})} \cdot \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.100}$$

Proof of Proposition 5. Fix the number of regular countries n. The state variables of the global bank are its wealth $\hat{w}$, a vector x that combines aggregate special country wealth $\underline{\hat{w}}$ and all other (w_i), and time t that summarizes all other variables. The evolution of individual wealth $\hat{w}$ is

$$d\hat{w} = (r_t \hat{w}_t - \hat{c}_t)dt + \sum_i f_{it} \hat{w}_t ((\mu_{it}^R - \sigma_{it}^R \xi_{it})dt + \sigma_{it}^R d\widetilde{Z}_{it}) + \hat{\theta}_t \hat{w}_t d\hat{R}_t \tag{A.101}$$

Combine all $(d\tilde{Z}_{it})$ into a vector $d\tilde{Z}_t$. Aggregate vector x evolves as

$$dx = \mu_x(x,t)dt + \sigma_x(x,t)d\tilde{Z}_t \tag{A.102}$$

The HJB equation for $\hat{V}(\hat{w},x,t)$ is, suppressing arguments,

$$\begin{aligned}\hat{\varrho}\hat{V} - \partial_t\hat{V} = \max_{\hat{c},\hat{\theta},(\hat{\theta}_i)}\min_{(\xi_i)}\ &\hat{\varrho}\log(\hat{c}) + \frac{\hat{\gamma}(t)}{2}\sum_i \eta(w_{it})\xi_i^2 + \mu_{\hat{w}}(\hat{w},x,t;\hat{c},\hat{\theta},(\hat{\theta}_i,\xi_i))\partial_{\hat{w}}\hat{V} + \mu_x(x,t)'\partial_x\hat{V}\\ &+ \frac{\sigma_{\hat{w}}(\hat{w},x,t;(\hat{\theta}_i))^2}{2}\partial_{\hat{w}\hat{w}}V + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\hat{w}x}(\underline{\hat{w}},x,t;(\hat{\theta}_i))'\partial_{\hat{w}x}V\end{aligned} \tag{A.103}$$

Here the drift and variance of $\hat{w}$ conditional on controls c, $\hat{\theta}$, $(\hat{\theta}_i)$, and (ξ_i) are

$$\mu_{\hat{w}}(\hat{w},x,t;\hat{c},\hat{\theta},(\hat{\theta}_i,\xi_i)) = (r(t) + \hat{\theta}\mathbb{E}_t[d\hat{R}_t])\hat{w} - c + \sum_i \hat{w}\hat{\theta}_i(\mu_{it}^R - \xi_i\sigma_{it}^R) \tag{A.104}$$

$$\sigma_{\hat{w}}(\hat{w},x,t;(\hat{\theta}_i))^2 = \hat{w}^2\sum_i \hat{\theta}_i^2(\sigma_{it}^R)^2 \tag{A.105}$$

$$\sigma_{\hat{w}x}(\hat{w},x,t;(\hat{\theta}_i)) = \hat{w}\sigma_x(x,t)v(x,t,(\hat{\theta}_i)) \tag{A.106}$$

The vector $v(x,t,(\hat{\theta}_i))$ in equation (A.106) collects the products $(\hat{\theta}_i\sigma_{it}^R)$.

The solution for the weight on the special country's tree $\hat{\theta}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I assume that this is the case. The optimal weight $\hat{\theta}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu_{\hat{w}}(\hat{w},x,t;\hat{c},\hat{\theta}^*,(\hat{\theta}_i,\xi_i^*))$ below.

The expressions for variance of $d\hat{w}$ and co-variance of $d\hat{w}$ and $d\hat{w}$ per unit of time in equation (A.105) and equation (A.106) use independence between $d\tilde{Z}_{it}$.

Solving the minimization problem over (ξ_i),

$$\xi_i^* = \frac{1}{\hat{\gamma}(t)\eta(w_{it})}\cdot\hat{\theta}_i\sigma_{it}^R\cdot\hat{w}\partial_{\hat{w}}\hat{V} \tag{A.107}$$

Plugging this into equation (A.123),

$$\mu_{\hat{w}}(\hat{w},x,t;\hat{c},(\hat{\theta}_i,\xi_i^*)) = r(t)\hat{w} - \hat{c} + \sum_i\left(\hat{w}\hat{\theta}_i\mu_{it}^R - \partial_{\hat{w}}\hat{V}\frac{(\hat{w}\sigma_{it}^R)^2}{\hat{\gamma}(t)\eta(w_{it})}\hat{\theta}_i^2\right) + \frac{w(t)}{\hat{w}(t)}\lambda\hat{w} \tag{A.108}$$

The problem in equation (A.103) is now, suppressing the arguments of $\varrho(x,t)$,

$$\begin{aligned}\hat{\varrho}\hat{V} - \partial_t\hat{V} = \max_{\hat{c},(\hat{\theta}_i)}\ &\hat{\varrho}\log(\hat{c}) + \frac{(\partial_{\hat{w}}\hat{V})^2}{2\hat{\gamma}(t)}\sum_i\frac{(\hat{w}\sigma_{it}^R)^2}{\eta(w_{it})}\hat{\theta}_i^2 + \mu_{\hat{w}}(\hat{w},x,t;\hat{c},(\hat{\theta}_i,\xi_i^*))\partial_{\hat{w}}\hat{V} + \mu_x(x,t)'\partial_x\hat{V}\\ &+ \frac{\sigma_{\hat{w}}(\hat{w},x,t;(\hat{\theta}_i))^2}{2}\partial_{\hat{w}\hat{w}}\hat{V} + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\hat{w}x}(\hat{w},x,t;(\hat{\theta}_i))'\partial_{\hat{w}x}\hat{V}\end{aligned} \tag{A.109}$$

Plugging equation (A.108) and equation (A.105) into this,

$$\begin{aligned}\hat{\varrho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c},(\hat{\theta}_i)} \ & \hat{\varrho}\log(\hat{c}) - \hat{c}\partial_{\hat{w}}\hat{V} + \partial_{\hat{w}}\hat{V}\sum_i \hat{w}\mu_{it}^R\hat{\theta}_i - \frac{(\partial_{\hat{w}}\hat{V})^2}{2\hat{\gamma}(t)}\sum_i \frac{(\hat{w}\sigma_{it}^R)^2}{\eta(w_{it})}\hat{\theta}_i^2 \\ & + r(t)\hat{w}\partial_{\hat{w}}\hat{V} + \hat{w}^2\sum_i \frac{\hat{\theta}_i^2(\sigma_{it}^R)^2}{2}\partial_{\hat{w}\hat{w}}V \\ & + \mu_x(x,t)'\partial_x\hat{V} + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}V\sigma_x(x,t)) + \sigma_{\hat{w}x}(\hat{w},x,t;(\hat{\theta}_i))'\partial_{\hat{w}x}V \end{aligned} \tag{A.110}$$

Guess that the value function $\hat{V}(\hat{w},x,t)$ has the following form

$$\hat{V}(\hat{w},x,t) = \log(\hat{\rho}\hat{w}) + \hat{\kappa}(x,t) \tag{A.111}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\varrho}(x,t)\hat{w} \tag{A.112}$$

Replacing this in equation (A.127) and using the functional form for $\hat{\varrho}(x,t)$,

$$\begin{aligned}\hat{\varrho}\hat{\kappa}(x,t) - \partial_t\hat{\kappa}(x,t) = \max_{(\hat{\theta}_i)} \sum_i \left(\mu_{it}^R\hat{\theta}_i - \frac{(\sigma_{it}^R)^2}{2\hat{\gamma}(t)\eta(w_{it})}\hat{\theta}_i^2 - \frac{(\sigma_{it}^R)^2}{2}\hat{\theta}_i^2\right) + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \\ + \mu_x(x,t)'\partial_x\hat{\kappa}(x,t) + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}\hat{\kappa}(x,t)\sigma_x(x,t))\end{aligned} \tag{A.113}$$

The optimal choice of portfolio weights $(\hat{\theta}_i)$ is

$$\hat{\theta}_i^* = \frac{\hat{\gamma}(t)\eta(w_{it})}{1+\hat{\gamma}(t)\eta(w_{it})}\cdot\frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.114}$$

This leads to the following expression for the optimal drift correction:

$$\xi_i^* = \frac{1}{1+\hat{\gamma}(t)\eta(w_{it})}\cdot\frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.115}$$

The differential equation for $\hat{\kappa}(t)$ becomes

$$\begin{aligned}\hat{\varrho}(x,t)\hat{\kappa}(x,t) - \partial_t\hat{\kappa}(\hat{w},t) = \frac{1}{2}\sum_i \frac{\hat{\gamma}(t)\eta(w_{it})}{1+\hat{\gamma}(t)\eta(w_{it})}\left(\frac{\mu_{it}^R}{\sigma_{it}^R}\right)^2 + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \\ + \mu_x(x,t)'\partial_x\hat{\kappa}(x,t) + \frac{1}{2}\mathrm{tr}(\sigma_x(x,t)'\partial_{xx'}\hat{\kappa}(x,t)\sigma_x(x,t))\end{aligned} \tag{A.116}$$

I use the steady-state value of $\hat{\kappa}$ as the terminal limit at $t \to \infty$. $\square$

F Alternative setup for the global banks

This section provides details of the bank's problem in the continuous limit and an alternative way to model the intermediary using a VAR-type constraint. To simplify notation, I do it in the setup with wealth-dependent discount rates. Everything is the same in the setup with migration.

F.1 Continuous limit

The problem of the bank in the continuous world is

$$\hat{V}_t = \max_{\{\hat{c}_s, \hat{\theta}_s, \boldsymbol{f}_s\}_{s\geq t}} \inf_{\{\boldsymbol{\xi}_s\}_{s\geq t}} \mathbb{E}_t^{Q(\xi)} \left[\int_t^\infty e^{\hat{\varrho}_s(s-t)} \left(\hat{\varrho}_t \log(\hat{c}_s) + \frac{\gamma_s}{2} \int_0^1 \eta(w_{is}) \xi_{is}^2 di \right) ds \right] \tag{A.117}$$

subject to the budget constraint

$$d\hat{w} = (r_t \hat{w}_t - \hat{c}_t)dt + \int_0^1 \hat{\theta}_{it} \hat{w}_t ((\mu_{it}^R - \sigma_{it}^R \xi_{it})dt + \sigma_{it}^R d\widetilde{Z}_{it})di + \hat{\theta}_t \hat{w}_t d\hat{R}_t \tag{A.118}$$

The bank has access to a continuum of uncorrelated risky assets, so its payoffs are deterministic. However, the bank has freedom to choose a model for uncertainty over each risky asset in its portfolio. Marginal benefits of considering scenarios with more substantial losses (that is, marginal benefits of increasing ξ_{it}) increase in portfolio weights $\hat{\theta}_{it}$. But as the bank entertains more pessimistic models, marginal benefits of raising $\hat{\theta}_{it}$ itself decline. This brings decreasing returns into the choice of portfolio weights, even though uncertainty can effectively be disregarded.

Proposition 6. *Given the paths of the global interest rate r_t, the special country's aggregate wealth $\hat{w}_t$, the drift and volatility of the excess return process $(\mu_{it}^R, \sigma_{it}^R)$ for all i, the value function of an individual special country saver is*

$$\hat{V}_t = \log(\hat{\varrho}_t \hat{w}_t) + \hat{\kappa}_t \tag{A.119}$$

The function $\hat{\kappa}_t$ solves a first-order ordinary differential equation. The choice of drift correction and portfolio weights for each country i is

$$\hat{\theta}_{it} = \gamma_t \eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.120}$$

$$\xi_{it} = \frac{1}{\gamma_t \eta(w_{it})} \cdot \hat{\theta}_{it} \sigma_{it}^R = \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.121}$$

Portfolio choice of the bank is similar to that of the local savers in regular countries, being proportional to mean over variance of returns. A common factor $\gamma_t \hat{w}_t$ applies to all regular countries.

Movements in γ_t map into shifts in foreign demand for risky assets in all countries at once. It can be interpreted as a risk-tolerance parameter. Infinite γ_t would make the bank essentially neutral to risk, willing to take an arbitrarily large position in any asset that offers excess returns.

Interpreted literally as a cost parameter, high γ_t makes the penalty for considering alternative models with large losses prohibitively high. Equation (A.138) shows that, for a fixed $\hat{\theta}_{it}$, high γ_t makes the bank stick closer to the baseline measure. With a smaller correction, marginal costs of increasing $\hat{\theta}_{it}$ are lower, and the bank demands more risky assets for a given mean-variance ratio.

Why do correction terms ξ_{it} pick up the volatility of idiosyncratic returns if idiosyncratic shocks wash out in aggregate? This is because alternative models chosen by the bank apply to the fundamental dividend shocks dZ_{it}. A mistake in evaluating the expectation of the dividend shock dZ_{it} translates to a mistake in evaluating the expectation of excess returns $dR_{it} = \mu^R_{it}dt + \sigma^R_{it}dZ_{it}$, which is what really matters for portfolio choice, and this latter mistake scales by σ^R_{it}.

Proof of Proposition 6. Since there is no aggregate uncertainty, the evolution of the aggregate wealth in the special country $\hat{w}_t$ and the distribution of (w_{it}) can be summarized by time. The HJB equation for $\hat{V}(\hat{w}, t)$ is, suppressing arguments,

$$\hat{\varrho}\hat{V} - \partial_t\hat{V} = \max_{\hat{c},\hat{\theta},(f_i)} \min_{(\xi_i)} \; \hat{\varrho}\log(\hat{c}) + \frac{\gamma(t)}{2}\int_0^1 \eta(w_{it})\xi_i^2 di + \mu^{\hat{w}}(\hat{w}, t; \hat{c}, \hat{\theta}, (f_i, \xi_i))\partial_{\hat{w}}\hat{V} \tag{A.122}$$

Here the drift of $\hat{w}$ conditional on controls c, $\hat{\theta}$, (f_i), and (ξ_i) is

$$\mu^{\hat{w}}(\hat{w}, t; \hat{c}, \hat{\theta}, (f_i, \xi_i)) = (r(t) + \mathbb{E}_t[\hat{\theta}d\hat{R}_t])\hat{w} - c + \int_0^1 \hat{w}f_i(\mu^R_{it} - \xi_i\sigma^R_{it})di \tag{A.123}$$

The solution for the weight on the special country's tree $\hat{\theta}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I will assume that this is the case henceforth. The optimal weight $\hat{\theta}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu^{\hat{w}}(\hat{w}, t; \hat{c}, \hat{\theta}^*, (f_i, \xi_i^*))$ below.

Solving the minimization problem over (ξ_i),

$$\xi_i^* = \frac{1}{\gamma(t)\eta(w_{it})} \cdot f_i\sigma^R_{it} \cdot \hat{w}\partial_{\hat{w}}\hat{V} \tag{A.124}$$

Plugging this into equation (A.123),

$$\mu^{\hat{w}}(\hat{w}, t; \hat{c}, (f_i, \xi_i^*)) = r(t)\hat{w} - \hat{c} + \int_0^1 \left(\hat{w}f_i\mu^R_{it} - \partial_{\hat{w}}\hat{V}\frac{(\hat{w}\sigma^R_{it})^2}{\gamma(t)\eta(w_{it})}f_i^2\right) di \tag{A.125}$$

The problem in equation (A.122) is now, suppressing arguments,

$$\hat{\varrho}\hat{V} - \partial_t\hat{V} = \max_{\hat{c},(f_i)} \; \hat{\rho}\log(\hat{c}) + \frac{(\partial_{\hat{w}}\hat{V})^2}{2\gamma(t)}\int_0^1 \frac{(\hat{w}\sigma^R_{it})^2}{\eta(w_{it})}f_i^2 di + \mu^{\hat{w}}(\hat{w}, t; \hat{c}, (f_i, \xi_i^*))\partial_{\hat{w}}\hat{V} \tag{A.126}$$

Plugging equation (A.125) into this,

$$\hat{\varrho}\hat{V} - \partial_t \hat{V} = \max_{\hat{c},(f_i)} \ \hat{\varrho}\log(\hat{c}) + (r(t)\hat{w} - \hat{c})\partial_{\hat{w}}\hat{V} + \partial_{\hat{w}}\hat{V}\int_0^1 \hat{w}\mu_{it}^R f_i di - \frac{(\partial_{\hat{w}}\hat{V})^2}{2\gamma(t)}\int_0^1 \frac{(\hat{w}\sigma_{it}^R)^2}{\eta(w_{it})} f_i^2 di \tag{A.127}$$

Guess that the value function $\hat{V}(\hat{w}, t)$ has the following form

$$\hat{V}(\hat{w}, t) = \log(\hat{\varrho}(t)\hat{w}) + \hat{\kappa}(t) \tag{A.128}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\varrho}(t)\hat{w} \tag{A.129}$$

Replacing this in equation (A.127) and plugging the functional form for $\varrho(t)$,

$$\hat{\varrho}(t)\hat{\kappa}(t) - \hat{\kappa}'(t) = \max_{(f_i)} \int_0^1 \left(\mu_{it}^R f_i - \frac{(\sigma_{it}^R)^2}{2\gamma(t)\eta(w_{it})} f_i^2 \right) di + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \tag{A.130}$$

The optimal choice of portfolio weights (f_i) is

$$f_i^* = \gamma(t)\eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.131}$$

The differential equation for $\hat{\kappa}(t)$ becomes

$$\hat{\varrho}(t)\hat{\kappa}(t) - \hat{\kappa}'(t) = \frac{\gamma(t)}{2}\int_0^1 \eta(w_{it}) \left(\frac{\mu_{it}^R}{\sigma_{it}^R} \right)^2 di + r(t) - \hat{\rho} - \hat{\lambda} + \frac{\lambda w(t)}{\hat{w}(t)} \tag{A.132}$$

The last remaining piece is a suitable initial or terminal condition. In practice, I will use the steady-state value of $\hat{\kappa}$ as the terminal limit at $t \to \infty$. $\square$

F.2 VAR-type constraint

The bank with a VAR-type constraint solves

$$\max_{(\hat{c}_s, \hat{\theta}_s, \boldsymbol{f}_s)_{s \geq t}} \mathbb{E}_t \left[\varrho_t \int_t^\infty e^{\hat{\varrho}_s(s-t)} \log(\hat{c}_s) ds \right] \tag{A.133}$$

subject to a budget constraint

$$d\hat{w}_t = (r_t\hat{w}_t - \hat{c}_t)dt + \int_0^1 \hat{\theta}_{it}\hat{w}_t dR_{it}di + \hat{\theta}_t\hat{w}_t d\hat{R}_t \tag{A.134}$$

and the following institutional or regulatory constraint:

$$\int_0^1 \mathbb{V}_t[\hat{\theta}_{it}dR_{it}]\eta(w_{it})^{-1}di \leq \gamma_t \int_0^1 \mathbb{E}_t[\hat{\theta}_{it}dR_{it}]di \tag{A.135}$$

The next proposition characterizes the solution.

PROPOSITION 7. *Given the path of the global interest rate r_t, the special country's aggregate wealth $\underline{\hat{w}}_t$, the drift and volatility of the excess return processes $(\mu_{it}^R, \sigma_{it}^R)$ for all i, the value function of an individual special country saver with a VAR-type constraint is*

$$\hat{V}_t = \log(\hat{\varrho}_t\hat{w}_t) + \hat{\kappa}_t \tag{A.136}$$

The function $\hat{\kappa}_t$ solves a first-order ordinary differential equation. The choice of drift correction and portfolio weights for each country i is

$$\hat{\theta}_{it} = \gamma_t\eta(w_{it}) \cdot \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.137}$$

$$\xi_{it} = \frac{1}{\gamma_t\eta(w_{it})} \cdot \hat{\theta}_{it}\sigma_{it}^R = \frac{\mu_{it}^R}{\sigma_{it}^R} \tag{A.138}$$

Proof of Proposition 7. Without aggregate uncertainty, the evolution of the aggregate special country wealth $\hat{w}_t$ and the cross-section of (w_{it}) can be summarized by time. The HJB equation for $\hat{V}(\hat{w}, t)$ is, suppressing arguments,

$$\hat{\varrho}\hat{V} - \partial_t\hat{V} = \max_{\hat{c},\hat{\theta},(\hat{\theta}_i)} \hat{\varrho}\log(\hat{c}) + \mu^{\hat{w}}(\hat{w}, t; \hat{c}, \hat{\theta}, (f_i, \xi_i))\partial_{\hat{w}}\hat{V} \tag{A.139}$$

$$\text{s.t.} \quad \int_0^1 f_i^2(\sigma_{it}^R)^2\eta(w_{it})^{-1}di \leq \gamma_t \int_0^1 f_i\mu_{it}^R di \tag{A.140}$$

Here the drift of $\hat{w}$ conditional on controls c, $\hat{\theta}$, (f_i), and (ξ_i) is

$$\mu^{\underline{\hat{w}}}(\hat{w}, t; \hat{c}, \hat{\theta}, (f_i, \xi_i)) = (r(t) + \mathbb{E}_t[\hat{\theta}d\hat{R}_t])\hat{w} - c + \int_0^1 \hat{w}f_i\mu_{it}^R di \tag{A.141}$$

The solution for the weight on the special country's tree $\hat{\theta}$ will only be finite if $\mathbb{E}_t[d\hat{R}_t] = 0$. I will assume that this is the case henceforth. The optimal weight $\hat{\theta}^*$ is not determined and does not affect the value of the objective, so I will omit it from the notation for $\mu^{\hat{w}}(\hat{w}, t; \hat{c}, \hat{\theta}^*, (f_i, \xi_i^*))$ below.

Guess that the value function $\hat{V}(\hat{w}, t)$ has the following form

$$\hat{V}(\hat{w}, t) = \log(\hat{\varrho}(t)\hat{w}) + \hat{\kappa}(t) \tag{A.142}$$

This immediately leads to the optimal choice of consumption:

$$\hat{c}^* = \hat{\varrho}(t)\hat{w} \tag{A.143}$$

Plugging this and the functional form of $\varrho(t)$ into equation (A.139),

$$\hat{\varrho}(t)\hat{\kappa}(t) - \hat{\kappa}'(t) = \max_{(\hat{\theta}_i)} \int_0^1 f_i \mu_{it}^R di + r(t) - \hat{\lambda} - \hat{\rho} + \lambda \frac{w(t)}{\hat{w}(t)} \tag{A.144}$$

$$\text{s.t.} \quad \int_0^1 f_i^2 (\sigma_{it}^R)^2 \eta(w_{it})^{-1} di \leq \gamma_t \int_0^1 f_i \mu_{it}^R di \tag{A.145}$$

Let the multiplier on the constraint be $\xi(t)$. The first-order condition for f_i is

$$f_i = \eta(w_{it}) \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \cdot \frac{1 + \xi(t)\gamma(t)}{2\xi(t)} \tag{A.146}$$

Plugging this into the constraint,

$$\frac{1 - \gamma(t)^2 \xi(t)^2}{4\xi(t)^2} \int_0^1 \eta(w_{it}) \left(\frac{\mu_{it}^R}{\sigma_{it}^R} \right)^2 di \leq 0 \tag{A.147}$$

This holds with equality if $\xi(t) > 0$, so $\xi(t) = 1/\gamma(t)$ and

$$f_i^* = \gamma(t)\eta(w_{it}) \frac{\mu_{it}^R}{(\sigma_{it}^R)^2} \tag{A.148}$$

Plugging this back into equation (A.144),

$$\hat{\varrho}(t)\hat{\kappa}(t) - \hat{\kappa}'(t) = \gamma(t) \int_0^1 \eta_{it} \left(\frac{\mu_{it}^R}{\sigma_{it}^R} \right)^2 di + r(t) - \hat{\lambda} - \hat{\rho} + \lambda \frac{w(t)}{\hat{w}(t)} \tag{A.149}$$

The only remaining bit is the suitable initial or terminal condition for $\hat{\kappa}(\cdot)$. In practice, I will use the steady state as a terminal condition as a limit at $t \to \infty$. $\square$

G Details of equilibrium

In this section, I provide details for the equilibrium section. First, I discuss the steady state and a useful benchmark of unlimited risk-taking capacity in which there is complete risk-sharing and no non-degenerate wealth distribution. This benchmark also illustrates the role of migration between countries. Then, I provide justification for the equilibrium condition in equation (9) and equation (A.150) that decomposes intermediary's consumption in steady state:

$$\hat{c} = \nu\hat{q} + \nu \cdot \int \hat{h}(w)dG(w) - r \cdot \int l(w)dG(w) + \int \mu_p(w)\hat{h}(w)dG(w) \tag{A.150}$$

I also provide a proof for Proposition 1.

Steady state. Define aggregate profit rates from risky assets π and $\hat{\pi}$:

$$\pi \int wdG(w) = \int w\theta(w)\mu_R(w)dG(w) \tag{A.151}$$

$$\hat{\pi}\hat{w} = \int \hat{w}f(w)\mu_R(w)dG(w) \tag{A.152}$$

Here π is the average expected excess return that regular countries receive on their trees, and $\hat{\pi}$ is the total expected excess return that the global bank receives on them. These profit rates allow me to express the steady-state interest rate and aggregate wealth of regular and special countries.

Proposition 8. *In the steady state, the interest rate is*

$$r = \frac{\rho + \lambda + \hat{\rho} + \hat{\lambda} - \pi - \hat{\pi} - \sqrt{(\rho + \lambda - \hat{\rho} - \hat{\lambda} - \pi + \hat{\pi})^2 + 4\lambda\hat{\lambda}}}{2} \tag{A.153}$$

It decreases in both π and $\hat{\pi}$ and is bounded between $\min\{\rho - \pi, \hat{\rho} - \hat{\pi}\}$ *and* $\max\{\rho - \pi, \hat{\rho} - \hat{\pi}\}$. *The aggregate wealth of regular countries and that of the special country are given by*

$$\int wdG(w) = \frac{\hat{\lambda}(\nu + \hat{q}\hat{\nu})}{\rho\hat{\lambda} + \hat{\rho}\lambda + \rho\hat{\rho} - \hat{\rho}(r + \pi)} \tag{A.154}$$

$$\hat{w} = \frac{\lambda(\nu + \hat{q}\hat{\nu})}{\rho\hat{\lambda} + \hat{\rho}\lambda + \rho\hat{\rho} - \rho(r + \hat{\pi})} \tag{A.155}$$

This proposition shows how the interest rate and aggregate wealth of regular and special countries depend on profit rates. This is useful for thinking about the benchmark limit $\gamma \to \infty$. In this limit, any positive expected excess returns lead the bank to assume an infinite position in risky assets. Expected excess returns $\mu_R(w)$ then have to be zero in equilibrium, and $\pi = \hat{\pi} = 0$. Since r decreases in both π and $\hat{\pi}$, limited risk-taking capacity depresses the global interest rate.

Holdings are well defined in this limit. Local savers are not willing to hold trees since there is fundamental risk in the dividends but expected excess returns are zero. The global bank holds all the risky assets and fully insures the regular countries. There is no wealth distribution among the regular countries because they are not exposed to idiosyncratic shocks. Equation (A.154) in this case shows wealth accumulated by each of them. Simplifying more, $\rho = \hat{\rho}$ leads to $r = \rho = \hat{\rho}$ and

$$\int w dG(w) = \frac{\hat{\lambda}}{\hat{\lambda} + \lambda} \frac{\nu + \hat{q}\hat{\nu}}{\rho} \tag{A.156}$$

$$\hat{w} = \frac{\lambda}{\hat{\lambda} + \lambda} \frac{\nu + \hat{q}\hat{\nu}}{\rho} \tag{A.157}$$

Risk is perfectly diversified and there is no difference in time preferences, so wealth is simply the present value of output split between the intermediary and the savers by migration. This is the only way in which migration affects the aggregates. The technical reason to include it is that, without it, $w = 0$ would be an absorbing state for regular countries, and the special country's income would be linear in its wealth $\hat{w}$, leaving no possibility for a well-defined invariant distribution. In my calibration, I set net migration in the steady state to zero but allow for some gross flows.

Proofs. I now formulate equation (9) and equation (A.150) as propositions and prove them. I also prove Proposition 1 and Proposition 8.

Before deriving the expression for the price and quantity of risk, it is useful to rearrange the market clearing conditions for risky assets:

$$p(w,t) = w \min\left\{\overline{\theta}, \frac{\mu_R(w,t)}{\sigma_R(w,t)^2}\right\} + \varphi(t)\eta(w)\frac{\mu_R(w,t)}{\sigma_R(w,t)^2} \tag{A.158}$$

Now I will show the equivalence between this and the price-quantity decomposition in equation (9).

Proposition 9. *Equation (A.158) implies equation (9). That is,*

$$p(w,t) = w \min\left\{\overline{\theta}, \frac{\mu^R(w,t)}{\sigma^R(w,t)^2}\right\} + \varphi(t)\eta(w)\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \Longrightarrow \tag{A.159}$$

$$\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} = \max\left\{\frac{p(w,t)}{\varphi(t)\eta(w) + w}, \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)}\right\} \tag{A.160}$$

Proof of Proposition 9. Suppose that equation (A.158) holds. Use the equivalence

$$\frac{p(w,t)}{\varphi(t)\eta(w) + w} \geq \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)} \Leftrightarrow \frac{p(w,t)}{\varphi(t)\eta(w) + w} \leq \overline{\theta} \tag{A.161}$$

Suppose $\mu^R(w,t)/\sigma^R(w,t)^2 \le \overline{\theta}$. Then, from equation (A.158) it follows that

$$\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} = \frac{p(w,t)}{\varphi(t)\eta(w) + w} \tag{A.162}$$

This means that $p(w,t)/(\varphi(t)\eta(w) + w) \le \overline{\theta}$, so, from equation (A.161),

$$\max\left\{\frac{p(w,t)}{\varphi(t)\eta(w) + w}, \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)}\right\} = \frac{p(w,t)}{\varphi(t)\eta(w) + w} = \frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \tag{A.163}$$

Now suppose that $\mu^R(w,t)/\sigma^R(w,t)^2 > \overline{\theta}$. From equation (A.158) it follows that

$$\frac{\mu^R(w,t)}{\sigma^R(w,t)^2} = \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)} \tag{A.164}$$

This means that $p(w,t) - \overline{\theta}w > \overline{\theta}\varphi(t)\eta(w)$, so $p(w,t)/(w + \varphi(t)\eta(w)) > \overline{\theta}$. Then, from equation (A.161) it follows that

$$\max\left\{\frac{p(w,t)}{\varphi(t)\eta(w) + w}, \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)}\right\} = \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)} = \frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \tag{A.165}$$

Thus, in any case,

$$\max\left\{\frac{p(w,t)}{\varphi(t)\eta(w) + w}, \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)}\right\} = \frac{\mu^R(w,t)}{\sigma^R(w,t)^2} \tag{A.166}$$

This completes the proof. □

Proof of Proposition 1. Start with plugging the expressions for $\mu^R(w,t)$ and $\sigma^R(w,t)$ into equation (9). Rewriting it yields a formula for the risk premium in terms of price dynamics:

$$\frac{\mu^p(w,t) + \nu(t)}{p(w,t)} - r(t) = \frac{(\sigma^p(w,t) + \sigma)^2}{p(w,t)^2} \cdot \max\left\{\frac{p(w,t)}{w + \varphi(t)\eta(w)}, \frac{p(w,t) - \overline{\theta}w}{\varphi(t)\eta(w)}\right\} \tag{A.167}$$

Using Itô's lemma,

$$\mu^p(w,t) = \mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww} p(w,t) + \partial_t p(w,t) \tag{A.168}$$

$$\sigma^p(w,t) = \sigma^w(w,t)\partial_w p(w,t) \tag{A.169}$$

Multiplying both sides of equation (A.167) by $p(w,t)$,

$$\mu^p(w,t) + \nu(t) - p(w,t)r(t) + \partial_t p(w,t) = (\sigma^p(w,t)+\sigma)^2 \cdot \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\overline{\theta}w}{p(w,t)}\right)\right\} \quad (A.170)$$

Plugging the drift and volatility of prices,

$$\begin{aligned}&\mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) + \nu(t) - p(w,t)r(t) + \partial_t p(w,t) \\ &\quad = (\sigma^w(w,t)\partial_w p(w,t)+\sigma)^2 \cdot \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\overline{\theta}w}{p(w,t)}\right)\right\}\end{aligned} \quad (A.171)$$

Now rewrite the process for a regular country's wealth in equation (A.41):

$$\begin{aligned}dw &= (r(t)-\rho)wdt + \theta(w,t)wdR(w,t) + (\hat{\lambda}\hat{w}(t) - \lambda w)dt \\ &= (r(t)-\rho-\lambda)wdt + \hat{\lambda}\hat{w}(t)dt + w\theta(w,t)\mu^R(w,t)dt + w\theta(w,t)\sigma^R(w,t)dZ \\ &= \hat{\lambda}\hat{w}(t)dt + \left(r(t)(1-\theta(w,t)) - \rho - \lambda + \theta(w,t)\frac{\mu^p(w,t)+\nu(t)}{p(w,t)}\right)wdt \\ &\quad + w\theta(w,t)\frac{\sigma^p(w,t)+\sigma}{p(w,t)}dZ\end{aligned} \quad (A.172)$$

From this, it follows that

$$\sigma^w(w,t) = w\theta(w,t)\frac{\sigma^p(w,t)+\sigma}{p(w,t)} \quad (A.173)$$

Plugging equation (A.169),

$$\begin{aligned}\sigma^w(w,t) &= w\theta(w,t)\frac{\sigma^w(w,t)\partial_w p(w,t)+\sigma}{p(w,t)} = \frac{\theta(w,t)w\sigma}{p(w,t) - w\theta(w,t)\partial_w p(w,t)} \\ &= \frac{\theta(w,t)w\sigma}{p(w,t)(1-\theta(w,t)\epsilon(w,t))}\end{aligned} \quad (A.174)$$

Here $\epsilon(w,t) = w/p(w,t)\cdot\partial_w p(w,t)$ is the wealth elasticity of the price. This implies

$$(\sigma^w(w,t)\partial_w p(w,t)+\sigma)^2 = \left(\frac{\sigma}{1-\theta(w,t)\epsilon(w,t)}\right)^2 \quad (A.175)$$

Plugging this into equation (A.171),

$$\mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) + \nu(t) - p(w,t)r(t) + \partial_t p(w,t) \tag{A.176}$$
$$= \left(\frac{\sigma}{1-\theta(w,t)\epsilon(w,t)}\right)^2 \cdot \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\overline{\theta}w}{p(w,t)}\right)\right\}$$

Define the risk-adjusted payoff $y(w,t)$ as

$$y(w,t) = \nu(t) - \left(\frac{\sigma}{1-\theta(w,t)\epsilon(w,t)}\right)^2 \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1-\frac{\overline{\theta}w}{p(w,t)}\right)\right\} \tag{A.177}$$

Plugging leads to

$$r(t)p(w,t) - \partial_w p(w,t) = y(w,t) + \mu^w(w,t)\partial_w p(w,t) + \frac{\sigma^w(w,t)^2}{2}\partial_{ww}p(w,t) \tag{A.178}$$

This is the Kolmogorov backward equation (A.214) for prices. The Kolmogorov forward equation (A.215) for wealth follows from the fact that the wealth process is a diffusion. □

Proof of Proposition 8. Take the evolution of the special country's wealth and integrate the evolution of the regular countries' wealth to get aggregate dynamics:

$$\begin{aligned} d\hat{w}(t) &= (r(t)-\hat{\rho})\hat{w}(t)dt + \int_0^1 \hat{w}(t)f(w,t)\mu^R(w,t)dG(w,t)dt \\ &\quad + \left(\lambda\int wdG(w,t) - \hat{\lambda}\hat{w}(t)\right)dt \end{aligned} \tag{A.179}$$

$$\begin{aligned} \int dwdG(w,t) &= (r(t)-\rho)\int wdG(w,t)dt + \int w\theta(w,t)\mu^R(w,t)dG(w,t)dt \\ &\quad + \left(\hat{\lambda}\hat{w}(t) - \lambda\int wdG(w,t)\right)dt \end{aligned} \tag{A.180}$$

In the steady state, the left-hand side is zero in both of these equations:

$$\begin{aligned} 0 &= (r-\hat{\rho})\hat{w} + \int_0^1 \hat{w}f(w)\mu^R(w)dG(w) + \left(\lambda\int wdG(w) - \hat{\lambda}\hat{w}\right) \\ &= (r-\hat{\rho})\hat{w} + \hat{\pi}\hat{w} + \lambda\int wdG(w) - \hat{\lambda}\hat{w} \end{aligned} \tag{A.181}$$

$$\begin{aligned} 0 &= (r-\rho)\int wdG(w) + \int w\theta(w)\mu^R(w)dG(w) + \left(\hat{\lambda}\hat{w} - \lambda\int wdG(w)\right) \\ &= (r-\rho)\int wdG(w) + \pi\int wdG(w) + \hat{\lambda}\hat{w} - \lambda\int wdG(w) \end{aligned} \tag{A.182}$$

This leads to

$$\hat{w} = \int w dG(w) \frac{\lambda}{\hat{\lambda} + \hat{\rho} - r - \hat{\pi}} = \hat{w} \frac{\hat{\lambda}}{\lambda + \rho - r - \pi} \frac{\lambda}{\hat{\lambda} + \hat{\rho} - r - \hat{\pi}} \tag{A.183}$$

Reorganize this as a quadratic equation

$$(r + \pi - \rho - \lambda)(r + \hat{\pi} - \hat{\rho} - \hat{\lambda}) = \lambda\hat{\lambda} \tag{A.184}$$

The solution is

$$r = \frac{\alpha + \hat{\alpha} \pm \sqrt{(\alpha - \hat{\alpha})^2 + 4\lambda\hat{\lambda}}}{2} \tag{A.185}$$

Here $\alpha = \rho + \lambda - \pi$ and $\hat{\alpha} = \hat{\rho} + \hat{\lambda} - \hat{\pi}$. Take the root with a plus and consider $r - \alpha$ and $r - \hat{\alpha}$:

$$r - \alpha = \frac{(\hat{\alpha} - \alpha) + \sqrt{(\hat{\alpha} - \alpha)^2 + 4\lambda\hat{\lambda}}}{2} > \hat{\alpha} - \alpha \tag{A.186}$$

$$r - \hat{\alpha} = \frac{(\alpha - \hat{\alpha}) + \sqrt{(\alpha - \hat{\alpha})^2 + 4\lambda\hat{\lambda}}}{2} > \alpha - \hat{\alpha} \tag{A.187}$$

These imply that $r > \alpha$ and $r > \hat{\alpha}$, which is not possible since $\hat{w}$ and $\int w dG(w)$ would be negative in this case. The right root is then that with a minus. Plugging this expression for the interest rate into

$$\hat{w} = \frac{\lambda}{\hat{\lambda} + \hat{\rho} - r - \hat{\pi}} \int w dG(w) \tag{A.188}$$

$$\int w dG = \frac{\hat{\lambda}}{\lambda + \rho - r - \pi} \hat{w} \tag{A.189}$$

completes the proof. □

Proposition 10. *In the steady state,*

$$\hat{c} = \nu\hat{q} + \nu \cdot \int \hat{h}(w) dG(w) - r \cdot \int (1 - \theta(w)) w dG(w) + \int \mu^p(w) \hat{h}(w) dG(w) \tag{A.190}$$

Proof of Proposition 10 The wealth accumulation of the global bank in the steady state can be rewritten as

$$
\begin{aligned}
d\hat{w} &= (r\hat{w} - \hat{c})dt + \int f(w)\hat{w}dR(w)dG(w) + (\lambda w - \hat{\lambda}\hat{w})dt \\
&= (r\hat{w} - \hat{c})dt - r\hat{w}dt \cdot \int f(w)dG(w) + \nu\hat{w}dt \cdot \int \hat{h}(w)dG(w) \\
&+ dt \cdot \int \mu^p(w)\hat{h}(w)dG(w) + (\lambda w - \hat{\lambda}\hat{w})dt
\end{aligned}
\tag{A.191}
$$

This uses the fact that $d\hat{R} = 0$ in equilibrium, so the special asset does not contribute to wealth accumulation. The second line simply applies the definition of $dR(w) = (\mu^p(w) + \nu)/p(w) - r$ and the definition of $h(w)$: $\hat{w}f(w) = p(w)h(w)$. The balance sheet of the global bank is

$$
\hat{w} = \int p(w)\hat{h}(w)dG(w) + \hat{p}\hat{q} - \int (1 - \theta(w))wdG(w) \tag{A.192}
$$

The bank's wealth is its consolidated position in risky assets and its position in the safe asset less bonds outstanding. Plugging this and using the steady-state relations $\hat{p} = \hat{\nu}/r$ (no risk premium on the safe asset), $d\hat{w} = 0$ (no fluctuations in the intermediary's wealth), and $\hat{\lambda}\hat{w} = \lambda w$ (no net migration),

$$
\hat{c} = \nu\hat{q} + \nu \cdot \int \hat{h}(w)dG(w) - r \cdot \int (1 - \theta(w))wdG(w) + \int \mu^p(w)\hat{h}(w)dG(w) \tag{A.193}
$$

This completes the proof. □

H Details for calibration and estimation

In this section, I explain the algorithm for calibration and estimation. I first calibrate the steady-state version of the model using four aggregate moments and a panel of external assets and liabilities from IFS data provided by the IMF. Then, I use two aggregate series and sequence-space Jacobians to estimate the parameters of aggregate shock processes by likelihood maximization.

Calibration. I construct the ratio of assets to liabilities:

$$R_{it} = \frac{A_{it}}{L_{it}} \tag{A.194}$$

I measure the moments of its distribution in the data using the following procedure:

- First, I take unbalanced panels for A_{it} and L_{it} starting in 1990.
- I then smooth out assets and liabilities by replacing the value in each quarter with the mean value over the last four quarters.
- For every country that eventually appears in the sample, I create a weight that is inversely proportional to the duration of its presence in the sample. This allows me to correct for the over-representation of advanced economies with relatively large assets and liabilities

Figure A.8 shows the model fit for the distribution.

Figure A.8: Model-generated distributions (red) and data for the ratio of external assets to external liabilities.

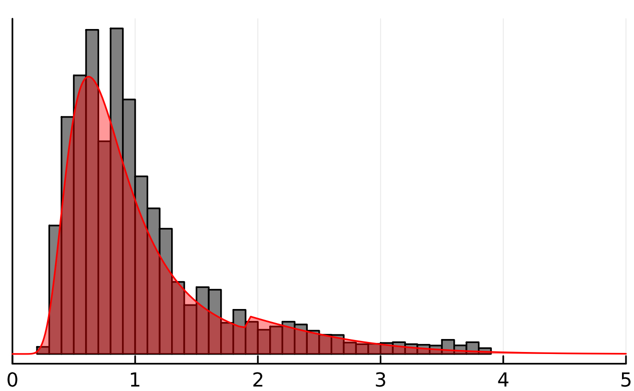

Table 17: Average excess returns in emerging markets and advanced economies. Emerging markets are those constrained: $\theta(w) = \overline{\theta}$

advanced economies	$0.676pp$
emerging markets	$2.893pp$
difference	$2.218pp$

I set the net migration flows to zero in the steady state:

$$\lambda \int w dG(w) = \hat{\lambda}\hat{w} \tag{A.195}$$

Given all other parameters, this pins down $\hat{\lambda}$. I also set $\nu = \hat{\nu}$, where $\hat{\nu}$ is the dividend rate in the special country. The level of dividends can be normalized using the following symmetry property. Consider one model parameterized by eleven parameters $(\rho, \hat{\rho}, \lambda, \hat{\lambda}, \nu, \hat{\nu}, \overline{\theta}, \sigma, \gamma, \hat{q}, \zeta)$ and another one parameterized by $(\rho, \hat{\rho}, \lambda, \hat{\lambda}, \nu^*, \hat{\nu}^*, \overline{\theta}, \sigma^*, \gamma^*, \hat{q}, \zeta^*)$ such that

$$\nu^* = \alpha\nu \tag{A.196}$$

$$\hat{\nu}^* = \alpha\hat{\nu} \tag{A.197}$$

$$\sigma^* = \alpha\sigma \tag{A.198}$$

$$\zeta^* = \frac{\alpha\zeta}{1 + (\alpha - 1)\zeta} \tag{A.199}$$

$$\gamma^* = \gamma\frac{1 + (\alpha - 1)\zeta}{\alpha} \tag{A.200}$$

The prices and quantities corresponding to the original and the starred model satisfy the following:

$$\hat{w}^*(t) = \alpha\hat{w}(t) \tag{A.201}$$

$$p^*(\alpha w, t) = \alpha p(w, t) \tag{A.202}$$

$$f^*(\alpha w, t) = f(w, t) \tag{A.203}$$

Multiplying expected dividends and their volatility by the same number simply rescales wealth and asset prices if γ and ζ are suitably transformed. The same transformation as in equation (A.202) applies to the loadings of prices and wealth processes $(\mu_p(w, t), \mu_w(w, t), \sigma_p(w, t), \sigma_w(w, t))$, while instantaneous returns $(\mu_R(w, t), \sigma_R(w, t))$ follow the pattern in equation (A.203). Interest rates remain the same. This can be verified by noting that

$$\varphi(t)\eta(\alpha w) = \hat{w}(t)\gamma(t)\eta + \hat{w}(t)\gamma(t)(1 - \eta) \cdot \alpha w \tag{A.204}$$

Applying the transformations above to equation (9),

$$\hat{w}^*(t)\gamma^*(t)\zeta^* = \alpha\hat{w}(t)\gamma(t)\zeta \tag{A.205}$$

$$\hat{w}^*(t)\gamma^*(t)(1 - \zeta^*) \cdot \alpha w = \hat{w}(t)\gamma(t)(1 - \zeta) \cdot \alpha w \tag{A.206}$$

This means

$$\begin{aligned}\frac{\mu_R^*(\alpha w, t)}{\sigma_R^*(\alpha w, t)} &= \max\left\{\frac{p^*(\alpha w, t)}{\varphi^*(t)\eta^*(\alpha w) + \alpha w}, \frac{p^*(\alpha w, t) - \overline{\theta}\alpha w}{\varphi^*(t)\eta^*(\alpha w)}\right\} \\ &= \max\left\{\frac{\alpha p(w, t)}{\alpha\varphi(t)\eta(w) + \alpha w}, \frac{\alpha p(w, t) - \overline{\theta}\alpha w}{\alpha\varphi(t)\eta(w)}\right\} = \frac{\mu_R(w, t)}{\sigma_R(w, t)}\end{aligned} \tag{A.207}$$

Portfolio choice for αw is hence the same under the starred parameterization as that for w under the original one. The upshot is that equation (A.201) and equation (A.202) imply that $(\mu_p(w,t), \mu_w(w,t), \sigma_p(w,t), \sigma_w(w,t))$ transform in the same way as in equation (A.202), which verifies equation (A.203) since consumption follows the same proportional rule under both parametrizations and migration scales with $\hat{w}(t)$. The evolution of $\hat{w}(t)$ then shows that equation (A.201) is satisfied, and equation (A.214) shows that equation (A.202) holds.

This symmetry means that choosing a number $\nu = \hat{\nu}$ is simply a normalization. The remaining parameters are an outcome of numerical optimization, where I look for a configuration that minimizes the quadratic distance between the targets and their model analogs in Table 1. All moments are assigned the same weight.

H.1 Calibration of the model in Section 2

The model in Section 2 is a special case of the full model with $\overline{\theta} = \infty$ and $\zeta = 1$, which makes the intermediary's weights constant: $\eta(w) = 1$ for all w. The special country's tree is also not in this version of the model: $\hat{q} = 0$. Other parameters are in Table 18.

Table 18: parameters for the model is Section 2.

parameter	value	meaning
regular countries		
ρ	0.0600	discount rate
λ	0.0200	emigration rate
ν	0.0500	output rate
σ	0.1000	output volatility
$\overline{\theta}$	∞	upper limit on risky asset share
special country		
$\hat{\rho}$	0.0960	discount rate
$\hat{\lambda}$	0.0811	emigration rate
$\hat{q}$	0.0000	asset stock
ζ	1.0000	country weight intercept
γ	1.4918	risk-taking capacity

I choose parameters for regular countries arbitrarily and then find parameters for the special country in order to set net migration flows to zero, like in the full model. The other two targets are the interest rate of 3% annually and the US wealth share of 20%.

The average risk premium comes out to be 4 percentage points, and the average foreign ownership share for the trees is 30%.

H.2 Details of estimation

My estimation procedure relies on the mapping from sequences of shocks $\{dW_t\}$ to the sequences of first-order deviations $\{\tilde{m}_t, \tilde{p}_t\}$. This mapping is given by the sequence-space Jacobians calculated at the steady state and parameters $\mathcal{P} = (\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2})$. Given a potentially large sequence $\{dW_t\}$ and a guess of $\mathcal{P}$, I can compute a large sequence $\{\tilde{m}_t, \tilde{p}_t\}$ and calculate its time-series moments $\mathcal{M}(\mathcal{P}, \{dW_t\})$. I can then update the guess of $\mathcal{P}$ to make these model-implied moments closer to $\mathcal{M}^{\text{data}}$, the moments from the data.

Getting a point estimate for parameters consists of these steps:

- compute the targeted moments in the data $\mathcal{M}^{\text{data}}$
- simulate a large sequence of shocks $\{dW(t)\}$
- given $\{dW(t)\}$, search over parameters $\mathcal{P} = (\mu_\gamma, \mu_\nu, \sigma_{\gamma 1}, \sigma_{\gamma 2}, \sigma_{\nu 2})$ that minimize the distance between model-implied moments $\mathcal{M}(\mathcal{P}, \{dW(t)\})$ and $\mathcal{M}^{\text{data}}$

Having obtained a point estimate $\hat{\mathcal{P}} = (\hat{\mu}_\gamma, \hat{\mu}_\nu, \hat{\sigma}_{\gamma 1}, \hat{\sigma}_{\gamma 2}, \hat{\sigma}_{\nu 2})$, I use bootstrap to estimate standard errors. This procedure replaces true data with simulations created using $\hat{\mathcal{P}}$ and re-estimates these parameters many times. The steps are the following: for b from 1 to B,

- simulate a sequence $\{dW_t\}_b$
- using $\{dW_t\}_b$ and the point estimate $\hat{\mathcal{P}}$, compute $\mathcal{M}^{\text{“data”},b} = \mathcal{M}(\hat{\mathcal{P}}, \{dW_t\}_b)$
- get a point estimate $\hat{\mathcal{P}}_b$ by minimizing the distance between $\mathcal{M}(\mathcal{P}, \{dW(t)\})$ and $\mathcal{M}^{\text{“data”},b}$ over parameters $\mathcal{P}$

When this is completed B times, I have a sample of estimates $\{\hat{\mathcal{P}}_b\}_{b=1}^{B}$. I then use the standard deviations of estimates in this sample as standard errors for the original point estimates $\hat{\mathcal{P}}$. Figure A.9 shows point estimates and bootstrapped marginal distributions.

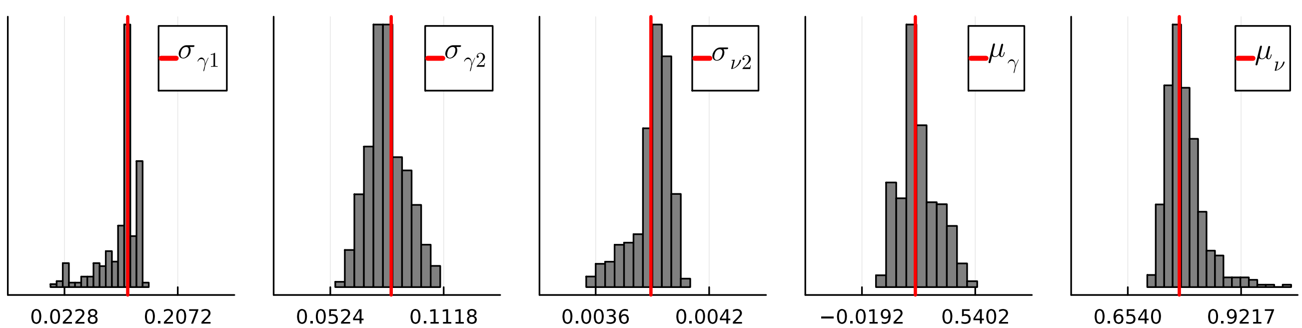

Figure A.9: Point estimates of parameters and the bootstrapped marginal distributions.

Quarterly magnitudes. To get a sense of magnitude for shocks to $\gamma(t)$ and $(\nu(t), \hat{\nu}(t))$ over discrete time periods, one can solve the stochastic differential equations:

$$\begin{aligned}\tilde{\gamma}(\tau) &= e^{-\mu_\gamma \tau}\tilde{\gamma}(0) + \int_0^\tau e^{-\mu_\gamma s}\sigma_{\gamma 1} dW_1(s) + \int_0^\tau e^{-\mu_\gamma s}\sigma_{\gamma 2} dW_2(s) \\ \tilde{\nu}(\tau) &= e^{-\mu_\nu \tau}\tilde{\nu}(0) + \int_0^\tau e^{-\mu_\nu s}\sigma_{\gamma 2} dW_2(s)\end{aligned} \tag{A.208}$$

Setting $\tau = 0.25$ yields a quarterly model and $\tau = 1$ makes it annual. Substituting the stochastic integrals random variables and defining parameters,

$$\tilde{\gamma}_{t+1} = \rho_\gamma \tilde{\gamma}_t + \varsigma_{\gamma 1}\varepsilon_{1,t+1} + \varsigma_{\gamma 2}\varepsilon_{2,t+1} \tag{A.209}$$

$$\tilde{\nu}_{t+1} = \rho_\nu \tilde{\nu}_t + \varsigma_{\nu 2}\varepsilon_{2,t+1} \tag{A.210}$$

Here the persistence parameters are $\rho_\gamma = e^{-\mu_\gamma \tau}$, $\rho_\nu = e^{-\mu_\nu \tau}$, random variables $(\varepsilon_{1,t+1}, \varepsilon_{2,t+1})$ are independent standard normals, and the volatilities of innovations are

$$\varsigma_{\gamma 1} = \sigma_{\gamma 1}\sqrt{\mathbb{V}\left[\int_0^\tau e^{-\mu_\gamma s} dW_1(s)\right]} = \sigma_{\gamma 1}\sqrt{\mathbb{E}\left[\int_0^\tau e^{-2\mu_\gamma s} ds\right]} = \sigma_{\gamma 1}\sqrt{\frac{1 - e^{-2\mu_\gamma \tau}}{2\mu_\gamma}} \tag{A.211}$$

Similarly,

$$\varsigma_{\gamma 2} = \sigma_{\gamma 2}\sqrt{\frac{1 - e^{-2\mu_\gamma \tau}}{2\mu_\gamma}} \tag{A.212}$$

$$\varsigma_{\nu 2} = \sigma_{\nu 2}\sqrt{\frac{1 - e^{-2\mu_\nu \tau}}{2\mu_\nu}} \tag{A.213}$$

It is more natural to report these parameters as a share of the steady-state values γ and ν. Table 19 reports them for the quarterly and annual frequencies.

Table 19: Finite time parameters.

	ρ_γ	ρ_ν	$\varsigma_{\gamma 1}$	$\varsigma_{\gamma 2}$	$\varsigma_{\nu 2}$
quarterly	0.941	0.824	$0.065 \cdot \gamma$	$-0.044 \cdot \gamma$	$0.022 \cdot \nu$
annual	0.783	0.460	$0.125 \cdot \gamma$	$0.084 \cdot \gamma$	$0.038 \cdot \nu$

The correlation between innovations to $\tilde{\gamma}_{t+1}$ and $\tilde{\nu}_{t+1}$ is 0.56 at all horizons.

I Numerical solution algorithm

Here I briefly describe the algorithm I use to solve the system of partial differential equations in Proposition 1. The system is for prices $p(w,t)$ and density $g(w,t)$:

$$r(t)p(w,t) - \partial_t p(w,t) = y(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{1}{2}\sigma_w(w,t)^2 \partial_{ww} p(w,t) \tag{A.214}$$

$$\partial_t g(w,t) = -\partial_w[\mu_w(w,t)g(w,t)] + \frac{1}{2}\partial_{ww}[\sigma_w(w,t)^2 p(w,t)] \tag{A.215}$$

Here the function $y(w,t)$ is the risk-adjusted payoff:

$$y(w,t) = \nu(t) - \left(\frac{\sigma}{1-\epsilon(w,t)\theta(w,t)}\right)^2 \max\left\{\frac{1}{w+\varphi(t)\eta(w)}, \frac{1}{\varphi(t)\eta(w)}\left(1 - \frac{\overline{\theta}w}{p(w,t)}\right)\right\} \tag{A.216}$$

with $\epsilon(w,t) = w/p(w,t) \cdot \partial_w p(w,t)$ being the wealth elasticity of price.

The partial differential equation (A.214) is non-linear. The price $p(w,t)$ is explicitly included in $y(w,t)$. Moreover, the drift and volatility of wealth $\mu_w(w,t)$ and $\sigma_w(w,t)$ depend on it. Plugging the optimal policy of investors,

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}\hat{w}(t) + \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \overline{\theta}\right\}\mu_R(w,t)w \tag{A.217}$$

$$\sigma_w(w,t) = \min\left\{\frac{\mu_R(w,t)}{\sigma_R(w,t)^2}, \overline{\theta}\right\}\sigma_R(w,t)w \tag{A.218}$$

Here the mean and volatility of returns are

$$\mu_R(w,t) = \frac{\mu_p(w,t) + \nu(t)}{p(w,t)} - r(t) \tag{A.219}$$

$$\sigma_R(w,t) = \frac{\sigma_p(w,t) + \sigma}{p(w,t)} \tag{A.220}$$

The drift and volatility of prices, $\mu_p(w,t)$ and $\sigma_p(w,t)$, in turn can be expressed in terms of $\mu_w(w,t)$ and $\sigma_w(w,t)$ using Itô's lemma:

$$\mu_p(w,t) = \partial_t p(w,t) + \mu_w(w,t)\partial_w p(w,t) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww} p(w,t) \tag{A.221}$$

$$\sigma_p(w,t) = \sigma_w(w,t)\partial_w p(w,t) \tag{A.222}$$

I next describe the iterative algorithm that I use to solve for $p(w,t)$ as a fixed point of equation (A.214). I first describe the steady state, where all functions just have w as their argument, and both equation (A.214) and equation (A.215) are ODE, not PDE. After that, I describe the transition dynamics.

My numerical solution requires a discretization of time and wealth spaces. In addition, I use a monotone transformation of the wealth scale from $[0, \infty)$ onto $[0, 1]$ by mapping w to another function $x(w) = 1 - e^{-w}$. This allows for better approximations at large levels of wealth, which is useful given that one of the boundary conditions for $p(w, t)$ is at infinity. All equations have to be corrected for this transformation using the chain rule and Itô's lemma.

Steady state. To compute the price functions in the steady state, I use a two-tier loop. In the outer loop, I solve for r, $\hat{w}$, and $G(\cdot)$, the distribution of w that produce all other quantities that clear markets. In the inner loop, I fix r, $\hat{w}$, and $\varphi = \gamma\hat{w}$ and solve for prices. Given these numbers, I iterate on the price functions in the following way:

- guess $p^{(n)}(w)$, $\mu_w^{(n)}(w)$, and $\sigma_w^{(n)}(w)$
- compute $y(w)$ and solve the time-invariant version of equation (A.214), which is on ODE instead of a PDE in the steady state, to get the new guess $p^{(n+1)}(w)$
- use the new guess $p^{(n+1)}(w)$ and old guesses $\mu_w^{(n)}(w)$ and $\sigma_w^{(n)}(w)$ in equation (A.221) and equation (A.222) get $\mu_p(w)$ and $\sigma_p(w)$
- use the new guess $p^{(n+1)}(w)$ and the newly computed $\mu_p(w)$ and $\sigma_p(w)$ to compute $\mu_R(w)$ and $\sigma_R(w)$, the mean and standard deviation of excess returns in equation (A.219) and equation (A.220)
- use the newly computed $\mu_R(w)$ and $\sigma_R(w)$ to compute the new guesses $\mu_w^{(n+1)}(w)$ and $\sigma_w^{(n+1)}(w)$ of the drift and volatility of wealth in equation (A.217) and equation (A.218)
- stop if old and new guesses of $p(w)$, $\mu_w(w)$, and $\sigma_w(w)$ are sufficiently close

In the outer loop, I use the last guesses for $\mu_w(w)$ and $\sigma_w(w)$ to solve equation (A.215). This allows me to compute the steady-state value of the average regular country wealth $\int w dG(w)$ and the total profits of the intermediary, which also takes in the last guess of $\mu_R(w)$. I then compute the steady-state value of $\hat{w}$ using the fact that wealth accumulation in the special country is zero: consumption $\hat{\rho}\hat{w}$ offsets profits, and net migration is zero. Given the new guess for $\hat{w}$, I compute the new guess for r using the intermediary's balance sheet:

$$\hat{w} = \frac{\nu}{r}\hat{q} + \int p(w)h(w)dG(w) - \int b(w)dG(w) \tag{A.223}$$

Here the ratio ν/r is the steady-state price of the safe asset.

Transition dynamics. I discretize the time and solve for sequences of $r(t)$ and $\hat{w}(t)$. Given guesses for these sequences, I also have a guess for the sequence $\varphi(t)$.

There is an inner loop at all nodes t of the time grid where I solve for the current price vector $p(w,t)$ and the vectors of wealth drift and volatilities $\mu_w(w,t)$ and $\sigma_w(w,t)$. This inner loop is exactly the same as in solving for the steady state price.

In the outer loop, I compute the flow profits of all investors, the evolution of wealth in all countries, and migration flows using $(\mu_p(w,t), \sigma_p(w,t), \mu_R(w,t), \sigma_R(w,t), \mu_w(w,t), \sigma_w(w,t))_{t\geq 0}$. This calculation leads to the new guess of the path of the special country's wealth $(\hat{w}(t))_{t\geq 0}$ and the global factor $(\varphi(t))_{t\geq 0}$. The new guess of the interest rate sequence $(r(t))_{t\geq 0}$ comes from differentiating the consumption goods market clearing condition with respect to time:

$$\rho \int \mu_w(w,t) dG(w,t) + \hat{\rho}\hat{w}'(t) = \nu'(t) + \hat{q}\hat{\nu}'(t) \tag{A.224}$$

The interest rate can be extracted from this equation given profit flows coming from expected excess returns $\mu_R(w,t)$. I then use the sequence space Jacobians to update the guesses.

J Details for impulse responses

The decomposition of the impact response of asset prices relies on the following fact. It is enough to know the future path of dividends $(\nu(t))_{t\geq 0}$, the interest rate $(r(t))_{t\geq 0}$, the global factor $(\varphi(t))_{t\geq 0}$, and the intermediary's wealth $(\hat{w}(t))_{t\geq 0}$ to calculate the whole path of $(p(w,t))_{t\geq 0}$. The intermediary's wealth only matters for migration flows, given the path of $(\varphi(t))_{t\geq 0}$.

Taking advantage of this, I produce decompositions on Figure 8 and the same decomposition for the shock to risk-taking capacity. First, for the shock to risk-taking capacity, I compute two counterfactual price sequences: one with $r(t)$ set at the steady-state r (this isolates the effect of $\varphi(t)$), and the other with $\varphi(t)$ set at the steady-state φ, isolates the effect of $r(t)$. In both cases, all other sequences are taken from the baseline general equilibrium transition dynamics.

For the output shock, I compute three counterfactual price sequences. One is with $r(t)$ and $\varphi(t)$ both held at the steady-state levels. This isolates the effect of $\nu(t)$ that directly enters the Kolmogorov backward equation for prices. Another holds $r(t)$ and $\nu(t)$ at the steady-state level, isolating the effect of $\varphi(t)$, and the last one isolates the effect of $r(t)$ by taking in constant (ν, φ).

These decompositions are not additive, since I consider relatively large shocks, and prices are highly non-linear in $r(t)$, $\varphi(t)$, and $\nu(t)$.

Expectations in cross-section. Panel (a) on Figure 7 provides cross-sections of expected holdings at three different points in time, $t = 0$, $t = 0.25$, and $t = 1$, conditional on w_0. The value of holdings at $t = 0$ conditional on w_0. Holdings as a function of (w, t) are known too. But wealth itself changes between $t = 0$ and $t = 0.25$, both due to aggregate drift and idiosyncratic shocks.

Consider any function $z(w,t)$. The time-s expectation at time t, denoted by $Z(w,t,s)$, is

$$Z(w,t,s) = \mathbb{E}\left[z(w_s, s)\middle| w_t = w\right] \tag{A.225}$$

This object satisfies the following HJB equation:

$$0 = \partial_t Z(w,t,s) + \mu_w(w,t)\partial_w Z(w,t,s) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww} Z(w,t,s) \tag{A.226}$$

The terminal condition is $Z(w,s,s) = z(w,s)$. I compute holdings of domestic assets τ ahead expected at $t = 0$ by solving this partial differential equation numerically and evaluating $Z(w, 0, \tau)$.

Another type of cross-section of expectations is an expected average over time. Panel (b) on Figure 7 shows the cross-section of expected wealth accumulation over the first quarter, decomposing it into parts. These expectations can be computed as follows. Take any time-varying function $\tilde{z}(w,t)$. The expected time average between t and s conditional on w_t, denoted by $\tilde{Z}(w,t,s)$, is

$$\tilde{Z}(w,t,s) = \mathbb{E}\left[\int_t^s \tilde{z}(w,\tau)d\tau \middle| w_t = w\right] \tag{A.227}$$

This object satisfies the following HJB equation:

$$0 = \tilde{z}(w,t) - \partial_t \tilde{Z}(w,t,s) + \mu_w(w,t)\partial_w \tilde{Z}(w,t,s) + \frac{\sigma_w(w,t)^2}{2}\partial_{ww}\tilde{Z}(w,t,s) \tag{A.228}$$

The terminal condition is $\tilde{Z}(w,s,s) = 0$. I evaluate $\tilde{Z}(w,0,\tau)$ for panel (b) on Figure 7.

Note that the expectation functions $Z(w,t,s)$ is essentially nested in $\tilde{Z}(w,t,s)$ if one is willing to consider the function $z(w,t)$ that incorporates Dirac's delta function:

$$\tilde{z}(w,t) = \delta(t-s)z(w,t) \tag{A.229}$$

Importantly, when plotting panel (a) and panel (b) on Figure 7, I account for initial wealth revaluation. Specifically, instead of $Z(w,0,\tau)$ and $\tilde{Z}(w,0,\tau)$, I plot $Z(W(w),0,\tau)$ and $\tilde{Z}(W(w),0,\tau)$, where the function $W(\cdot)$ maps wealth just before the shock hits into the level after revaluation.

Revaluation. Wealth revaluation in regular countries happens because asset prices jump on impact. The function $W(\cdot)$ solves the following functional equation:

$$W(w) = b(w) + p(W(w),0)h(w) \tag{A.230}$$

After revaluation, wealth consists of the old, steady-state level of bond holdings $b(w)$ and old holdings of risky assets $h(w)$ evaluated at the new price $p(W(w),0)$. The holdings have to be taken from just before the shock since revaluation happens before portfolios can be rebalanced. The absence of t as an argument in $h(w)$ and $l(w)$ means that these are steady-state functions. The price is evaluated at $W(w)$ since the country instantly becomes one with wealth $W(w)$ instead of w.

Wealth revaluation thus comes from two sources. First, price as a function of wealth changes relative to the steady state: $p(w,0) \neq p(w)$. Second, wealth itself changes because changing prices revalue it: $W(w) \neq w$. In my numerical procedure, I solve for $W(w)$ given $p(\cdot)$ as a function of (w,t) by iterating on guesses $W^n(\cdot)$:

$$W^{n+1}(w) = b(w) + p(W^n(w),0)h(w) \tag{A.231}$$

I evaluate the new guess $W^{n+1}(\cdot)$ on the grid by interpolation and do it until convergence.

K Additional details for shocks

This section provides additional details for the shock to risk-taking capacity of the intermediaries and the output shock.

Adjustment to the risk-taking capacity shock in the special country. The special country's net foreign assets position (NFA) falls on impact because of the losses it makes on its risky portfolio. Panel (a) on Figure A.10 shows the path of NFA over the special country's GDP over time after the shock to γ.

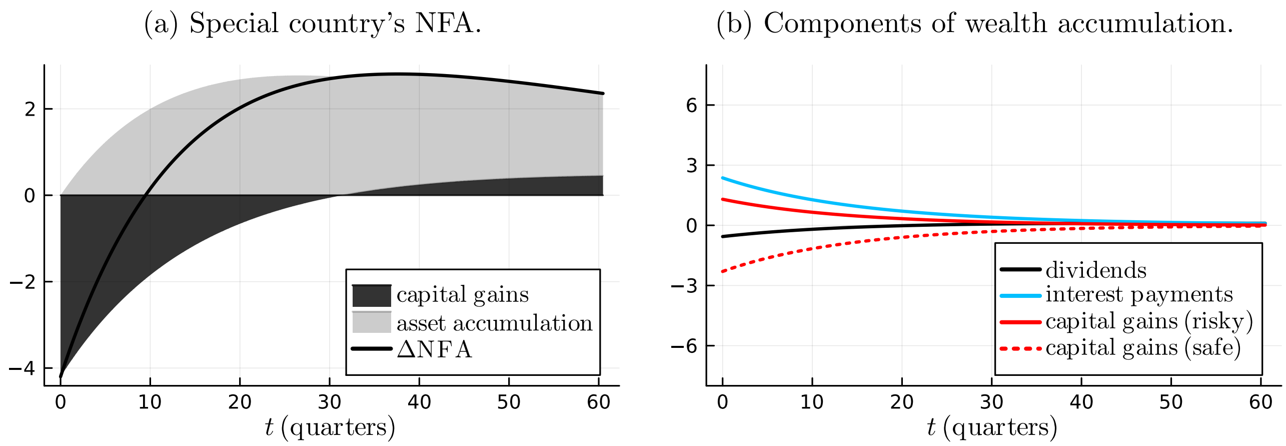

Figure A.10: Responses of the special country's NFA and components of net income, percent of GDP. Panel (a): change in NFA decomposed into price changes and net assets accumulation. Panel (b): changes in dividends from regular countries, interest payments to them, and asset prices.

The evolution after $t = 0$ is due to changes in asset prices and accumulation of assets net of incurrence of liabilities. The capital gains component $k(t)$ is given by a $k(0) < 0$ and

$$\dot{k}(t) = \int \mu_p(w,t)\hat{h}(w,t)dG(w,t) - \int \mu_p(w)\hat{h}(w)dG(w)$$

The subtracted term corresponds to the steady state. Capital gains start off negative at $t = 0$, reflecting the losses made on impact. NFA then reverses, and valuation changes contribute to it.

Asset accumulation accounts for purchases of shares in risky assets that do not result in additional deposits made by regular countries. Panel (b) sheds light on the sources of these purchases. It plots the changes in four components of the intermediary's wealth accumulation: dividends from assets in regular countries, interest payments (this component contributes negatively to net income), capital gains on risky assets, and capital gains on the safe asset.

Interest payments to regular countries decline by more than dividends flowing out of those countries. The intermediary uses this difference to buy back the shares that local investors in rich countries purchased on impact while retrenching. Changes in consumption are much smaller in magnitude, so almost all of this extra income goes to financing asset purchases. Dividends from the safe asset remain constant and are not shown in the picture. Capital gains on risky assets are positive, and those on the safe asset are negative as prices revert back to normal.

Shock to output. Figure A.11 and Figure A.12 show density-weighted losses made on assets as a percentage of global GDP. The distributions are remarkably similar. Losses on risky assets are shared between regular countries and the intermediary in roughly the same proportions after both shocks, and the fall in the safe asset price $\hat{p}(t)$ is only marginally larger (relative to other assets) when its dividends are hit.

This shows how important the global intermediary is for risk-sharing. Even though it is the only owner of the safe asset, the fall in the safe asset's dividends has largely the same consequences for the global wealth distribution as a shock to ν, adjusting for size. The intermediary's exposure to risky assets generates contagion. This force is the reverse side of insurance that it provides to other countries by absorbing a part of their losses.

Figure A.11: Shock to ν in regular countries: gains and losses on of the intermediary and local savers on impact (percent of global GDP, weighted by density)

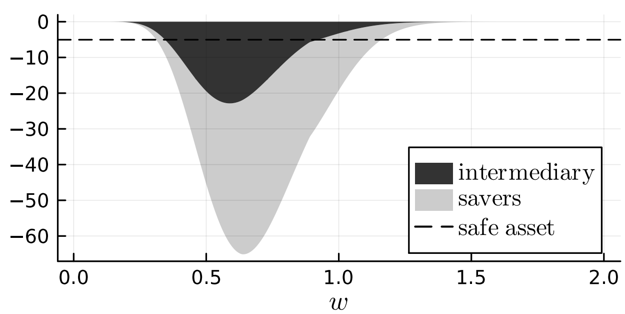

Figure A.12: Shock to ν in special country: gains and losses on of the intermediary and local savers on impact (percent of global GDP, weighted by density)

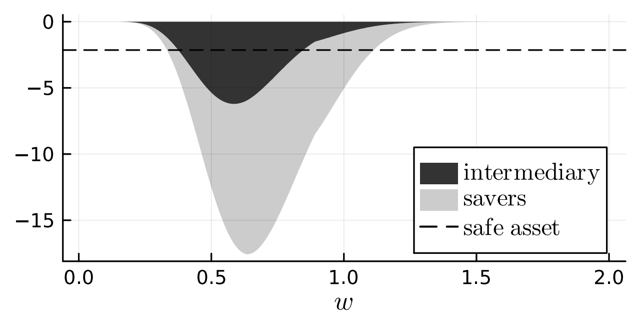

Table 20: Shock to ν in regular countries: gains on impact (percent of global GDP). Low w countries are those constrained in steady state, $\theta(w) = \overline{\theta}$

intermediary (safe asset)	−15.71%
intermediary (risky assets, low w)	−12.76%
intermediary (risky assets, high w)	−0.90%
savers in low w countries	−23.71%
savers in high w countries	−4.37%

Table 21: Shock to ν in special country: gains on impact (percent of global GDP). Low w countries are those constrained in steady state, $\theta(w) = \overline{\theta}$

intermediary (safe asset)	−5.33%
intermediary (risky assets, low w)	−3.85%
intermediary (risky assets, high w)	−0.27%
savers in low w countries	−7.14%
savers in high w countries	−1.30%

Table 22 collects the differences between the shocks to γ and ν. The shock to the global intermediary's risk-taking capacity is essential to generate large capital flows on impact, and these flows only happen in deep markets with rich and unconstrained domestic investors. The shock to output is essential to generate a fall in asset prices that is not confined to poor countries with a shallow investor base. The safe asset appreciates when the intermediary loses appetite for risk and depreciates when the interest rate rises to accommodate a fall in production.

Table 22: Summarized qualitative facts about negative shocks to γ and ν

	fall in $\gamma(t)$	fall in $\nu(t)$ or $\hat{\nu}(t)$
interest rate	-	+
safe asset	+	-
risky assets, rich countries	+	-
risky assets, poor countries	-	-
retrenchment flows, rich countries	+	0
retrenchment flows, poor countries	0	0

L Details for linearization

In this section, I explain the linearization procedure. The notation convention is that a function $z(w)$ with only one argument corresponds to the steady state. Functions with tildes, like $\tilde{z}(w,t)$, correspond to the first-order deviations. I also denote the intermediary's net worth $\hat{w}(t)$ by $n(t)$ to avoid excessive use of hats.

Step 1: constraint is slack. Take values of w for which the constraint is slack in the steady state. Start with the drift and volatility of the wealth:

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \mu_R(w,t)\theta(w,t)w \tag{A.232}$$

$$\sigma_w(w,t) = \sqrt{\sigma_R(w,t)}\theta(w,t)w \tag{A.233}$$

Using the optimal choice, $\theta(w,t) = \mu_R(w,t)/\sigma_R(w,t)$, replace the returns in the drift:

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \frac{\sigma_w(w,t)^2}{w} \tag{A.234}$$

This leads to

$$\tilde{\mu}_w(w,t) = w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) + \frac{2\sigma_w(w)}{w}\tilde{\sigma}_w(w,t) \tag{A.235}$$

Now using the definition $\sqrt{\sigma_R(w,t)} = (\sigma_p(w,t) + \sigma_y)/p(w,t)$ and $\sigma_p(w,t) = \partial_w p(w,t)\sigma_w(w,t)$,

$$\sigma_w(w,t) = \frac{\sigma_y\theta(w,t)w}{p(w,t) - \theta(w,t)\partial_w p(w,t)w} \tag{A.236}$$

Using market clearing $\mu_R(w,t)/\sigma_R(w,t) = p(w,t)/(\varphi(t)\eta(w) + w)$ and $\theta(w,t) = \mu_R(w,t)/\sigma_R(w,t)$,

$$\sigma_w(w,t) = \frac{\sigma_y w}{\varphi(t)\eta(w) + w - \partial_w p(w,t)w} \tag{A.237}$$

Expanding,

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y w}(w\partial_w\tilde{p} - \eta(w)\tilde{\varphi}(t)) \tag{A.238}$$

The market-clearing condition is

$$\mu_p(w,t) + \nu - r(t)p(w,t) = (\sigma_p(w,t) + \sigma_y)^2\frac{1}{\varphi(t)\eta(w) + w} \tag{A.239}$$

Expanding,

$$\tilde{\mu}_p - \tilde{r}(t)p(w) - r\tilde{p} = \tilde{\sigma}_p \frac{2(\sigma_p(w)+\sigma_y)}{\varphi(t)\eta(w)+w} - \tilde{\varphi}(t)\eta(w)\frac{(\sigma_p(w)+\sigma_y)^2}{(\varphi(t)\eta(w)+w)^2} \tag{A.240}$$

Now using $\sigma_w(w) = w\theta(w)(\sigma_p(w)+\sigma_y)/p(w) = w(\sigma_p(w)+\sigma_y)/(\varphi\eta(w)+w)$,

$$\tilde{\mu}_p - \tilde{\sigma}_p\frac{2\sigma_w(w)}{w} - r\tilde{p} = \tilde{r}(t)p(w) - \tilde{\varphi}(t)\eta(w)\frac{\sigma_w(w)^2}{w^2} \tag{A.241}$$

The expressions for $\tilde{\mu}_p$ and $\tilde{\sigma}_p$ are

$$\begin{aligned}
\tilde{\mu}_p &= \partial_t\tilde{p} + \mu_w(w)\partial_w\tilde{p} + p'(w)\tilde{\mu}_w + \tilde{\sigma}_w\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} && \text{(A.242)}\\
&= \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \mu_w\partial_w\tilde{p} + p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) + \frac{2\sigma_w(w)}{w}\tilde{\sigma}_w\right) + \tilde{\sigma}_w\sigma_w(w)p''(w)\\
\tilde{\sigma}_p &= \partial_w\tilde{p}\sigma_w(w) + p'(w)\tilde{\sigma}_w && \text{(A.243)}
\end{aligned}$$

Combining,

$$\begin{aligned}
\tilde{\mu}_p - \tilde{\sigma}_p\frac{2\sigma_w(w)}{w} &= p'(w)(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t)) + \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) - \frac{2\sigma_w(w)^2}{w}\right)\partial_w\tilde{p}\\
&\quad + \tilde{\sigma}_w\sigma_w(w)p''(w)\\
&= p'(w)(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t)) - \tilde{\varphi}(t)p''(w)\frac{\sigma_w(w)^3\eta(w)}{\sigma_y w}\\
&\quad + \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) - \frac{2\sigma_w(w)^2}{w} + p''(w)\frac{\sigma_w(w)^3}{\sigma_y}\right)\partial_w\tilde{p} && \text{(A.244)}
\end{aligned}$$

Plugging this into equation (A.241),

$$\begin{aligned}
\frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) - \frac{2\sigma_w(w)^2}{w} + p''(w)\frac{\sigma_w(w)^3}{\sigma_y}\right)\partial_w\tilde{p} - r\tilde{p} &= \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t)\\
+\left(p''(w)\frac{\sigma_w(w)^3\eta(w)}{\sigma_y w} - \eta(w)\frac{\sigma_w(w)^2}{w^2}\right)\tilde{\varphi}(t) - \partial_t\tilde{p} & && \text{(A.245)}
\end{aligned}$$

Step 2: the constraint binds. Take values of w for which the constraint binds in the steady state. The drift and volatility of the wealth are

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \mu_R(w,t)\overline{\theta}w \tag{A.246}$$

$$\sigma_w(w,t) = \overline{\theta}\sqrt{\sigma_R(w,t)}w \tag{A.247}$$

Using market clearing $\mu_R(w,t)/\sigma_R(w,t) = (p(w,t) - \overline{\theta}w)/(\varphi(t)\eta(w))$ and the definition of $\sigma_R(w,t)$,

$$\mu_w(w,t) = (r(t) - \rho - \lambda)w + \hat{\lambda}n(t) + \frac{\sigma_w(w,t)^2}{\varphi(t)\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right) \tag{A.248}$$

$$\sigma_w(w,t) = \frac{\overline{\theta}w\sigma_y}{p(w,t) - \overline{\theta}w\partial_w p(w,t)} \tag{A.249}$$

Expanding,

$$\begin{aligned}\tilde{\mu}_w &= \tilde{r}(t)w + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t) \\ &\quad + \frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\sigma}_w + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p} \end{aligned} \tag{A.250}$$

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y}\partial_w\tilde{p} - \frac{\sigma_w(w)^2}{\overline{\theta}w\sigma_y}\tilde{p} \tag{A.251}$$

The expressions for expanded drift and volatility of the price process are

$$\begin{aligned}\tilde{\mu}_p &= \partial_t\tilde{p} + \mu_w(w)\partial_w\tilde{p} + p'(w)\tilde{\mu}_w + \tilde{\sigma}_w\sigma_w(w)p''(w) + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} \\ &= \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \mu_w\partial_w\tilde{p} + p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t)\right) \\ &\quad + p'(w)\left(\frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\sigma}_w + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p}\right) + p''(w)\sigma_w(w)\tilde{\sigma}_w \end{aligned} \tag{A.252}$$

$$\tilde{\sigma}_p = \sigma_w(w)\partial_w\tilde{p} + p'(w)\tilde{\sigma}_w \tag{A.253}$$

The market clearing condition is

$$\mu_p(w,t) + y - r(t)p(w,t) = (\sigma_p(w,t) + \sigma_y)^2\frac{1}{\varphi(t)\eta(w)}\left(1 - \frac{\overline{\theta}w}{p(w,t)}\right) \tag{A.254}$$

Expanding,

$$\begin{aligned}\tilde{\mu}_p - p(w)\tilde{r}(t) - r\tilde{p} &= 2\tilde{\sigma}_p\frac{\sigma_p(w) + \sigma_y}{\varphi\eta(w)}\left(1 - \frac{\overline{\theta}w}{p(w)}\right) - \frac{(\sigma_p(w) + \sigma_y)^2}{\varphi^2\eta(w)}\left(1 - \frac{\overline{\theta}w}{p(w)}\right)\tilde{\varphi}(t) \\ &\quad + \frac{(\sigma_p(w) + \sigma_y)^2}{\varphi\eta(w)}\frac{\overline{\theta}w}{p(w)^2}\tilde{p} \end{aligned} \tag{A.255}$$

Now using $\sigma_p(w) + \sigma_y = \sigma_w(w)p(w)/(\overline{\theta}w)$,

$$\begin{aligned}\tilde{\mu}_p - p(w)\tilde{r}(t) - r\tilde{p} = \tilde{\sigma}_p \frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right) - \frac{\sigma_w(w)^2 p(w)}{\varphi^2\overline{\theta}w(w)w}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t) \\ + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p}\end{aligned} \tag{A.256}$$

Now compute

$$\begin{aligned}\tilde{\mu}_p - \tilde{\sigma}_p\frac{2\sigma_w(w)}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right) &= \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \mu_w\partial_w\tilde{p} \\ &+ p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t)\right) \\ &- \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\partial_w\tilde{p} + p'(w)\frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\tilde{p} + p''(w)\sigma_w(w)\tilde{\sigma}_w \\ &= \partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} \\ &+ \left(\mu_w - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y}\right)\partial_w\tilde{p} \\ &+ p'(w)\left(w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t)\right) \\ &+ \left(p'(w)\frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w} - p''(w)\frac{\sigma_w(w)^3}{\overline{\theta}w\sigma_y}\right)\tilde{p}\end{aligned} \tag{A.257}$$

Plugging,

$$\begin{aligned}\partial_t\tilde{p} + \partial_{ww}\tilde{p}\frac{\sigma_w(w)^2}{2} + \partial_w\tilde{p}\left(\mu_w(w) + \frac{\sigma_w(w)^3}{\sigma_y}p''(w) - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\right) \\ + \tilde{p}\left(\frac{\sigma_w(w)^2(p'(w) - 1)}{\varphi\overline{\theta}\eta(w)w} - \frac{\sigma_w(w)^3 p''(w)}{\overline{\theta}w\sigma_y} - r\right) \\ = (p(w) - p'(w)w)\tilde{r}(t) - p'(w)\hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\left(\frac{p(w)}{\overline{\theta}w} - p'(w)\right)\tilde{\varphi}(t)\end{aligned} \tag{A.258}$$

Finally, acknowledging that $p(w)/(\overline{\theta}w) - p'(w) = \sigma_y/\sigma_w(w)$,

$$\begin{aligned}\partial_t\tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y} - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\right)\partial_w\tilde{p} \\ + \tilde{p}\left(\frac{\sigma_w(w)^2(p'(w) - 1)}{\varphi\overline{\theta}\eta(w)w} - \frac{\sigma_w(w)^3 p''(w)}{\overline{\theta}w\sigma_y} - r\right) \\ = \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)\sigma_y}{\varphi^2\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t)\end{aligned} \tag{A.259}$$

Step 3: putting it together. Now rewrite the equations together. When the constraint is slack,

$$\partial_t \tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y} - \frac{2\sigma_w(w)^2}{w}\right)\partial_w \tilde{p} - r\tilde{p} \\ = \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t) + \left(p''(w)\frac{\sigma_w(w)^3\eta(w)}{\sigma_y w} - \frac{\sigma_w(w)^2\eta(w)}{w^2}\right)\tilde{\varphi}(t) \tag{A.260}$$

When it binds:

$$\partial_t \tilde{p} + \frac{\sigma_w(w)^2}{2}\partial_{ww}\tilde{p} + \left(\mu_w(w) + p''(w)\frac{\sigma_w(w)^3}{\sigma_y} - \frac{2\sigma_w(w)^2}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\right)\partial_w \tilde{p} \\ + \tilde{p}\left(\frac{\sigma_w(w)^2(p'(w)-1)}{\varphi\overline{\theta}\eta(w)w} - \frac{\sigma_w(w)^3 p''(w)}{\overline{\theta}w\sigma_y} - r\right) \\ = \tilde{r}(t)(p(w) - p'(w)w) - p'(w)\hat{\lambda}\tilde{n}(t) - \frac{\sigma_w(w)\sigma_y}{\varphi^2\eta(w)}\left(\frac{p(w)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t) \tag{A.261}$$

At the boundary $\overline{w}$,

$$\lim_{w\to\overline{w}+0}\frac{1}{\varphi\eta(w)}\left(\frac{p(w)}{\overline{\theta}} - w\right) = 1 \tag{A.262}$$

The coefficient on $\partial_w \tilde{p}$ is continuous. Subtracting the $\overline{w}+$ and $\overline{w}-$ limits of A.260 and A.261,

$$\frac{\sigma_w(\overline{w})^2}{2}\left(\partial_{ww}\tilde{p}(\overline{w}+) - \partial_{ww}\tilde{p}(\overline{w}-)\right) = \left[p''(\overline{w})\frac{\sigma_w(\overline{w})^3\eta(\overline{w})}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2\eta(\overline{w})}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi^2\eta(\overline{w})}\left(\frac{p(\overline{w})}{\overline{\theta}\overline{w}} - 1\right)\right]\tilde{\varphi}(t) \\ + \left[\frac{\sigma_w(\overline{w})^2(p'(\overline{w})-1)}{\varphi\overline{\theta}\eta(\overline{w})\overline{w}} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}\overline{w}\sigma_y}\right]\tilde{p} \tag{A.263}$$

Now using the fact that $p'(\overline{w}) = p(\overline{w})/(\overline{\theta}\overline{w}) - \sigma_y/\sigma_w(\overline{w})$, rewrite the coefficient on $\tilde{p}$:

$$\frac{\sigma_w(\overline{w})^2(p'(\overline{w})-1)}{\varphi\overline{\theta}\eta(\overline{w})\overline{w}} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}w\sigma_y} = \frac{\sigma_w(\overline{w})^2}{\overline{\theta}\overline{w}\varphi\eta(\overline{w})}\left(\frac{p(\overline{w})}{\overline{\theta}\overline{w}} - 1\right) - \frac{\sigma_w(\overline{w})\sigma_y}{\overline{\theta}\overline{w}\varphi\eta(\overline{w})} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}\overline{w}\sigma_y} \\ = \frac{\sigma_w(\overline{w})^2}{\overline{\theta}\overline{w}} - \frac{\sigma_w(\overline{w})\sigma_y}{\overline{\theta}\overline{w}\varphi\eta(\overline{w})} - \frac{\sigma_w(\overline{w})^3 p''(\overline{w})}{\overline{\theta}\overline{w}\sigma_y} \equiv -\frac{1}{\theta}C \tag{A.264}$$

Rewriting the coefficient on $\tilde{\varphi}(t)$,

$$\eta(\overline{w})\left[\frac{p''(\overline{w})\sigma_w(\overline{w})^3}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi^2\eta(\overline{w})^2}\left(\frac{p(\overline{w})}{\overline{\theta}\overline{w}} - 1\right)\right] = \eta(\overline{w})\left[\frac{p''(\overline{w})\sigma_w(\overline{w})^3}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi\eta(\overline{w})}\right] \\ = \eta(\overline{w})C \tag{A.265}$$

Plugging this into the boundary condition equation (A.263),

$$\frac{\sigma_w(\overline{w})^2}{2}\left(\partial_{ww}\tilde{p}(\overline{w}+) - \partial_{ww}\tilde{p}(\overline{w}-)\right.$$
$$= \left(\eta(\overline{w})\tilde{\varphi}(t) - \frac{\tilde{p}(\overline{w})}{\overline{\theta}}\right)\left[\frac{p''(\overline{w})\sigma_w(\overline{w})^3}{\sigma_y\overline{w}} - \frac{\sigma_w(\overline{w})^2}{\overline{w}^2} + \frac{\sigma_w(\overline{w})\sigma_y}{\varphi\eta(\overline{w})}\right] \tag{A.266}$$

Step 4: distribution. Consider the KFE for $g(w,t)$:

$$\partial_t g(w,t) = -\partial_w(\mu_w(w,t)g(w,t)) + \frac{1}{2}\partial_{ww}(\sigma_w(w,t)^2 g(w,t)) \tag{A.267}$$

Expanding,

$$\partial_t\tilde{g} = -\partial_w(\mu_w(w)\tilde{g}) + \frac{1}{2}\partial_{ww}(\sigma_w(w)^2\tilde{g}) - \partial_w(\tilde{\mu}_w(w)g(w)) + \partial_{ww}(\tilde{\sigma}_w\sigma_w(w)g(w)) \tag{A.268}$$

Having computed $\tilde{p}$, one can plug $\tilde{\mu}_w$ and $\tilde{\sigma}_w$. When the constraint is slack,

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y}\partial_w\tilde{p} - \frac{\sigma_w(w)^2\eta(w)}{\sigma_y w}\tilde{\varphi}(t) \tag{A.269}$$
$$\tilde{\mu}_w = \frac{2\sigma_w(w)^3}{\sigma_y w}\partial_w\tilde{p} - \frac{2\sigma_w(w)^3\eta(w)}{\sigma_y w^2}\tilde{\varphi}(t) + w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t) \tag{A.270}$$

When it binds,

$$\tilde{\sigma}_w = \frac{\sigma_w(w)^2}{\sigma_y}\partial_w\tilde{p} - \frac{\sigma_w(w)^2}{\overline{\theta}w\sigma_y}\tilde{p} \tag{A.271}$$
$$\begin{aligned}\tilde{\mu}_w = {} & \frac{2\sigma_w(w)^3}{\sigma_y\varphi\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\partial}_w\tilde{p} - \frac{\sigma_w(w)^2}{\varphi^2\eta(w)}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\tilde{\varphi}(t) + w\tilde{r}(t) + \hat{\lambda}\tilde{n}(t)\\ & + \frac{\sigma_w(w)^2}{\varphi\overline{\theta}\eta(w)w}\left[1 - \frac{2\sigma_w(w)}{\sigma_y}\left(\frac{p(w,t)}{\overline{\theta}w} - 1\right)\right]\tilde{p}\end{aligned} \tag{A.272}$$

Aggregate regular country and intermediary wealth deviations are

$$\tilde{w}(t) = \int \tilde{g}w\,dw \tag{A.273}$$
$$\tilde{n}(t) = -\frac{\rho}{\hat{\rho}}\tilde{w}(t) \tag{A.274}$$

The US tree price satisfies

$$\hat{p}(t)q = n(t) + w(t) - \int p(w,t)g(w,t)dt \tag{A.275}$$

Expanding,

$$\tilde{\hat{p}}q = \tilde{n}(t)\left(1 - \frac{\rho}{\hat{\rho}}\right) - \int \tilde{p}g(w)dw - \int \tilde{g}p(w)dw \tag{A.276}$$

The deviations in the interest rate and the global factor satisfy

$$\tilde{r}(t) = \frac{\partial_t \tilde{\hat{p}}}{\hat{p}} - \frac{r\tilde{\hat{p}}}{\hat{p}} \tag{A.277}$$

$$\tilde{\varphi}(t) = \gamma\tilde{n}(t) + n\tilde{\gamma}(t) \tag{A.278}$$

These close the linearized model.

Step 5: numerical procedure. The discrete approximation of the PDE for $\tilde{p}$ can be written as

$$\left(A_p - \frac{1}{dt}\right)\mathbf{p}(t) = J_{rhs,r}\,\mathbf{r}(t) + J_{rhs,n}\,\mathbf{n}(t) + J_{rhs,\varphi}\,\boldsymbol{\varphi}(t) - \frac{1}{dt}\,\mathbf{p}(t+1) \tag{A.279}$$

Denoting $M_1 = (A_p - 1/dt)^{-1}$, $M_2 = 1/dt$, and $M_3 = -M_1M_2$,

$$\begin{aligned}
\mathbf{p}(t) &= M_1 J_{rhs,r}\,\mathbf{r}(t) + M_1 J_{rhs,n}\,\mathbf{n}(t) + M_1 J_{rhs,\varphi}\,\boldsymbol{\varphi}(t) + M_3\,\mathbf{p}(t+1) \\
&= \sum_{s=0}^{T-t}(M_3)^s M_1 J_{rhs,r}\,\mathbf{r}(t+s) + \sum_{s=0}^{T-t}(M_3)^s M_1 J_{rhs,n}\,\mathbf{n}(t+s) + \sum_{s=0}^{T-t}(M_3)^s M_1 J_{rhs,\varphi}\,\boldsymbol{\varphi}(t+s) \\
&= \sum_{s=0}^{T-t} J_{p,r}(s)\,\mathbf{r}(t+s) + \sum_{s=0}^{T-t} J_{p,n}(s)\,\mathbf{n}(t+s) + \sum_{s=0}^{T-t} J_{p,\varphi}(s)\,\boldsymbol{\varphi}(t+s)
\end{aligned} \tag{A.280}$$

Here M_1, M_2, and M_3 have to be corrected to incorporate the boundary condition at $\overline{w}$.

The discrete approximation of the KFE for $\tilde{g}$ is

$$\left(A_g + \frac{1}{dt}\right)\mathbf{g}(t+1) = -A_1\,\boldsymbol{\mu}_w(t) + A_2\,\boldsymbol{\sigma}_w(t) + \frac{1}{dt}\,\mathbf{g}(t) \tag{A.281}$$

Here the matrix A_1 discretizes the operator $\partial_w \cdot g(w)\cdot$, and A_2 discretizes $\partial_{ww} \cdot \sigma_w(w) \cdot g(w)\cdot$.

Denoting $M_4 = (A_g + 1/dt)^{-1}$, $M_5 = -M_4A_1$, $M_6 = M_4A_2$, and $M_g = M_4M_2$,

$$\begin{aligned}
\mathbf{g}(t+1) &= M_5\,\boldsymbol{\mu}_w(t) + M_6\,\boldsymbol{\sigma}_w(t) + M_g\,\mathbf{g}(t) \\
&= \sum_{s=0}^{t-1}(M_g)^s M_5\,\boldsymbol{\mu}_w(t-s) + \sum_{s=0}^{t-1}(M_g)^s M_6\,\boldsymbol{\sigma}_w(t-s) + (M_g)^{t-1}\,\mathbf{g}(0) \\
&= \sum_{s=0}^{t-1} J_{g,\mu}(s)\,\boldsymbol{\mu}_w(t-s) + \sum_{s=0}^{t-1} J_{g,\sigma}(s)\,\boldsymbol{\sigma}_w(t-s) + (M_g)^{t-1}\,\mathbf{g}(0)
\end{aligned} \tag{A.282}$$

The drift deviations can be written as

$$
\begin{aligned}
\boldsymbol{\mu}_w(t) &= J_{\mu,p}\,\mathbf{p}(t) + J^{direct}_{\mu,r}\,\mathbf{r}(t) + J^{direct}_{\mu,n}\,\mathbf{n}(t) + J^{direct}_{\mu,\varphi}\,\boldsymbol{\varphi}(t) = \left(\sum_{s=0}^{T-t} J_{p,r}(s) + J^{direct}_{\mu,r}\delta_{s,0}\right)\mathbf{r}(t+s) \\
&+ \left(\sum_{s=0}^{T-t} J_{p,n}(s) + J^{direct}_{\mu,n}\delta_{s,0}\right)\mathbf{n}(t+s) + \left(\sum_{s=0}^{T-t} J_{p,\varphi}(s) + J^{direct}_{\mu,\varphi}\delta_{s,0}\right)\boldsymbol{\varphi}(t+s) \\
&= \sum_{s=0}^{T-t} J_{\mu,r}(s)\,\mathbf{r}(t+s) + \sum_{s=0}^{T-t} J_{\mu,n}(s)\,\mathbf{n}(t+s) + \sum_{s=0}^{T-t} J_{\mu,\varphi}(s)\,\boldsymbol{\varphi}(t+s)
\end{aligned}
\tag{A.283}
$$

The volatility deviations are

$$
\begin{aligned}
\boldsymbol{\sigma}_w(t) &= J_{\sigma,p}\,\mathbf{p}(t) + J^{direct}_{\sigma,r}\,\mathbf{r}(t) + J^{direct}_{\sigma,n}\,\mathbf{n}(t) + J^{direct}_{\sigma,\varphi}\,\boldsymbol{\varphi}(t) = \left(\sum_{s=0}^{T-t} J_{p,r}(s) + J^{direct}_{\sigma,r}\delta_{s,0}\right)\mathbf{r}(t+s) \\
&+ \left(\sum_{s=0}^{T-t} J_{p,n}(s) + J^{direct}_{\sigma,n}\delta_{s,0}\right)\mathbf{n}(t+s) + \left(\sum_{s=0}^{T-t} J_{p,\varphi}(s) + J^{direct}_{\sigma,\varphi}\delta_{s,0}\right)\boldsymbol{\varphi}(t+s) \\
&= \sum_{s=0}^{T-t} J_{\sigma,r}(s)\,\mathbf{r}(t+s) + \sum_{s=0}^{T-t} J_{\sigma,n}(s)\,\mathbf{n}(t+s) + \sum_{s=0}^{T-t} J_{\sigma,\varphi}(s)\,\boldsymbol{\varphi}(t+s)
\end{aligned}
\tag{A.284}
$$

Plugging,

$$
\begin{aligned}
\mathbf{g}(t+1) &= \sum_{s=0}^{t-1} J_{g,\mu}(s) \sum_{u=0}^{T-t+s} J_{\mu,r}(u)\,\mathbf{r}(t-s+u) + \sum_{s=0}^{t-1} J_{g,\mu}(s) \sum_{u=0}^{T-t+s} J_{\mu,n}(u)\,\mathbf{n}(t-s+u) \qquad \text{(A.285)} \\
&+ \sum_{s=0}^{t-1} J_{g,\mu}(s) \sum_{u=0}^{T-t+s} J_{\mu,\varphi}(u)\,\boldsymbol{\varphi}(t-s+u) + \sum_{s=0}^{t-1} J_{g,\sigma}(s) \sum_{u=0}^{T-t+s} J_{\sigma,r}(u)\,\mathbf{r}(t-s+u) \\
&+ \sum_{s=0}^{t-1} J_{g,\sigma}(s) \sum_{u=0}^{T-t+s} J_{\sigma,n}(u)\,\mathbf{n}(t-s+u) + \sum_{s=0}^{t-1} J_{g,\sigma}(s) \sum_{u=0}^{T-t+s} J_{\sigma,\varphi}(u)\,\boldsymbol{\varphi}(t-s+u) \\
&+ (M_g)^{t-1}\,\mathbf{g}(0)
\end{aligned}
$$

Changing the order of summation,

$$
\begin{aligned}
\mathbf{g}(t+1) &= \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,r}(u+s-t)\,\mathbf{r}(s) + \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,n}(u+s-t)\,\mathbf{n}(s) \qquad \text{(A.286)} \\
&+ \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,\varphi}(u+s-t)\,\boldsymbol{\varphi}(s) + \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,r}(u+s-t)\,\mathbf{r}(s) \\
&+ \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,n}(u+s-t)\,\mathbf{n}(s) + \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,\varphi}(u+s-t)\,\boldsymbol{\varphi}(s) + (M_g)^{t-1}\,\mathbf{g}(0)
\end{aligned}
$$

Initial revaluation $\mathbf{g}(0)$ comes from the jump in prices: $\mathbf{g}(0) = A^{reval}\,\mathbf{p}(0)$, so $\mathbf{g}(t+1)$ is

$$\begin{aligned}\mathbf{g}(t+1) = &\sum_{s=1}^{T} J^{accum}_{g,\mu,r}(t,s)\,\mathbf{r}(s) + \sum_{s=1}^{T} J^{accum}_{g,\mu,n}(t,s)\,\mathbf{n}(s) + \sum_{s=1}^{T} J^{accum}_{g,\mu,\varphi}(t,s)\,\boldsymbol{\varphi}(s) \\ &+ \sum_{s=1}^{T} J^{accum}_{g,\sigma,r}(t,s)\,\mathbf{r}(s) + \sum_{s=1}^{T} J^{accum}_{g,\sigma,n}(t,s)\,\mathbf{n}(s) + \sum_{s=1}^{T} J^{accum}_{g,\sigma,\varphi}(t,s)\,\boldsymbol{\varphi}(s) \\ &+ \sum_{s=1}^{T} J^{reval}_{g,r}(t,s)\,\mathbf{r}(s) + \sum_{s=1}^{T} J^{reval}_{g,n}(t,s)\,\mathbf{n}(s) + \sum_{s=1}^{T} J^{reval}_{g,\varphi}(t,s)\,\boldsymbol{\varphi}(s) \end{aligned} \tag{A.287}$$

Here the accumulation Jacobians are

$$J^{accum}_{g,\mu,r}(t,s) = \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,r}(u+s-t) = J^{accum}_{g,\mu,r}(t+1,s+1) - J_{g,\mu}(t) J_{\mu,r}(s) \tag{A.288}$$

$$J^{accum}_{g,\mu,n}(t,s) = \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,n}(u+s-t) = J^{accum}_{g,\mu,n}(t+1,s+1) - J_{g,\mu}(t) J_{\mu,n}(s) \tag{A.289}$$

$$J^{accum}_{g,\mu,\varphi}(t,s) = \sum_{u=0}^{t-1} J_{g,\mu}(u) J_{\mu,\varphi}(u+s-t) = J^{accum}_{g,\mu,\varphi}(t+1,s+1) - J_{g,\mu}(t) J_{\mu,\varphi}(s) \tag{A.290}$$

$$J^{accum}_{g,\sigma,r}(t,s) = \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,r}(u+s-t) = J^{accum}_{g,\sigma,r}(t+1,s+1) - J_{g,\sigma}(t) J_{\sigma,r}(s) \tag{A.291}$$

$$J^{accum}_{g,\sigma,n}(t,s) = \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,n}(u+s-t) = J^{accum}_{g,\sigma,n}(t+1,s+1) - J_{g,\sigma}(t) J_{\sigma,n}(s) \tag{A.292}$$

$$J^{accum}_{g,\sigma,\varphi}(t,s) = \sum_{u=0}^{t-1} J_{g,\sigma}(u) J_{\sigma,\varphi}(u+s-t) = J^{accum}_{g,\sigma,\varphi}(t+1,s+1) - J_{g,\sigma}(t) J_{\sigma,\varphi}(s) \tag{A.293}$$

The revaluation Jacobians are

$$J^{reval}_{g,r}(t,s) = (M_g)^{t-1} A^{reval} J_{p,r}(s) \tag{A.294}$$

$$J^{reval}_{g,n}(t,s) = (M_g)^{t-1} A^{reval} J_{p,n}(s) \tag{A.295}$$

$$J^{reval}_{g,\varphi}(t,s) = (M_g)^{t-1} A^{reval} J_{p,\varphi}(s) \tag{A.296}$$

The accumulation Jacobians can be computed recursively. This recursive procedure is the analog of the fake-news algorithm. The initial conditions are $J^{accum}_{g,\mu,r}(t,1) = J_{g,\mu}(t-1)J_{\mu,r}(0)$ and $J^{accum}_{g,\mu,r}(1,s) = J_{g,\mu}(0)J_{\mu,r}(s-1)$ for $\mathbf{r}$ and analogously for $\mathbf{n}$ and $\boldsymbol{\varphi}$. The initial conditions for $\boldsymbol{\sigma}$ parts of the Jacobian are identical.

With these Jacobians at hand, it is possible to compute the Jacobian of any moment of the distribution and any function of w. Specifically, aggregate wealth of the regular countries is

$$\mathbf{w}(t) = \sum_{s=1}^{T} W' J_{g,r}^{total}(s,t)\, \mathbf{r}(s) + \sum_{s=1}^{T} W' J_{g,n}^{total}(s,t)\, \mathbf{n}(s) + \sum_{s=1}^{T} W' J_{g,\varphi}^{total}(s,t)\, \boldsymbol{\varphi}(s) \tag{A.297}$$

The deviation of the US tree price is

$$\begin{aligned}
\hat{\mathbf{p}}(t) &= \mathbf{w}(t)\frac{1}{q}\left(1 - \frac{\hat{\rho}}{\rho}\right) - \frac{1}{q}P'\,\mathbf{g}(t) - \frac{1}{q}G'\,\mathbf{p}(t) \\
&= \frac{1}{q}\sum_{s=1}^{T}\left[\left(\left(1 - \frac{\hat{\rho}}{\rho}\right)W' - P'\right) J_{g,r}^{total}(s,t) - G' J_{p,r}(s-t)\right]\mathbf{r}(s) \\
&+ \frac{1}{q}\sum_{s=1}^{T}\left[\left(\left(1 - \frac{\hat{\rho}}{\rho}\right)W' - P'\right) J_{g,n}^{total}(s,t) - G' J_{p,n}(s-t)\right]\mathbf{n}(s) \\
&+ \frac{1}{q}\sum_{s=1}^{T}\left[\left(\left(1 - \frac{\hat{\rho}}{\rho}\right)W' - P'\right) J_{g,\varphi}^{total}(s,t) - G' J_{p,\varphi}(s-t)\right]\boldsymbol{\varphi}(s)
\end{aligned} \tag{A.298}$$

Here G is the vectorized steady-state density, P is the vectorized steady-state price, and W is the wealth grid. The convention is that $J_{p,r}(t-s) = 0$ whenever $s > t$, and the same for n and φ.

Shocks to output. All equations are very similar when there are shocks to output $\tilde{\nu}(t)$ and $\tilde{\tilde{\nu}}(t)$ instead of risk-taking capacity $\tilde{\gamma}(t)$. Indeed, $\tilde{\gamma}(t)$ only appears explicitly at the very end in $\tilde{\varphi}(t) = \gamma\tilde{n}(t) + n\tilde{\gamma}(t)$. In case of output shocks, deviations $\tilde{\varphi}(t)$ and $\tilde{n}(t)$ still enter all equations separately but the relationship is simpler, $\tilde{\varphi}(t) = \gamma\tilde{n}(t)$.

One substantial difference is that $\tilde{\nu}(t)$ directly enters the Kolmogorov backward equation for risky asset prices, while $\tilde{\tilde{\nu}}(t)$ enters the pricing equation for the safe asset. It is straightforward to add these deviations to all equations.